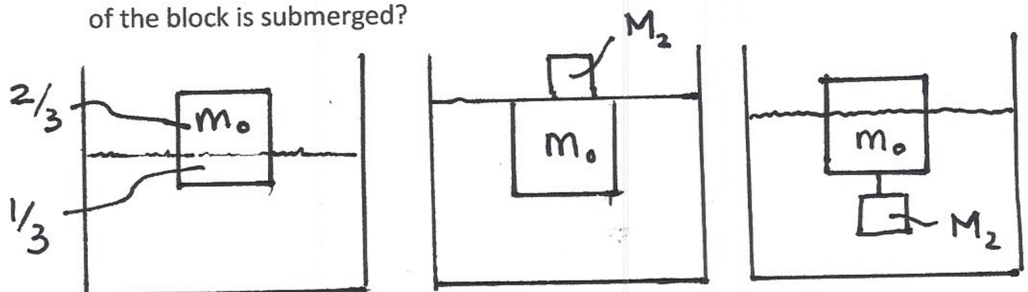
Physics 4C Spring 2017 – Dickson Name SOLUTION

Exam 1

Show all your work for full credit. No scratch papers, note cards, cell phones or other electronic devices are allowed. You have one (1) hour.

1. . A block of mass m_o floats in liquid of density ρ_L with one-third of its volume submerged, thus $\rho_o = {}^1/_3\rho_L$. A mass M_2 with density twice that of the liquid $\rho_2 = 2\rho_L$ is placed on top of the object and then the liquid level is at the top edge of the block as shown. If M_2 is tied with an ideal string to the underside of the block, what fraction of the block is submerged?



(a) (b) (c)

Part (a)

No need to apply N's Laws since $\rho_o = \frac{1}{3}\rho_L$ is given

Part (b)

system: $M_2 + m_o$

$\vec{F}_{Bo}$ $\uparrow +$

$\vec{F}_{g2}$ $\vec{F}_{go}$

$$\vec{F}_{net} = (m_o + M_2)\vec{a}^{\,0}$$

$$\vec{F}_{Bo} + \vec{F}_{g2} + \vec{F}_{go} = 0$$

$$+: F_{Bo} - F_{g2} - F_{go} = 0$$

here $V_L = V_o$

$$\rho_L V_L \cancel{g} - \rho_2 V_2 \cancel{g} - \rho_o V_o \cancel{g} = 0$$

$$\cancel{\rho_L} V_o - 2\cancel{\rho_L} V_2 - \tfrac{1}{3}\cancel{\rho_L} V_o = 0$$

$$2V_2 = \frac{2}{3}V_o$$

$$V_2 = \frac{1}{3}V_o$$

use this

Part (c) system $M_2 + m_o$

$\vec{F}_{B2}$ $\vec{F}_{Bo}$ $+\uparrow$ $\vec{F}_{g2}$ $\vec{F}_{go}$

$$\vec{F}_{net} = (M_2 + m_o)\vec{a}^{\,0}$$

$$\vec{F}_{Bo} + \vec{F}_{B2} + \vec{F}_{go} + \vec{F}_{g2} = 0$$

$$+: F_{Bo} + F_{B2} - F_{go} - F_{g2} = 0$$

$$\rho_L V_{Lo} g + \rho_L V_{L2} g - \rho_o V_o g - \rho_2 V_2 g = 0$$

$\rho_o = \frac{1}{3}\rho_L$ and $\rho_2 = 2\rho_L$

use this in both

here $V_{Lo} = V_S$ ← what we want

and

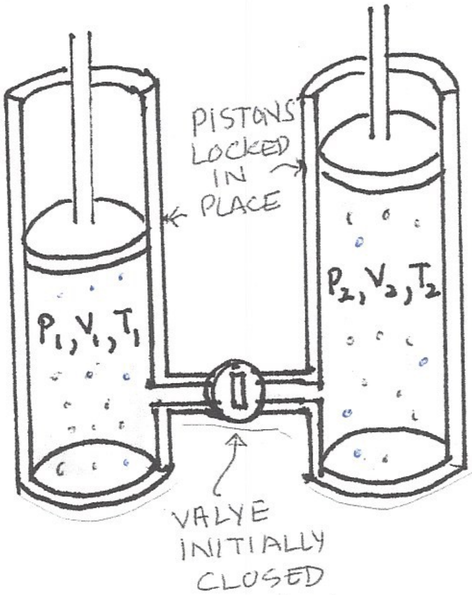
$$V_{L2} = V_2$$

$$\cancel{\rho_L} V_S + \cancel{\rho_L} V_2 - \tfrac{1}{3}\cancel{\rho_L} V_o - 2\cancel{\rho_L} V_2 = 0$$

$$V_S + \frac{1}{3}V_o - \frac{1}{3}V_o - \frac{2}{3}V_o = 0$$

$$\boxed{\frac{V_S}{V_o} = \frac{2}{3}}$$

2. Two thermally insulated vessels are connected by a narrow tube fitted with a valve that is initially closed as shown. One vessel of volume V_1 contains oxygen at temperature T_1 and pressure P_1. The other vessel of volume $V_2 = {}^3/_2V_1$ contains oxygen initially at temperature $T_2 = {}^3/_2T_1$ and pressure $P_2 = 2P_1$. The pistons are locked in place. When the valve is opened, the two vessels mix and the temperature and pressure become uniform throughout, but the final volume of each vessel is the same as the initial volume. What is the final temperature and final pressure of the gas? (Recall, m = Mn and Q = mcΔT.)



$$P_1V_1 = n_1RT_1$$
$$P_2V_2 = n_2RT_2$$
$$P_{f1}V_1 = n_1'RT_{f1}$$
$$P_{f2}V_2 = n_2'RT_{f2}$$

$$P_{f1} = P_{f2} = P_f$$
$$T_{f1} = T_{f2} = T_f$$

$$n_1 = \frac{P_1V_1}{RT_1} \qquad n_2 = \frac{P_2V_2}{RT_2} = \frac{2P_1\cancel{\frac{3}{2}}V_1}{R\cancel{\frac{3}{2}}T_1} = \frac{2P_1V_1}{RT_1}$$

$$\boxed{n_2 = 2n_1} \qquad n_f = n_1' + n_2' = n_i = n_1 + n_2 = n_1 + 2n_1 = 3n_1$$

$$Q_1 + Q_2 = 0$$
$$m_1\cancel{c}\Delta T_1 + m_2\cancel{c}\Delta T_2 = 0$$
$$n_1\cancel{M}(T_{1f} - T_{1i}) + n_2\cancel{M}(T_{2f} - T_{2i}) = 0$$
$$n_1T_{1f} + n_2T_{2f} = n_1T_{1i} + n_2T_{2i}$$
$$T_f(n_1 + n_2) = n_1T_1 + 2n_1\left(\frac{3}{2}T_1\right)$$
$$T_f = \frac{\cancel{n_1}4T_1}{3\cancel{n_1}}$$
$$\boxed{T_f = \frac{4}{3}T_1}$$

$$n_1' = \frac{P_fV_1}{RT_f} \qquad n_2' = \frac{P_fV_2}{RT_f}$$

$$\frac{P_fV_1}{\cancel{R}T_f} + \frac{P_fV_2}{\cancel{R}T_f} = \frac{3P_1V_1}{\cancel{R}T_1}$$

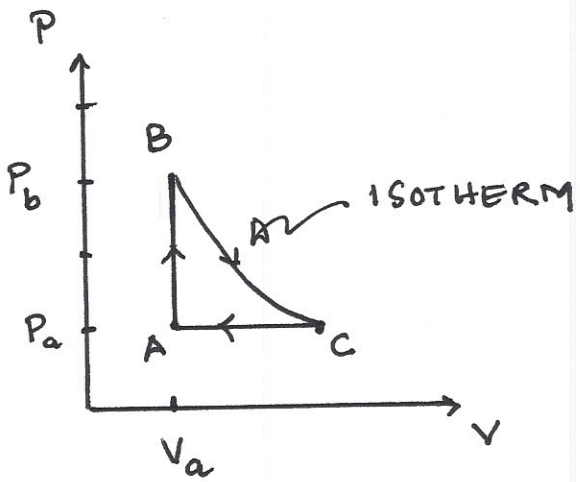
$$P_f\left(\frac{V_1}{\frac{4}{3}\cancel{T_1}} + \frac{\frac{3}{2}V_1}{\frac{4}{3}\cancel{T_1}}\right) = \frac{3P_1V_1}{\cancel{T_1}}$$

$$P_f\left(\frac{\cancel{3}}{4}\cancel{V_1} + \frac{\cancel{3}}{4}\cdot\frac{3}{2}\cancel{V_1}\right) = \cancel{3}P_1\cancel{V_1}$$

$$P_f\left(\frac{1}{4} + \frac{3}{8}\right) = P_1 \qquad \boxed{P_f = \frac{8}{5}P_1}$$

$$P_f\left(\frac{5}{8}\right) = P_1$$

3. One mole of a monatomic gas, undergoes the process ABCA shown. The pressure at A is P_a and the volume is V_a. It is then heated at constant volume until the pressure is $P_b = 3P_a$, then the gas expands isothermally until the pressure is $P_c = P_a$. Finally, the gas is compressed at constant pressure until back to the original state. Find the work done by the gas on the surround in one cycle. Please express your answer in terms of P_a and V_a.



$$P_a V_a = nRT_a$$

$$P_b V_b = nRT_b = P_b V_a = 3P_a V_a$$

$$P_c V_c = nRT_c = nRT_b = P_a V_c$$

$$T_b = 3T_a$$

$$P_a V_c = nR3T_a$$

$$V_c = 3V_a$$

$$W_{TOTAL} = W_{AB} + W_{BC} + W_{CA}$$

$$W_{TOTAL} = P\cancel{\Delta V}^{\,0}_{AB} + \int_B^C P\,dV + P_a \Delta V_{CA}$$

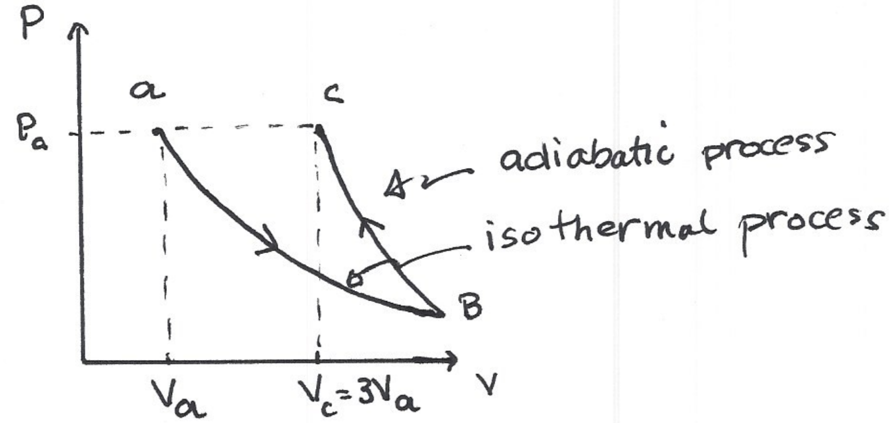
$$W_{TOTAL} = nRT_B \int_{V_B}^{V_C} \frac{dV}{V} + P_a(V_a - V_c)$$

$$W_{TOTAL} = nRT_B \ln\left(\frac{V_c}{V_B}\right) + P_a(V_a - 3V_a)$$

$$W_{TOTAL} = 3P_aV_a \ln\left(\frac{3V_a}{V_a}\right) + P_aV_a(-2)$$

$$\boxed{W_{TOTAL} = P_aV_a\left(3\ln 3 - 2\right)}$$

4. A system consists of n moles of an ideal gas which undergoes two reversible processes shown: Starting from pressure P_a and volume V_a, it expands isothermally to point B, then contracts adiabatically to reach point C with pressure $P_c = P_a$ and volume $V_c = 3\,V_a$. Find the change in entropy for this gas during this process.



Since entropy is a state variable, any path connecting the end points along a reversible process will work to calculate.

The easy way:

$$\Delta S_{ac} = \int_a^c dS$$

$$\Delta S_{ac} = \int_a^c \frac{dQ}{T}$$

$$\Delta S_{ac} = \int_a^c \frac{nC_p\,dT}{T}$$

$$\Delta S_{ac} = nC_p \ln \frac{T_c}{T_a}$$

$$P_a V_a = nRT_a$$

$$P_c V_c = nRT_c = P_a 3V_a$$

$$T_c = 3T_a$$

$$\boxed{\Delta S_{ac} = nC_p \ln 3}$$

The harder way

$$\Delta S_{ac} = \int_a^b dS + \int_b^c dS$$

$$\Delta S_{ac} = \int_a^b \frac{dQ}{T} + \cancel{\int_b^c \frac{dQ}{T}}^{\,0} \quad \text{adiabatic}$$

$$\Delta S_{ac} = \int_a^b \frac{(\cancel{dU}^{\,0} + dW)}{T}. \quad \text{isotherm}$$

$$\Delta S_{ac} = \int_a^b \frac{P\,dV}{T_a}$$

$$\Delta S_{ac} = \int_a^b \frac{nR\cancel{T_a}}{\cancel{T_a}} \frac{dV}{V}$$

$$\Delta S_{ac} = nR \ln\left(\frac{V_b}{V_a}\right) = nR \ln 3^{\frac{C_p}{R}}$$

but what is V_b?

$$\frac{V_b}{V_a} = 3^{\gamma/\gamma-1} \; ; \; \frac{\gamma}{\gamma-1} = \frac{C_p/C_v}{C_p/C_v - 1} \cdot \frac{C_v}{C_v}$$

$$\frac{V_b}{V_a} = \frac{C_p}{R}$$

$$\boxed{\Delta S_{ac} = nC_p \ln 3}$$

easier

$$T_c V_c^{\gamma-1} = T_b V_b^{\gamma-1}$$

$$P_c V_c^{\gamma} = P_b V_b^{\gamma}$$

$$3\cancel{T_a}(3V_a)^{\gamma-1} = \cancel{T_a} V_b^{\gamma-1}$$

$$\left(\frac{V_b}{V_a}\right)^{\gamma-1} = 3 \cdot 3^{\gamma-1} = 3^{\gamma}$$