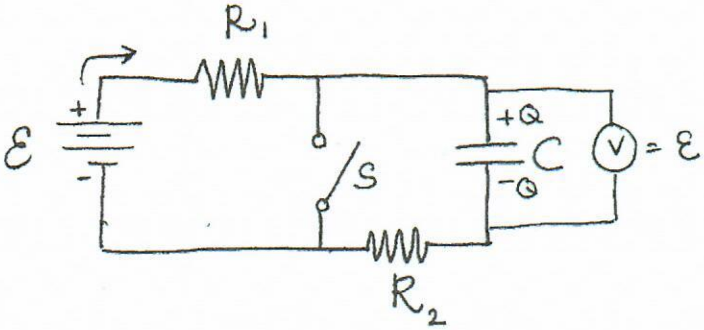
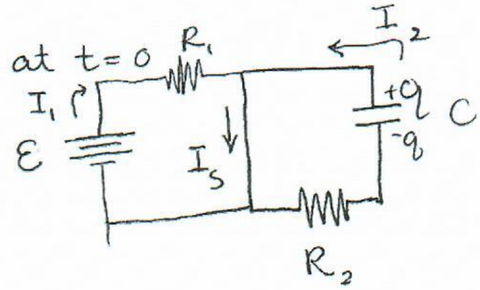
4. In the circuit shown, the switch has been open for a long time. It is then suddenly closed at t = 0. Determine the current in the switch as a function of time.





Junction Rule:

$I_s = I_1 + I_2$

Loop Rule:

1) $\mathcal{E} - I_1 R_1 = 0$ not necessary!

2) $\mathcal{E} - I_1 R_1 + \frac{Q}{C} - I_2 R_2 = 0$

3) $\frac{Q}{C} - I_2 R_2 = 0$

Finding Q (t):

$\frac{Q}{C} = I_2 R_2$

$$\frac{Q_0 - q_{left}}{R_2 C} = \frac{dq_{left}}{dt}$$

in order to get e^{-t/R_2C} form

$$\left(\frac{Q_0 - q_{left}}{R_2 C} = \frac{dq_{left}}{dt}\right)(-1)$$

$$\int_0^t \frac{-dt}{R_2 C} = \int_0^{Q(t)} \frac{dq_{left}}{q_{left} - Q_0}$$

charge leaving capacitor is 0 at start.

$$-\frac{t}{R_2 C} = \ln(q_{left} - Q_0)\Big|_0^{Q(t)}$$

$$e^{-t/R_2C} = \frac{Q(t) - Q_0}{-Q_0}$$

$$Q(t) = Q_0(1 - e^{-t/R_2C}) \quad *$$

Finding $I_2(t)$ from $*$:

$$Q(t) = Q_0(1 - e^{-t/R_2C})$$

$$\frac{dQ}{dt} = \frac{d}{dt}\left(Q_0(1 - e^{-t/R_2C})\right)$$

$$I_2(t) = 0 + Q_0\left(\frac{-1}{R_2C}\right)(-e^{-t/R_2C})$$

$$I_2(t) = \frac{Q_0}{R_2C} e^{-t/R_2C}$$

Recall: $\mathcal{E} = \frac{Q}{C}$

$$I_2(t) = \frac{\mathcal{E}}{R_2} e^{-t/R_2C}$$

Finding I_1 using 1):

$$\mathcal{E} = I R_1$$

$$I_1 = \frac{\mathcal{E}}{R_1}$$

Using Junction Rule to find $I_s(t)$:

$$I_s = I_1 + I_2$$

$$\boxed{I_s(t) = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} e^{-t/R_2C}}$$