Problem set 5 – Mechanical Waves

1. A string fastened at both ends has successive resonances with wavelengths of 0.54 m for the n^{th} harmonic and 0.48 for the $(n+1)^{th}$ harmonic. (a) Which harmonics are these? (b) What is the length of the string?

2. A coiled spring, such as s Slinky, is stretched to a length *L*. It has a force constant *k* and a mass *m*. Show that the velocity of longitudinal compression waves along the spring is given by $= L\sqrt{k/m}$.

Hint: University page 499 outlines the approach. The main difference is that you are looking at the velocity in the propagation direction. Hope that helps.

3. A metal wire, with density ρ and Young's modiulus *Y*, is stretched between rigid supports. At temperature *T*, the speed of a transverse wave is found to be v_1. When the temperature is increased to $T + \Delta T$, the speed decreases to $v_2 < v_1$. Determine the coefficient of linear expansion of the wire.

4. Show that the standing wave function $A'sin(kx)\ cos(\omega t + \delta)$ can be written as the sum of two harmonic wave functions – one for a wave traveling the positive x direction and the other for a wave traveling in the negaitive x direction. The traveling waves each have the same wave number and angular frequency as does the standing wave.

5. A rope hangs vertically under its own weight. Show that the wavespeed at a distance y from the lower end is given by $\sqrt{gy}$. Note that the wavespeed is independent of the rope's linear density; hence, this relation applies even if the rope is not uniform.

6. Three pieces of string, each of length L are joined together end to end to make a combined string of length 3L. The first piece of string has mass per unit length μ_1, the second piece has mass per unit length $\mu_2 = 4\mu_1$, and the third has mass per unit length $\mu_3 = ¼\ \mu_1$. (a) If the combined string is under tension Ft, how much time does it take a transverse wave to travel the entire length 3L?

7. A uniform rope with length L and mass *m* is held at one end and whirled in a horizontal circle with angular velocity ω. You may ignore the force of gravity on the rope. Find the time required for a transverse wave to travel from one end of the rope to the other.

8. A horizontal string tied at both ends is vibrating in its fundamental mode. The traveling waves have speed *v*, frequency *f,* amplitude *A* and wavelength λ. Calculate (a) the maximum transverse velocity and (b)maximum transverse acceleration of points located at $x = \lambda/2, \lambda/4$ and $\lambda/8$. (c) At each point what is the amplitude of the motion?