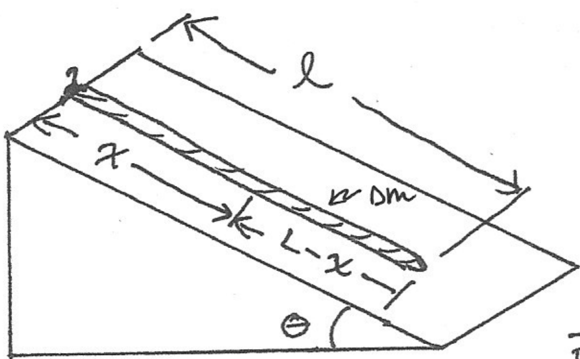
4C Spring 2106 Exam 2 Name SOLUTIONS

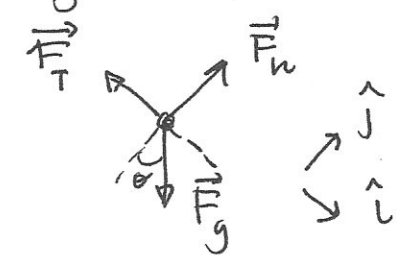
Dickson

Show all your work for full credit. You have one (1) hour for all four problems. No scratch papers, note cards, calculators or electronic devices are allowed.

1. A rope of mass *m* and length *l* and uniform mass density μ is attached at the top of a frictionless inclined plane which makes an angle θ with the horizontal. The rope is parallel to the incline as shown in the picture. What is the time required for a wave pulse to travel the length of the rope?



System: bottom section of rope $\underline{\Delta m}$



Find the tension as a function of x before the pulse

$$\frac{M}{L} = \mu = \frac{\Delta m}{L-x}$$

$$\Delta m = \mu(L-x)$$

$$\vec{F}_{net} = \Delta m \vec{a}$$

$$\vec{F}_n + \vec{F}_g + \vec{F}_T = \Delta m \vec{a}$$

$$\hat{i}: \quad F_T - F_g \sin\theta = \Delta m \cancel{a_x}^{0}$$

$$F_T = \Delta m g \sin\theta$$

$$v_x = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{\Delta m g \sin\theta}{\mu}}$$

$$v_x = \sqrt{\frac{\cancel{\mu}(L-x) g \sin\theta}{\cancel{\mu}}}$$

$$\frac{dx}{dt} = \sqrt{(L-x) g \sin\theta}$$

$$\frac{dx}{\sqrt{L-x}} = \sqrt{g\sin\theta}\, dt$$

$$\int \frac{-du}{\sqrt{u}} = \int \sqrt{g\sin\theta}\, dt$$

let $u = L-x$

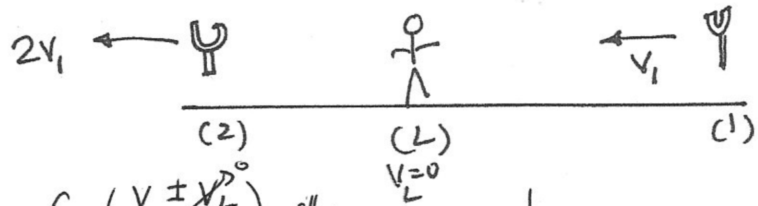
$du = -dx$

$$-\frac{1}{\frac{1}{2}}(L-x)^{1/2}\Big|_0^L = \sqrt{g\sin\theta}\, t$$

$$-(0 - 2L^{1/2}) = \sqrt{g\sin\theta}\, t$$

$$\boxed{t = 2\sqrt{\frac{L}{g\sin\theta}}}$$

2a. A stationary listener is positioned between two tuning forks which emit the same frequency f_o. The first tuning fork is approaching the listener with speed V_1. The second tuning fork is receeding from the listener with speed $V_2 = 2V_1$. What is the beat frequency f_b heard by the listener? Assume the speed of sound in air is V.



$f_b = f'' - f'$

① $f'' = f_0\left(\frac{V \pm \cancel{V_L}^{0}}{V \pm V_S}\right)$ $f''\uparrow (-)$

② $f' = f_0\left(\frac{V \pm \cancel{V_L}}{V \pm V_S}\right)$ $f'\downarrow (+)$

$f_b = f'' - f_0$

$f_b = f_0\left(\frac{V}{V - V_1}\right) - f_0\left(\frac{V}{V + 2V_1}\right)$

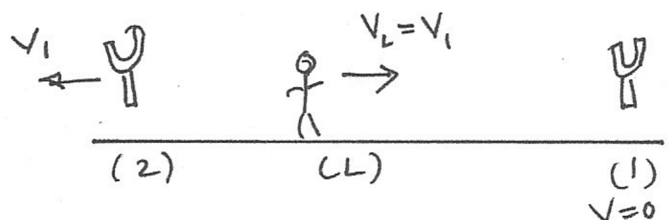
$f_b = f_0\left[\left(\frac{\cancel{V^2} + 2V_1V - (\cancel{V^2} - V_1V)}{(V - V_1)(V + 2V_1)}\right)\right]$

$\boxed{f_b = f_0 \frac{3V_1V}{(V - V_1)(V + 2V_1)}}$

or

$f_b = f_0 \frac{3V_1V}{V^2 + V_1V - 2V_1^2}$

b. The first tuning fork is now held still and the listener is moving, approaching tuning fork 1 with speed $V_L = V_1$. The second tuning fork is still receeding from the listener, but now it is moving away from the fixed tuning fork with speed $V_2 = V_1$. What is the beat frequency f_b heard by the listener?



$f_b = f'' - f'$

① $f'' = f_0\left(\frac{V \pm V_L}{V \pm \cancel{V_S}}\right) f''\uparrow (+)$

② $f' = f_0\left(\frac{V \pm V_L}{V \pm V_S}\right) f'\downarrow \frac{(-)}{(+)}$

$f_b = f'' - f'$

$f_b = f_0\left(\frac{V + V_1}{V}\right) - f_0\left(\frac{V - V_1}{V + V_1}\right)$

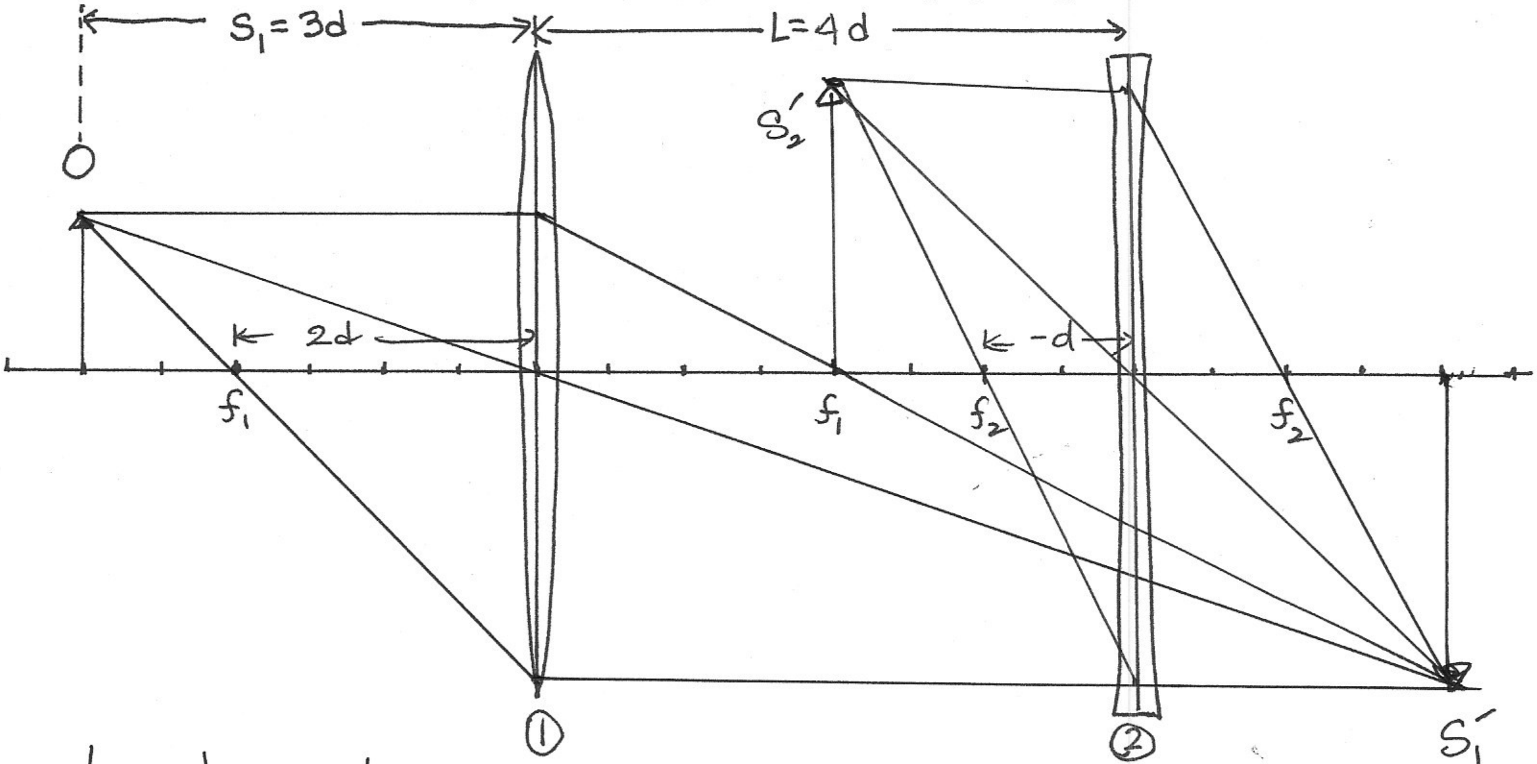
$f_b = f_0\left(\frac{\cancel{V^2} + 2V_1V + V_1^2 - (\cancel{V^2} - V_1V)}{V(V + V_1)}\right)$

$\boxed{f_b = f_0\left(\frac{V_1^2 + 3V_1V}{V(V + V_1)}\right)}$

or

$f_b = f_0\left(\frac{V_1^2 + 3V_1V}{V^2 + V_1V}\right)$

3. A real object is located distance $3d$ to the left of a converging lens of focal length $f_1 = 2d$. A diverging lens of focal length $f_2 = -d$ is placed to the right of the first lens by amount $L = 4d$. Find the position of the final image and the overall magnification, M. A ray diagram must accompany an algebraic solution for full credit.



$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1}$$

$$\frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s_1}$$

$$\frac{1}{s_1'} = \frac{1}{2d} - \frac{1}{3d}$$

$$\frac{1}{s_1'} = \frac{3}{6d} - \frac{2}{6d}$$

$s_1' = 6d$ as shown

$$S_2 = L - s_1'$$

$$S_2 = 4d - 6d$$

$$S_2 = -2d$$

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2}$$

$$\frac{1}{s_2'} = \frac{1}{f_2} - \frac{1}{s_2}$$

$$\frac{1}{s_2'} = -\frac{1}{d} + \frac{1}{+2d}$$

$$\frac{1}{s_2'} = \frac{-2}{2d} + \frac{1}{2d}$$

$$\frac{1}{s_2'} = -\frac{1}{2d}$$

$$\boxed{s_2' = -2d}$$

To the left of lens 2

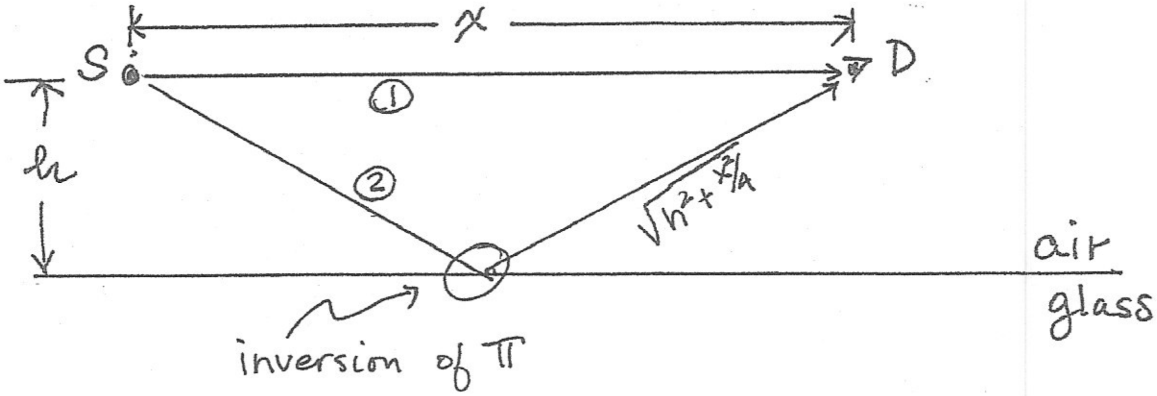
$$M = \frac{-s_1'}{s_1} \cdot \frac{-s_2'}{s_2}$$

$$M = \frac{-6d}{3d} \cdot \frac{-(-2d)}{-2d}$$

$$\boxed{M = 2}$$

upright.

4. A source S of monochromatic light and a detector D are both located in air a distance h above a horizontal plane sheet of glass, and are separated by a horizontal distance x. Waves reaching D directly from S interfere with waves that reflect off the glass as shown in the diagram. Find the condition for destructive interference. [Recall $\sin a \pm \sin b = 2 \sin \frac{1}{2}(a \pm b) \cos \frac{1}{2}(a \mp b)$]



$$E = E_1 + E_2$$

$$E = E_0 \sin(kx_1 - \omega t) + E_0 \sin(kx_2 - \omega t + \pi)$$

$$E = 2E_0 \sin\tfrac{1}{2}(kx_1 - \omega t + kx_2 - \omega t + \pi)\cos\tfrac{1}{2}\big((kx_1 - \omega t) - (kx_2 - \omega t + \pi)\big)$$

$$E = 2E_0 \sin\left(\tfrac{1}{2}k(x_1 + x_2) - \omega t + \tfrac{\pi}{2}\right)\cos\left(\tfrac{1}{2}k(x_1 - x_2) - \tfrac{\pi}{2}\right)$$

$$E = \underbrace{2E_0 \cos\left(k\frac{\Delta x}{2} - \frac{\pi}{2}\right)}_{\text{Amplitude}} \sin\left(k\bar{x} - \omega t + \frac{\pi}{2}\right)$$

destructive condition:

$$\cos\left(\frac{k\Delta x}{2} - \frac{\pi}{2}\right) = 0$$

or

$$\cos\left(\frac{\pi}{2} - \frac{k\Delta x}{2}\right) = 0$$

with one inversion, the condition is that the path difference $\Delta x = m\lambda$ for destructive

since $\cos a = \cos(-a)$

Thus:

$$\frac{\pi}{2} - \frac{k\Delta x}{2} = \left(\frac{2m+1}{2}\right)\pi \qquad m = 0, 1, 2, 3 \ldots$$

$$\pi - k\Delta x = (2m+1)\pi$$

$$-k\Delta x = (2m\pi)$$

$$-k(x_1 - x_2) = 2m\pi$$

$$\frac{\cancel{2\pi}}{\lambda}(x_2 - x_1) = \cancel{2m\pi}$$

$$\boxed{x_2 - x_1 = m\lambda}$$

The path difference = an integer * λ

$$\boxed{2\sqrt{h^2 + \frac{x^2}{4}} - x = m\lambda}$$

If you like, you could solve for x, or h