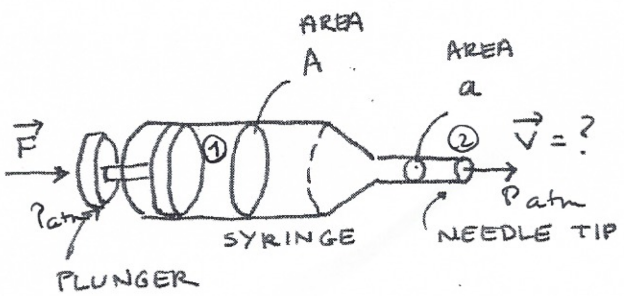
Physics 4C Spring 2019 – Dickson Name SOLUTIONS

Exam 1

Show all your work for full credit. No scratch papers, note cards, cell phones or other electronic devices are allowed. You have one (1) hour.

1. The hypodermic syringe shown contains a medicine with the density ρ. The barrel has a cross-sectional area of A, the needle has a cross sectional area of $a = 0.1A$. In the absence of the force on the plunger, the pressure everywhere is equal to atmosphere. A force $\vec{F}$ acts on the plunger. What is the speed, V, of the medicine as it leaves the needle tip? (Your answer may be in terms of A, F, and ρ)



Continuity equation: $A_1 V_1 = A_2 V_2$

Bernoulli's equation: $P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2$

$P_1 = P_0 + \frac{F}{A}$ $\qquad h_1 = h_2 = 0$

$P_2 = P_0$ $\qquad V_1 = \frac{a V_2}{A} = \frac{0.1 A V_2}{A} = 0.1 V_2$

$V_1 = \frac{1}{10} V_2$

$$\cancel{P_0} + \frac{F}{A} + 0 + \frac{1}{2}\rho\left(\frac{1}{10}V_2\right)^2 = \cancel{P_0} + 0 + \frac{1}{2}\rho V_2^2$$

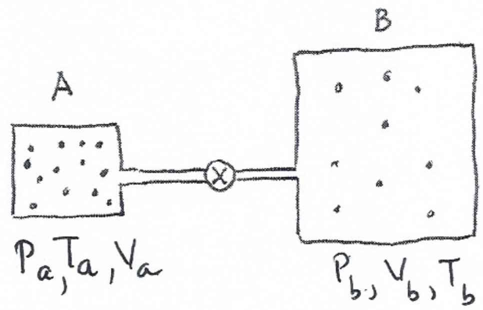
$$\frac{2F}{A} = \rho V_2^2\left(1 - \frac{1}{100}\right)$$

$$\frac{99}{100} V_2^2 = \frac{2F}{\rho A}$$

$$\boxed{V_2 = \sqrt{\frac{200 F}{99 \rho A}} = \frac{10}{3}\sqrt{\frac{2F}{11\rho A}}}$$

either

2. Chamber *A* shown in the figure holds an ideal gas at pressure P_a and temperature T_a. It is connected to chamber B by a thin tube and closed valve. The chambers contain identical gas molecules. However, $P_a = 5\,P_b$, $V_b = 4V_a$ and $T_b = {}^4/_3\,T_a$. When the valve is opened, the pressure is allowed to equalize, but the temperature of each container is maintained. **Find the final pressure.** Please put your final answer in terms of Pa.



initial

$P_a V_a = n_a R T_a$

$P_b V_b = n_b R T_b$

replace

$\dfrac{\left(\frac{1}{5}P_a\right)(4V_a)}{R\left(\frac{4}{3}T_a\right)} = n_b$

$n_b = \dfrac{3}{5}\dfrac{P_a V_a}{RT_a} = \dfrac{3}{5} n_a$

$n = n_a + n_b$

$n = \dfrac{P_a V_a}{RT_a} + \dfrac{3}{5}\dfrac{P_a V_a}{RT_a}$

$n = \dfrac{8}{5}\dfrac{P_a V_a}{RT_a}$

Final

$P_a' = P_b' = P'$

$n_a + n_b = n_a' + n_b'$

$P_a' V_a = n_a' R T_a$

$P_b' V_b = n_b' R T_b$

$P' V_a = n_a' R T_a$

$P' 4V_a = n_b' R \frac{4}{3} T_a$

$n_b' = \dfrac{3P' V_a}{RT_a} = 3n_a'$

$n = n_a' + n_b'$

$n = n_a' + 3n_a'$

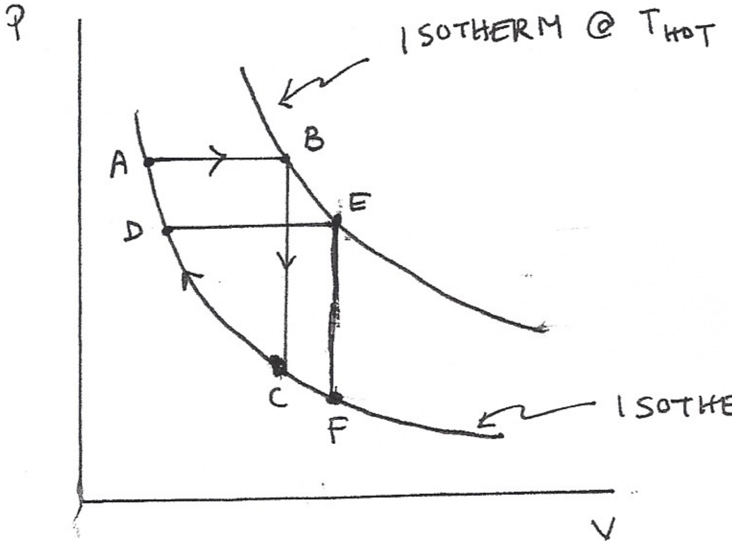
$n = 4n_a'$

$n_a' = \frac{1}{4} n = \frac{1}{4}\left(\frac{8}{5}\frac{P_a V_a}{RT_a}\right)$

$\dfrac{P' V_a}{RT_a} = \dfrac{2}{5}\dfrac{P_a V_a}{RT_a}$

$\boxed{P' = \frac{2}{5} P_a}$

3. Shown on this P-V diagram are two isotherms, T_{hot} and T_{cold}. Cycle *ABCA* joining the isotherms consists of a constant pressure process expansion to T_{hot}, followed by a constant volume process to T_{cold}, followed by an isothermal compression back to the original state. **Prove that the work done by the gas in one cycle is independent of the starting pressure.** (To clarify, is the work done by the gas along ABCA is the same as DEFD, for example?)



$$P_A V_A = nRT_A = nRT_{COLD}$$
$$P_B V_B = nRT_B = nRT_{HOT} = P_A V_B$$
$$P_C V_C = nRT_C = nRT_{COLD}$$
$$P_A = P_B$$

$$W_{ABCA} = W_{AB} + \cancel{W_{BC}}^{0} + W_{CA}$$

$$PV = nRT_{ISO}$$
$$P = \frac{nRT_{COLD}}{V}$$

$$W_{ABCA} = P_A(V_B - V_A) + \int_C^A P\,dV$$

$$= nRT_{HOT} - nRT_{COLD} + nRT_{COLD}\int_{V_C}^{V_A}\frac{dV}{V}$$

$$= nR(T_{HOT} - T_{COLD}) + nRT_{COLD}\ln\frac{V_A}{V_C} \quad \Big\}\text{ but } V_B = V_C$$

$$= nR(T_{HOT} - T_{COLD}) + nRT_{COLD}\ln\frac{V_A}{V_B} \quad \Big\{\text{ but } V_A = \frac{nRT_{COLD}}{P_A}$$

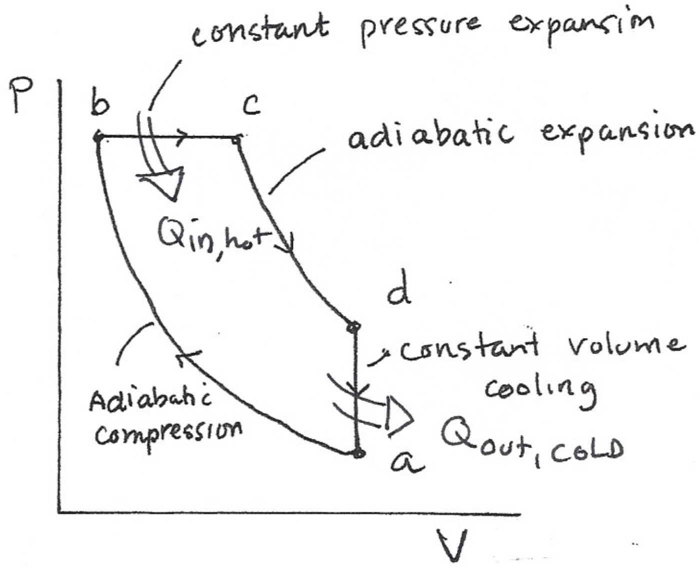
$$V_B = \frac{nRT_{HOLD}}{P_A}$$

$$= nR(T_{HOT} - T_{COLD}) + nRT_{COLD}\ln\left(\frac{\frac{\cancel{nR}T_{COLD}}{\cancel{P_A}}}{\frac{\cancel{nR}T_{HOT}}{\cancel{P_A}}}\right)$$

$$W_{ABCA} = nR\left[(T_{HOT} - T_{COLD}) - T_{COLD}\ln\frac{T_{HOT}}{T_{COLD}}\right]$$

independent of P_A

4. The diesel cycle shown consists of an adiabatic compression from *a* to *b*, a constant pressure expansion from *b* to *c*, an adiabatic compression from *c* to *d* and a constant volume cooling from *d* to *a*. **Find the thermal efficiency** of the cycle in terms of the four volumes: V_a, V_b, V_c, and V_d



$$P_a V_a = nRT_a$$

$$P_b V_b = nRT_b \qquad P_b = P_c$$

$$P_c V_c = nRT_c \qquad V_c = V_a$$

$$P_d V_d = nRT_d$$

$$T_c V_c^{\gamma-1} = T_d V_d^{\gamma-1}$$

$$T_b V_b^{\gamma-1} = T_a V_a^{\gamma-1}$$

$$P_a V_a^{\gamma} = P_b V_b^{\gamma} \rightarrow P_a = P_b \frac{V_b^{\gamma}}{V_a^{\gamma}}$$

$$P_c V_c^{\gamma} = P_d V_d^{\gamma} \rightarrow P_d = P_c \frac{V_c^{\gamma}}{V_d^{\gamma}}$$

$$\varepsilon_{TH} = 1 - \frac{|Q_{out,COLD}|}{Q_{in,hot}}$$

$$Q_{out,COLD} = Q_{da}$$

$$Q_{out,COLD} = nC_v(T_a - T_d) = \frac{\cancel{n}C_v}{\cancel{n}R}\left(P_a V_a - P_d V_d\right)$$

$$Q_{in,hot} = Q_{bc}$$

$$Q_{in,hot} = nC_p(T_c - T_b) = \frac{\cancel{n}C_p}{\cancel{n}R}\left(P_c V_c - P_b V_b\right)$$

$$\varepsilon_{TH} = 1 - \frac{\left|\frac{C_v}{R}\left(P_a V_a - P_d V_d\right)\right|}{\frac{C_p}{R}\left(P_c V_c - P_b V_b\right)}$$

$$\varepsilon_{TH} = 1 - \frac{C_v}{C_p}\left(\frac{P_d V_d - P_a V_a}{P_b(V_c - V_b)}\right) = 1 - \frac{C_v}{C_p}\left(\frac{\cancel{P_c}\left(\frac{V_c}{V_d}\right)^{\gamma} V_d - \cancel{P_b}\left(\frac{V_b}{V_a}\right)^{\gamma} V_a}{\cancel{P_b}(V_c - V_b)}\right)$$

$$\boxed{\varepsilon_{TH} = 1 - \frac{C_v}{C_p}\left(\frac{V_c^{\gamma} V_d^{1-\gamma} - V_b^{\gamma} V_a^{1-\gamma}}{V_c - V_b}\right) = 1 - \frac{C_v}{C_p} V_a^{1-\gamma}\left(\frac{V_c^{\gamma} - V_b^{\gamma}}{V_c - V_b}\right)}$$