4C Spring 2015 Exam 2 Name Solution

Show all your work for full credit. No calculators, note cards, electronic devices or scratch paper is allowed. You have one (1) hour.

1. A string on a musical instrument is held under tension F_T and extends from the point $x = 0$ to $x = L$. The string is overwound with wire in such a way that its mass per unit length $\mu(x) = \mu_o(1 + x/L)$. **Find the time it takes for a pulse to travel the length of the string.** Your answer should be in terms of L, μ_o and F_T.

$$V = \sqrt{\frac{F_T}{\mu}}$$

$$\frac{dx}{dt} = \sqrt{\frac{F_T}{\mu_o(1+x/L)}\left(\frac{L}{L}\right)}$$

$$\frac{dx}{dt} = \sqrt{\frac{F_T L}{\mu_o(L+x)}}$$

$$\int (L+x)^{1/2}dx = \int\sqrt{\frac{F_T L}{\mu_o}}\,dt$$

let $u = L+x$

$du = dx$

$$\int u^{1/2} = \frac{u^{3/2}}{3/2}$$

$$\left.\frac{(L+x)^{3/2}}{3/2}\right|_0^L = \left.\sqrt{\frac{F_T L}{\mu_o}}\right|_0^t$$

$$\frac{(2L)^{3/2} - (L)^{3/2}}{3/2} = \sqrt{\frac{F_T L}{\mu_o}}\,t \quad *$$

$$\frac{L^{3/2}}{3/2}\left(2^{3/2} - 1\right) = \sqrt{\frac{F_T L}{\mu_o}}\,t$$

$$t = \frac{2}{3}\sqrt{\frac{\mu_o L^{\cancel{3}2}}{F_T \cancel{L}}}\left(2^{3/2} - 1\right)$$

$$\boxed{t = \frac{2L}{3}\sqrt{\frac{\mu_o}{F_T}}\left(2^{3/2} - 1\right)}$$

Bonus question: (5 points extra credit)

At a certain frequency, standing waves are observed and one node is located in the wire between x = 0 and x = L. Find the position of the node in terms of L. The condition is met where the time the pulse has travel is $\frac{1}{2}t_{TOTAL}$

$$\left.\frac{(L+x)^{3/2}}{3/2}\right|_0^{x'} = \left.\sqrt{\frac{F_T L}{\mu_o}}\,t\right|_0^{t/2}$$

$$\frac{(L+x')^{3/2} - L^{3/2}}{3/2} = \frac{1}{2}\sqrt{\frac{F_T L}{\mu_o}}\,t \quad \text{compare w/ } *$$

$$\frac{1}{2}\,\frac{(2L)^{3/2} - L^{3/2}}{\cancel{3/2}} = \frac{(L+x')^{3/2} - L^{3/2}}{\cancel{3/2}}$$

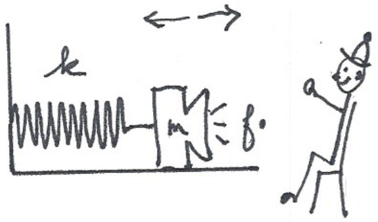
$$(L+x')^{3/2} = \frac{(2L)^{3/2}}{2} - \frac{L^{3/2}}{2} + L^{3/2}$$

$$(L+x')^{3/2} = \left(\frac{2^{3/2}}{2} + \frac{1}{2}\right)L^{3/2}$$

$$x' = \left(\sqrt{2} + \tfrac{1}{2}\right)^{2/3}(L) - L$$

$$\boxed{x' = L\left[\left(\sqrt{2} + \tfrac{1}{2}\right)^{2/3} - 1\right]}$$

2. A person is seated near a speaker emitting sound waves of frequency f_o. The speaker has mass m and is connected to a spring of spring constant k. The mass-spring system oscillates with amplitude A. **What is the highest frequency heard by the person? At what point along the path of the speaker does the person hear the highest frequency and in which direction is the speaker moving at that instant?** Your answer should be in terms of f_o, m, k, A, and the speed of sound in air v. Recall $\omega = \sqrt{(k/m)}$



syst: spring/speaker

$W_{net} = \Delta E$

$0 = \Delta K + \Delta U_s$

$0 = K_f - \cancel{K_i}^{\,0} + \cancel{U_{sf}} - U_{si}$

let $K_f \to x = 0$

$U_{sf} \to x = 0$

$\cancel{\tfrac{1}{2}} m v_{max}^2 = \cancel{\tfrac{1}{2}} k A^2$

$v_{max}^2 = \frac{k}{m} A^2$

same

S.H. CONDITION

$x(t) = A \sin(\omega t)$

$v = \frac{dx}{dt} = -A\omega \cos(\omega t)$

$v_{max} = |A\omega| = A\omega = A\sqrt{\frac{k}{m}}$

$f' = f_o \left(\frac{1 \pm v_L/v}{1 \pm v_s/v} \right)$

$v_L = 0$

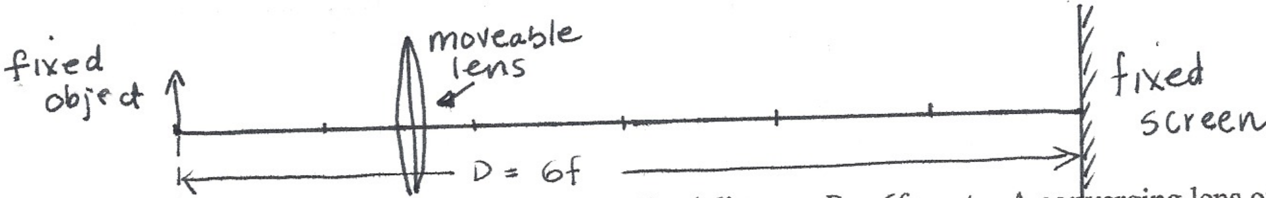
$v_{s_{max}} = A\sqrt{\frac{k}{m}}$

highest $f' \Rightarrow$ choose $(-)$

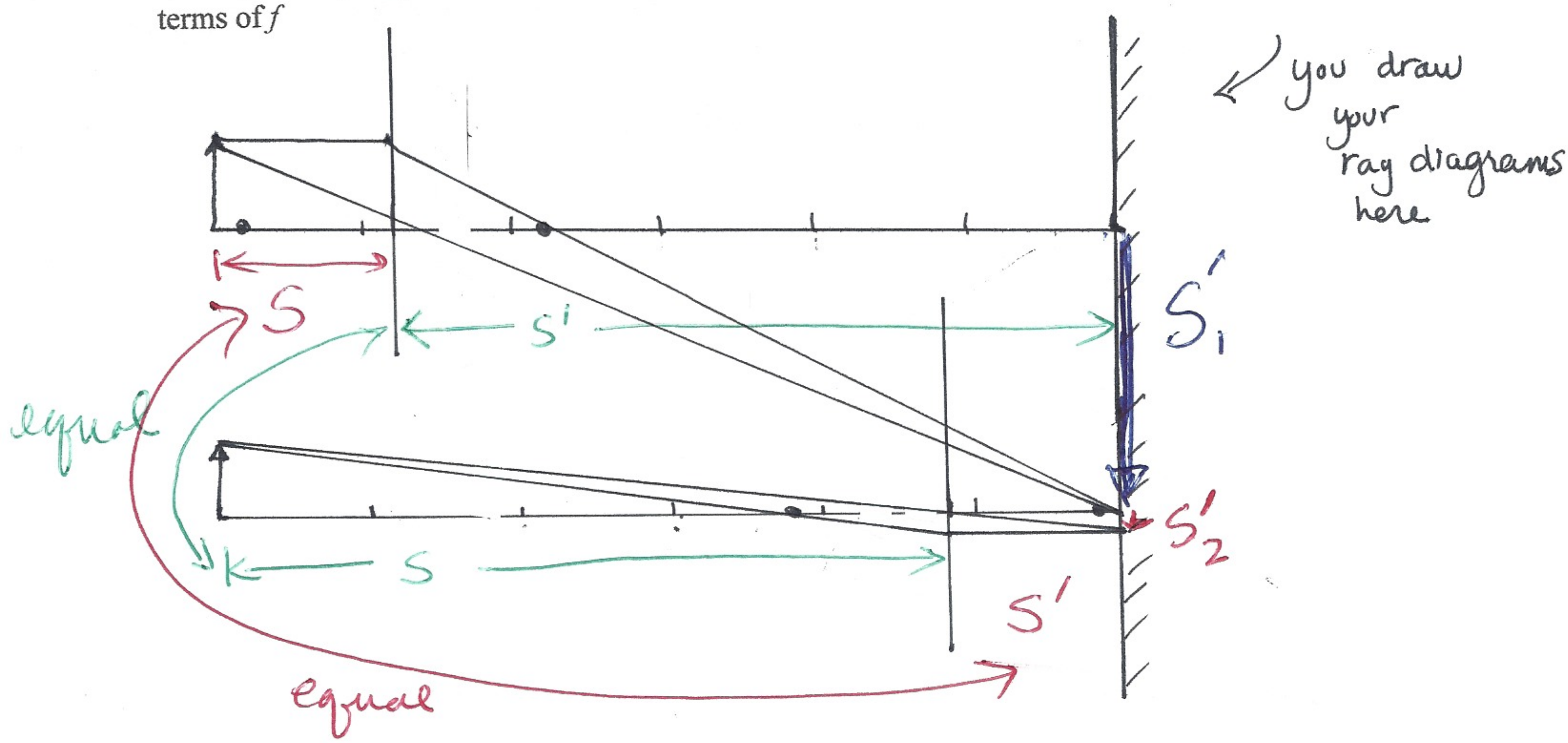
$$\boxed{f' = f_o \frac{v}{v - A\sqrt{\frac{k}{m}}}}$$

at the equilibrium position 2

approaching. 2



3. A luminous object and a screen are a fixed distance $D = 6f$ apart. A converging lens of focal length f is placed between them and it is observed that a real image is formed for two different positions of the lens. **Find the distance between the two lens positions.**
A complete ray diagram for the two cases is required for full credit. Your answer should be in terms of f



$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$s + s' = D = 6f$$

$$s = 6f - s'$$

$$\frac{1}{6f - s'} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{\cancel{s'} + 6f - \cancel{s'}}{6fs' - s'^2} = \frac{1}{f}$$

$$6f^2 = 6fs' - s'^2$$

$$s'^2 - 6fs' + 6f^2 = 0$$

$$s' = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s_1' - s_2' = \frac{-b + \sqrt{b^2 - 4ac}}{2a} - \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$

$$\Delta s' = \frac{\cancel{2}\sqrt{b^2 - 4ac}}{\cancel{2}a}$$

$$\Delta s' = \sqrt{(-6f)^2 - 4(1)(6f^2)}$$

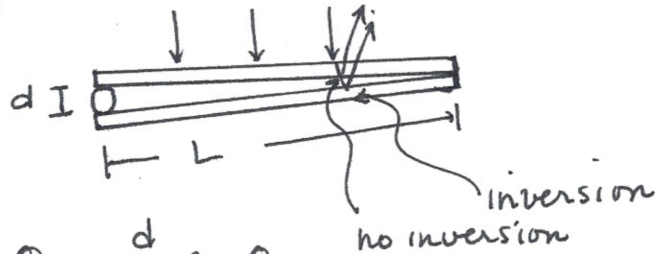
$$\Delta s' = f\sqrt{36 - 24}$$

$$\boxed{\Delta s' = 2\sqrt{3}\, f}$$

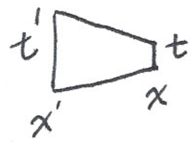
A little bigger than 3f

≈ 1.7 from the center each lens positi

4. An air wedge is formed between two glass plates of length L separated at one edge by a very fine wire of circular cross section as shown. When the wedge is illuminated from above with light of wavelength λ, adjacent interference bands are separated by Δx. What is the diameter of the wire?



$$\tan \theta = \frac{d}{L} \approx \theta$$



$$\Delta x = x' - x$$

$$\theta = \frac{t}{x} = \frac{t'}{x}$$

$$2t = m\lambda$$

$$2t' = (m+1)\lambda$$

$$\frac{2t'}{\lambda} = \frac{2t}{\lambda} + 1$$

$$t' = t + \frac{\lambda}{2}$$

$$\theta = \frac{t' - t}{x' - x} = \frac{t + \lambda/2 - t}{\Delta x}$$

$$\theta = \frac{\lambda}{2\Delta x} = \frac{d}{L}$$

$$\boxed{d = \frac{L\lambda}{2\Delta x}}$$