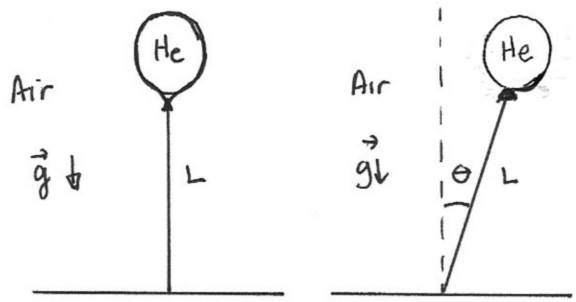
Physics 4C Spring 2016 Name Solution

Exam 1

Show all your work for full credit. You have one (1) hour for four, equally weighted questions. No scratch papers, note cards or electronic devices are allowed.

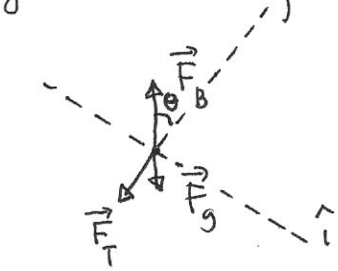
1. A light balloon filled with helium of density ρ_{He} is tied to a string of length L. The string is tied to the ground forming an inverted pendulum as shown. If the balloon is displaced slightly from equilibrium as shown and released, **show that the subsequent motion is simple harmonic and find the period, *T*.** Let the density of the air be ρ_{air}. Assume the air applies a buoyant force on the balloon, but does not otherwise affect its motion. Recall that the period $T = 2\pi/\omega$. Your answer should be in terms of the two densities, L and g.

* It is entirely possibee to use $\vec{\tau} = I\vec{\alpha}$, but a bit more effort



(a) (b)

System: balloon



$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_B + \vec{F}_g + \vec{F}_T = m\vec{a}$$

$$\hat{i}: -F_B \sin\theta + F_g \sin\theta = ma_x$$

$$\hat{j}: F_B \cos\theta - F_g \cos\theta - F_T = ma_y$$

$$m_{air} g \sin\theta - m_{balloon} g \sin\theta = -ma_x$$

$$(\rho_{air} V_o g - \rho_{He} V_o g)\sin\theta = -ma_x$$

Let $\sin\theta \cong \theta$ for small θ

Let $s = L\theta$

$$\frac{ds}{dt} = L\frac{d\theta}{dt}$$

$$a_x = \frac{d^2s}{dt^2} = L\frac{d^2\theta}{dt^2}$$

$$(\rho_{air} - \rho_{He})\cancel{V_o} g\,\theta = -\rho_{He}\cancel{V_o} L\frac{d^2\theta}{dt^2}$$

$$* \quad \boxed{\left(\frac{\rho_{air}}{\rho_{He}} - 1\right)\frac{g}{L}\theta = -\frac{d^2\theta}{dt^2}}$$

S.H.O. FORM.

Let $\theta = A\cos\omega t$

$$\frac{d\theta}{dt} = -A\omega\sin\omega t$$

$$\frac{d^2\theta}{dt^2} = -A\omega^2\cos\omega t$$

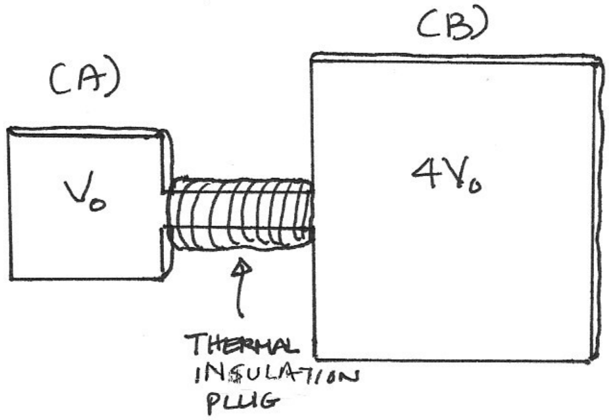
Substitute into *

$$\left(\frac{\rho_{air}}{\rho_{He}} - 1\right)\frac{g}{L}\cancel{A\cos\omega t} = +\cancel{A}\omega^2\cancel{\cos\omega t}$$

$$\omega = \sqrt{\omega^2} = \sqrt{\left(\frac{\rho_{air}}{\rho_{He}} - 1\right)\frac{g}{L}}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}\left(\frac{\rho_{He}}{\rho_{air} - \rho_{He}}\right)}}$$

2. In a chemical processing plant, a reaction chamber (A) of fixed volume V_o is connected to a reservoir chamber (B) of fixed volume $4V_o$ by a passage containing a thermally insulating porous plug. Gas molecules are free to move from chamber to chamber, so that the pressure is the same in both, but the temperatures can differ. Initially, the gas in both chambers is at temperature T_i and pressure P_i. Then the intake and exhaust valves are closed. The reservoir (B) stays at T_i, but the reaction chamber (A) is heated to $T_{Af} = 6\,T_i$. **What is the final pressure in both chambers** after this is done? Your answer should be in terms of P_i.



Initially $\quad P_i V_i = nRT_i$

$$P_i 5V_o = nRT_i$$

$$n = \frac{5P_iV_o}{RT_i}$$

Finally (A) $\quad P_f V_o = n_a R T_{af} \rightarrow n_a = \frac{P_f V_o}{RT_{af}}$

(B) $\quad P_f 4V_o = n_b R T_{bf} \rightarrow n_b = \frac{4P_f V_o}{RT_{bf}}$

* $n = n_a + n_b$

$$T_{bf} = T_i$$

$$T_{af} = 6T_i$$

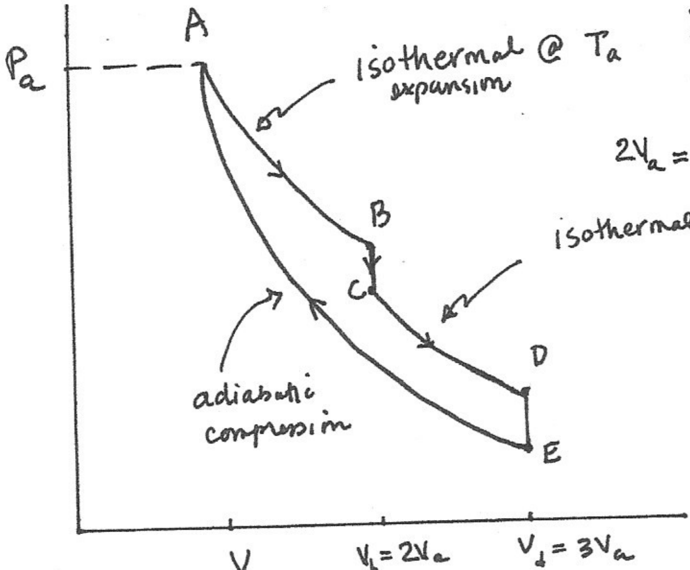
* $\dfrac{5P_i \cancel{V_o}}{\cancel{R}\cancel{T_i}} = \dfrac{P_f \cancel{V_o}}{\cancel{R}\,6\cancel{T_i}} + \dfrac{4P_f\cancel{V_o}}{\cancel{R}\cancel{T_i}}$

$$5P_i = P_f\left(\frac{1}{6} + 4\right)$$

$$\frac{\cancel{25}^{\,5}}{6} P_f = \cancel{5}\, P_i$$

$$\boxed{P_f = \frac{6}{5} P_i}$$

3. The P-V diagram shown depicts one mole of an ideal diatomic gas moving through the cycle ABCDEA. The system starts at pressure P_a, V_a, Ta and isothermally expands to $V_b = 2V_a$. Next there is an isovolumetric exhaustion of heat to temperature T_c. From point C to D, there is an isothermal expansion to $V_d = 3V_a$. From D to E is a constant volume exhaustion of heat to E. From E to A is an adiabatic compression back to the original state. **Find the work done by the gas on the surround in one cycle.** Your answer may include *T_a, T_c, R, C_v, and δ.*



$P_A V_A = nRT_a = P_b V_b$

$P_c V_c = nRT_c = P_D V_D$

$2V_a = V_b = V_c$; $V_D = V_E = 3V_a$

isothermal expansion @ T_c

$P = \frac{nRT}{V}$

$P_A V_A^\gamma = P_E V_E^\gamma = PV^\gamma$

$T_A V_A^{\gamma-1} = T_E V_E^{\gamma-1}$

$P = \frac{P_A V_A^\gamma}{V^\gamma}$

(ISO VOLUMETRIC)

$W_{ABCDE} = W_{AB} + \cancel{W_{BC}}^{0} + W_{CD} + \cancel{W_{DE}}^{0} + W_{EA}$

$$\boxed{W_{ABCDE} = RT_a \ln 2 + RT_c \ln \tfrac{3}{2} + C_v T_a (3^{1-\gamma} - 1)}$$

$C_p = C_v + R \rightarrow C_v - C_p = -R$

$\gamma = \frac{C_p}{C_v}$

$W_{ab} = \int_a^b P\,dV$

$W_{ab} = \int_a^b \frac{nRT_a}{V}\,dV$

$W_{ab} = nRT_a \int_a^b \frac{dV}{V}$

$W_{ab} = nRT_a \ln \frac{V_b}{V_a}$

$$\boxed{W_{ab} = (1) RT_a \ln 2}$$

$W_{CD} = \int_C^D P\,dV$

$W_{CD} = \int_C^D \frac{nRT_c}{V}\,dV$

$W_{CD} = nRT_c \int_C^D \frac{dV}{V}$

$W_{CD} = nRT_c \ln \frac{V_d}{V_c}$

$W_{CD} = (1) RT_c \ln \frac{3V_a}{2V_a}$

$$\boxed{W_{CD} = RT_c \ln \tfrac{3}{2}}$$

2 ways 2 Find W_{EA}

① $W_{EA} = \int_E^A P\,dV$

$W_{EA} = \int_E^A \frac{P_A V_A^\gamma}{V^\gamma}\,dV$

$W_{EA} = P_A V_A^\gamma \int_E^A \frac{dV}{V^\gamma}$

$W_{EA} = P_A V_A^\gamma \frac{1}{1-\gamma} V^{1-\gamma} \Big|_{V_E}^{V_A}$

$W_{EA} = P_A V_A^\gamma \frac{1}{1-\gamma} \left[V_A^{1-\gamma} - V_E^{1-\gamma} \right]$

$W_{EA} = \frac{1}{1-\gamma} \left[P_A V_A - P_A V_A^\gamma V_E^{1-\gamma} \right]$

$W_{EA} = \frac{1}{1-\gamma} \left[P_A V_A - P_A V_A^\gamma [3V_A]^{1-\gamma} \right]$

$W_{EA} = \frac{1}{1-\gamma} \left[P_A V_A - 3^{1-\gamma} \left[P_A V_A^\gamma \cdot V_A^{1-\gamma} \right] \right]$

$W_{EA} = \frac{1}{1-\gamma} \left[P_A V_A - 3^{1-\gamma} (P_A V_A) \right]$

$$\boxed{W_{EA} = P_A V_A \left[\frac{1}{1-\gamma} (1 - 3^{1-\gamma}) \right]}$$

① cont: $\frac{1}{1-\gamma} = \frac{C_v}{C_v - C_p}$

$W_{EA} = nRT_A \left(\frac{C_v}{C_v - C_p} \right) \left[1 - 3^{1-\gamma} \right]$

$W_{EA} = -(1) C_v T_a (1 - 3^{1-\gamma})$

method ②

$\cancel{Q}^{0} = \Delta U_{EA} + W_{EA}$

$W_{EA} = -\Delta U_{EA}$

$W_{EA} = -nC_v \Delta T_{EA}$

$W_{EA} = -nC_v (T_A - T_E)$

$= -C_v \left(T_A - \frac{T_A V_A^{\gamma-1}}{V_E^{\gamma-1}} \right)$

$= -C_v T_a \left(1 - \left(\frac{1}{3} \right)^{\gamma-1} \right)$

$$\boxed{= -C_v T_a (1 - 3^{1-\gamma})}$$

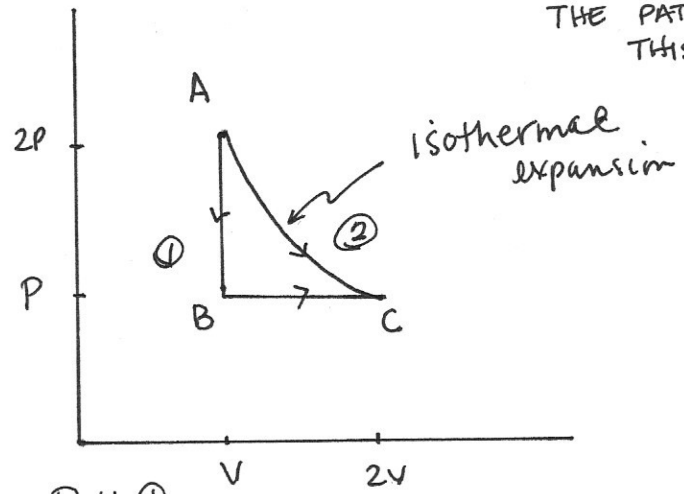
same!

4. The P-V diagram shown is an ideal gas undergoing three processes. From A to B is a constant volume decrease in pressure from *2P* to *P*. The path B to C is a constant pressure expansion from *V* to *2V*. The path AC is an isothermal expansion from *2P, V* to *P, 2V*.

Prove that the entropy change for path ABC is the same as the entropy change for path AC. That is, show that ΔS for A -> B -> C equals ΔS for A -> C. You answer may contain *n* and *R*. Recall $C_p = C_v + R$

(This is the Arindam Theorem.)

NOTE: MY ARROWS INDICATE THE PATH FOR CALCULATING ΔS THIS IS NOT A CYCLE.



$P_A V_A = nRT_A$

$P_B V_B = nRT_B$

$P_C V_C = nRT_C$

$T_A = T_C \quad P_A V_A = P_C V_C$

$P_B = \frac{1}{2} P_A \rightarrow T_B = \frac{1}{2} T_A$

Path①

$\Delta S_{ABC} = \Delta S_{AB} + \Delta S_{BC}$

$\Delta S_{ABC} = \int_A^B dS + \int_B^C dS$

$\Delta S_{ABC} = \int_A^B \frac{dQ}{T} + \int_B^C \frac{dQ}{T}$

$\Delta S_{ABC} = \int_A^B \frac{nC_v\,dT}{T} + \int_B^C \frac{nC_p\,dT}{T}$

$\Delta S_{ABC} = nC_v \int_A^B \frac{dT}{T} + nC_p \int_B^C \frac{dT}{T}$

$\Delta S_{ABC} = nC_v \ln T \Big|_A^B + nC_p \ln T \Big|_B^C$

$\Delta S_{ABC} = nC_v \ln \frac{T_b}{T_a} + nC_p \ln \frac{T_c}{T_B}$

$\Delta S_{ABC} = nC_v \ln \frac{\frac{1}{2}T_a}{T_a} + nC_p \ln \frac{T_a}{\frac{1}{2}T_a}$

$\Delta S_{ABC} = nC_v \ln \frac{1}{2} + nC_p \ln 2$

$\boxed{\Delta S_{ABC} = n(C_p - C_v) \ln 2 = nR \ln 2}$

Path②

$\Delta S_{AC} = \int_A^C dS$

$\Delta S_{AC} = \int_A^C \frac{dQ}{T}$ isotherm

$\Delta S_{AC} = \cancel{\int_A^C \frac{dU}{T}}^{\,0} + \int_A^C \frac{dW}{T}$

$\Delta S_{AC} = \int_A^C \frac{P\,dV}{T}$

$PV = nRT \rightarrow P = \frac{nRT}{V}$

$\Delta S_{AC} = \int_A^C \frac{nR\cancel{T}}{\cancel{T}} \frac{dV}{V}$

$\Delta S_{AC} = nR \int_A^C \frac{dV}{V}$

$\Delta S_{AC} = nR \ln \frac{V_c}{V_a}$

$\boxed{\Delta S_{AC} = nR \ln 2}$

Same