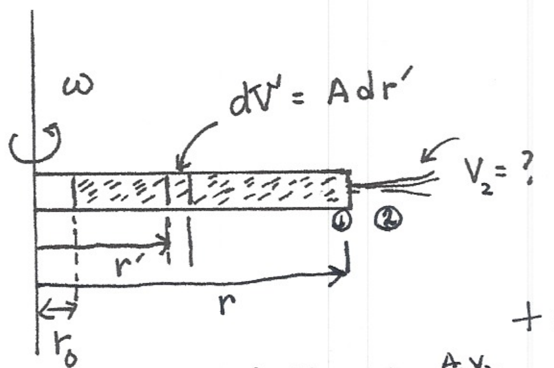
Physics 4C Spring 2018 – Dickson

Name SOLUTIONS

Exam 1

Show all your work for full credit. No scratch papers, note cards, cell phones or other electronic devices are allowed. You have one (1) hour.

1. An incompressible fluid with density ρ is in a horizontal test tube of inner cross-sectional area A. The test tube spins in a horizontal circle at angular speed ω. Gravitational forces are negligible. Consider a volume element of the fluid of area *A* and width *dr'* located at *r'* from the rotational axis. The pressure difference on the two faces is *P* and *P + dp*. Let the surface of the fluid be at radius r_o and the pressure at r_o be $P_{atm} = P_o$. A small hole at the opposite end allows the fluid to come out as shown in the picture. Find the speed of the fluid leaving the tube.



System: $dm' = \rho \, dV'$

$\overset{+}{\leftarrow} \quad \vec{F} + d\vec{F} \quad \longleftrightarrow \quad \vec{F}$

$\vec{F}_{net} = dm' \vec{a}$

$\vec{F} + (\vec{F} + d\vec{F}) = dm' \vec{a}$

$+: \quad \cancel{F} + dF - \cancel{F} = dm' a'_c$

$dP \cdot A = \rho \, dV' \dfrac{v'^2}{r'}$ but $\dfrac{v'^2}{r'} = \omega^2 r'$

$dP \cdot \cancel{A} = \rho \cancel{A} dr' \omega^2 r'$

$\displaystyle\int_{P_0}^{P} dP = \int_{r_0}^{r} \rho \, dr' \omega^2 r'$

$P - P_0 = \rho \omega^2 \dfrac{r'^2}{2} \Big|_{r_0}^{r}$

$P = P_0 + \rho \dfrac{\omega^2}{2}(r^2 - r_0^2) = P_1$ for *

$A_1 V_1 = A_2 V_2 \; ; \; V_1 = \dfrac{A_2 V_2}{A_1}$

$* \quad P_1 + \cancel{\rho g h_1} + \cancel{\tfrac{1}{2}\rho V_1^2} = P_0 + \cancel{\rho g h_2} + \tfrac{1}{2}\rho V_2^2$

$h_1 = h_2 = 0 \qquad \left(\dfrac{A_2}{A_1}\right)^2 \ll 1$

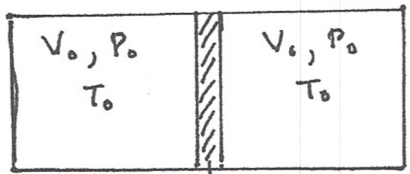
so $V_1 \Rightarrow 0$ { the fluid may be spinning, but ⊥ to V_2 not the streamline

$\cancel{P_0} + \cancel{\rho} \dfrac{\omega^2}{\cancel{2}}(r^2 - r_0^2) = \cancel{P_0} + \cancel{\tfrac{1}{2}\rho} V_2^2$

$V_2^2 = \omega^2 (r^2 - r_0^2)$

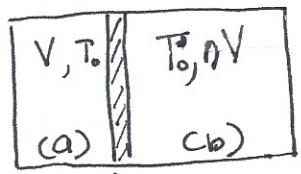
$$\boxed{V = \omega\sqrt{r^2 - r_0^2}}$$

2. A piston can move freely inside a horizontal cylinder closed on both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V_o containing an ideal gas at identical pressure P_o and temperature T_o. What work must an external agent do to move the piston slowly and at constant temperature until the volume of one part of the gas is n times the volume of the other part of the gas?



moveable piston

initial state



final state

$$P_0 V_0 = nRT_0 = PV$$

$$P = \frac{nRT_0}{V}$$

$$nV + V = V_0 + V_0$$

$$(n+1)V = 2V_0$$

$$V = \frac{2V_0}{n+1}$$

$$W_{\text{on the gas}} = -W_{\text{by the gas}}$$

$$W_{ext} = -\left(W_{\text{by (a)}} + W_{\text{by (b)}}\right)$$

$$W_{ext} = -\left[\int_{V_0}^{V_{fa}} P\,dV + \int_{V_0}^{V_{fb}} P\,dV\right]$$

$$W_{ext} = -\left[\int_{V_0}^{V} nRT_0 \frac{dV}{V} + \int_{V_0}^{nV} nRT_0 \frac{dV}{V}\right]$$

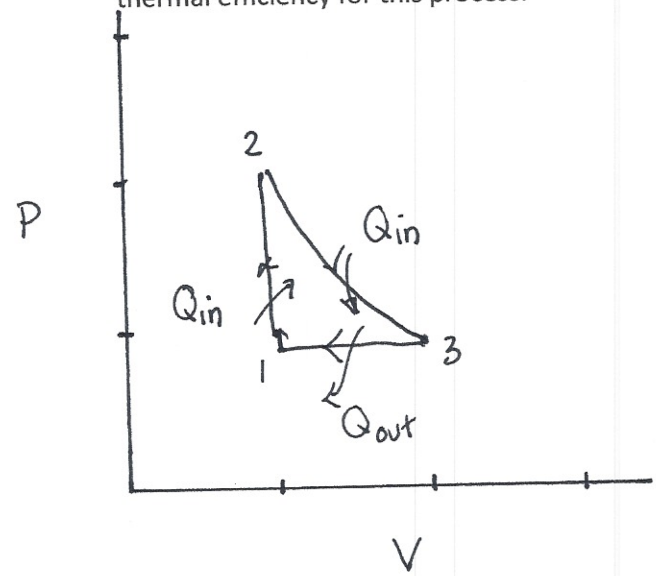
$$W_{ext} = -nRT_0\left(\ln\frac{V}{V_0} + \ln\frac{nV}{V_0}\right)$$

$$W_{ext} = -nRT_0\left(\ln\frac{2\cancel{V_0}}{(n+1)\cancel{V_0}} + \ln\left(\frac{n}{\cancel{V_0}}\left(\frac{2\cancel{V_0}}{n+1}\right)\right)\right)$$

$$W_{ext} = nRT_0\left(\ln\left(\frac{n+1}{2}\cdot\left(\frac{n+1}{2n}\right)\right)\right)$$

$$\boxed{W_{ext} = P_0 V_0 \ln\frac{(n+1)^2}{4n}}$$

3. An ideal gas is initially at volume V_1 and pressure P_1. It is then heated at constant ~~pressure~~ volume until $P_2 = 2P_1$. Next, it is expanded at constant temperature until the pressure returns to P_1. It is then cooled at constant ~~volume~~ pressure until it is back to the original state. Draw the P-V diagram for this process and find the thermal efficiency for this process.



$$P_1V_1 = nRT_1$$

$$P_2V_2 = nRT_2 = 2P_1V_1 \qquad T_2 = 2T_1$$

$$P_3V_3 = nRT_3 = nRT_2 = P_1V_3 \qquad V_3 = 2V_1$$

$$\varepsilon_{TH} = \frac{W}{Q_{in,hot}} = 1 - \frac{|Q_{out,c}|}{Q_{in,hot}}$$

$$Q_{12} = \Delta U_{12} + \cancel{W_{12}}^{0}$$

$$Q_{23} = \cancel{\Delta U_{23}}^{0} + W_{23}$$

$$Q_{in,hot} = Q_{12} + Q_{23}$$

$$= nC_V \Delta T_{12} + \int_2^3 P\,dV$$

$$= nC_V(T_2 - T_1) + nRT_2 \int_{V_2}^{V_3} \frac{dV}{V}$$

$$= nC_V(2T_1 - T_1) + nR\,2T_1 \ln\frac{V_3}{V_2}$$

$$= nC_VT_1 + 2nRT_1 \ln\frac{2V_1}{V_1}$$

$$= nC_VT_1 + 2nRT_1 \ln 2$$

$$Q_{out,c} = Q_{31}$$

$$= nC_p \Delta T_{31}$$

$$= nC_p(T_1 - T_3)$$

$$= nC_p(T_1 - 2T_1)$$

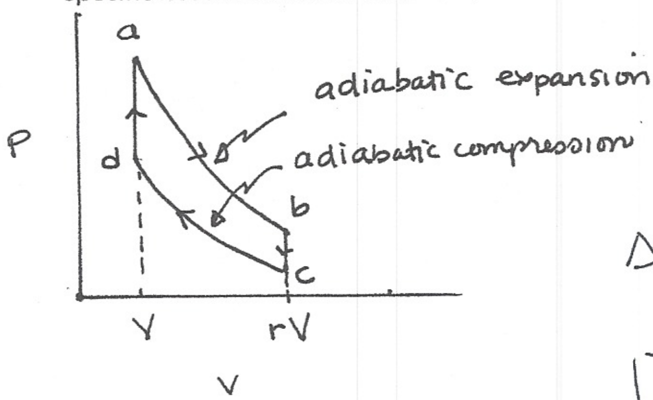
$$= -nC_pT_1$$

$$\varepsilon_{TH} = 1 - \frac{|+\cancel{n}C_p\cancel{T_1}|}{\cancel{n}C_v\cancel{T_1} + 2\cancel{n}R\cancel{T_1}\ln 2}$$

$$\boxed{\varepsilon_{TH} = 1 - \frac{C_p}{C_v + 2R\ln 2}}$$

4. The Otto cycle joins two adiabatic processes with two constant volume processes. The ratio of the volumes is called the compression ratio and for this ideal gas engine, let that ratio be 1:r. That is, let $V_a = V_d = r\,V_b = r\,V_c$.

a) calculate the changes in entropy of the gas for both of the constant-volume processes: (b -> c and d -> a) in terms of the temperatures T_a, T_b, T_c, T_d, the number of moles n, and the molar specific heat at constant volume C_v.



$$\Delta S_{da} = \int_d^a \frac{dQ}{T} = \int_d^a \frac{nC_v\,dT}{T}$$

$$\boxed{\Delta S_{da} = nC_v \ln \frac{T_a}{T_d}}$$

$$\Delta S_{bc} = \int_b^c \frac{dQ}{T} = \int_b^c \frac{nC_v\,dT}{T}$$

$$\boxed{\Delta S_{bc} = nC_v \ln \frac{T_c}{T_d}}$$

b) compute the entropy change for one complete cycle by adding the contributions for all four processes. Simplify your answer as much as possible and explain the meaning of the resultant entropy change.

$$\Delta S = \cancel{\Delta S_{ab}}^{\,0} + \Delta S_{bc} + \cancel{\Delta S_{cd}}^{\,0} + \Delta S_{da}$$

ΔS along an adiabatic process is ∅

$$\Delta S = nC_v\left(\ln\frac{T_c}{T_b} + \ln\frac{T_a}{T_d}\right)$$

but

$$T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1}$$
$$T_d V_a^{\gamma-1} = T_c V_c^{\gamma-1}$$

so

$$\frac{T_a}{T_d} = \frac{T_b}{T_c} \quad \text{and} \quad \frac{T_c}{T_b} = \frac{T_d}{T_a}$$

$$\Delta S = nC_v\left(\ln\frac{T_c}{T_b} - \ln\frac{T_d}{T_a}\right)$$

$$\Delta S = nC_v\left(\ln\left(\frac{T_c}{T_b}\right) - \ln\left(\frac{T_c}{T_b}\right)\right)$$

$$\boxed{\Delta S = 0}$$

Since entropy is a state variable (depending only on the state of the system and not the processes) when the system is back at the original state $\Delta S = 0$ as required!