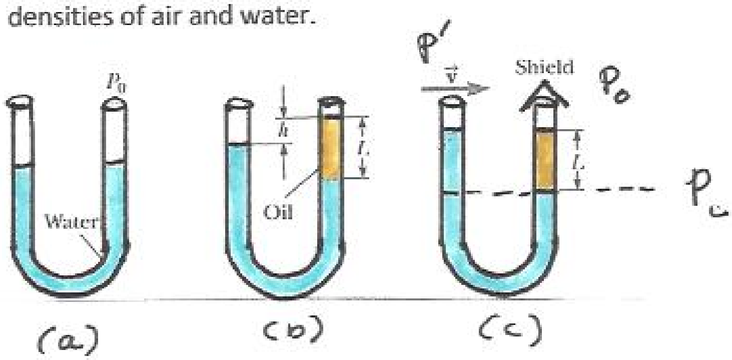
Physics 4C Spring 2014 – Dickson   Name *Solutions*

Exam 1

Show all your work for full credit. No scratch papers, note cards, cell phones or other electronic devices are allowed. You have one (1) hour.

1. A U-tube manometer open at both end is partially filled with water. Oil, of density $\rho_{oil} = \frac{3}{4}\rho_{water}$ is poured into the right side. It forms a column L high as shown in diagram (b). The right side of the manometer is then shielded from air motion and air is blown across the top of the left side until the surfaces of the two liquids are the same height as shown in diagram (c). Let the density of air be ρ_{air}. What is the speed of the air blown across the left arm? Your answer may be in terms of g, L and the densities of air and water.



The elevation change for the air is zero Thus

For the air Bernoulli's eqn: $P' + \frac{1}{2}\rho_{air}v^2 = P_0 + \frac{1}{2}\cancel{\rho_{air}v^2}^{\,0}$ shielded side

$$P' + \frac{1}{2}\rho_{air}v^2 = P_0 \rightarrow P' = P_0 - \frac{1}{2}\rho_{air}v^2$$

For the fluid

$$P_{c_{Left}} = P_{c\,RIGHT}$$

$$P_0 + \rho_{oil}gL + \frac{1}{2}\cancel{\rho_{oil}v_{oil}^2}^{\,0} = P' - \rho_w gL + \frac{1}{2}\cancel{\rho_w v_{water}^2}^{\,0}$$

$$\cancel{P_0} + \frac{3}{4}\rho_w gL = \cancel{P_0} - \frac{1}{2}\rho_{air}v^2 + \rho_w gL$$

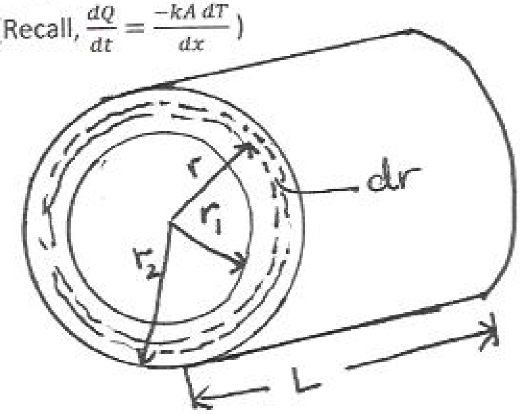
$$\rho_w gL\left(-1 + \frac{3}{4}\right) = -\frac{1}{2}\rho_{air}v^2$$

$$+\frac{1}{4}\rho_w gL = +\frac{1}{2}\rho_{air}v^2$$

$$\boxed{v = \sqrt{v^2} = \sqrt{\frac{\rho_w gL}{2\rho_{air}}}}$$

2. A cylindrical pipe of length L, inner radius r_1, outer radius r_2 and thermal conductivity k conducts heat radially outward at the constant rate H. Both the inner and outer cylinders are maintained constant temperature. Find the temperature difference ΔT. You answer may be in terms of H, r_1, r_2, L and k.

(Recall, $\frac{dQ}{dt} = \frac{-kA\,dT}{dx}$)



$$H = \frac{dQ}{dt} = \frac{-kA\,dT}{dx}$$

$$\frac{H\,dx}{kA} = -dT \qquad dx \equiv dr$$

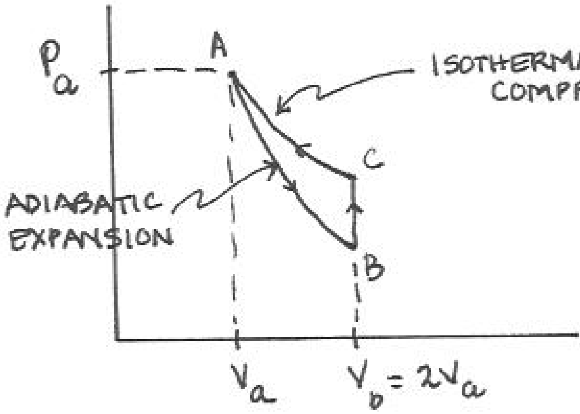
$$A = 2\pi r L$$

$$\int_{r_1}^{r_2} \frac{H\,dr}{k\,2\pi r L} = -\int_{T_{inner}}^{T_{outer}} dT$$

$$\frac{H}{2\pi L k} \ln\frac{r_2}{r_1} = -(T_{outer} - T_{inner})$$

$$\boxed{\Delta T = \frac{H}{2\pi L k} \ln\frac{r_1}{r_2}}$$

3. One mole of an ideal monatomic gas undergoes the cycle ABCA: Starting from pressure P_a and volume V_a, it is expanded adiabatically until its volume $V_b = 2V_a$, then it is heated at constant pressure to point C, finally, it is compressed isothermally back to its original state. Find the work done **by** the gas during one cycle. Your answer may be in terms of P_a and V_a (For full credit please start with the first law in the form $Q = \Delta U + W_{by\ sytem}$)



$$P_aV_a = nRT_a \rightarrow T_a = \frac{P_aV_a}{nR}$$

$$P_bV_b = nRT_b$$

$$P_cV_c = nRT_c = nRT_a$$

$$T_aV_a^{\gamma-1} = T_bV_b^{\gamma-1}$$

$$T_b = T_a\left(\frac{V_a}{V_b}\right)^{\gamma-1} = \left(\tfrac{1}{2}\right)^{\gamma-1} T_a$$

$$\cancel{Q}_{AB}^{\,0} = \Delta U_{AB} + W_{AB}$$

$$W_{AB} = -\Delta U_{AB}$$

$$W_{AB} = -nC_v(T_b - T_a)$$

$$W_{AB} = C_v\left(T_a - \left(\tfrac{1}{2}\right)^{0.6} T_a\right) = C_vT_a\left[1 - 2^{-0.6}\right]$$

$$W_{AB} = \frac{3}{2}\cancel{R}\,\frac{P_aV_a}{\cancel{R}}\left[1 - 2^{-0.6}\right]$$

$$W_{AB} = \frac{3}{2}P_aV_a\left[1 - \tfrac{1}{2}^{\gamma-1}\right] = 3P_aV_a\left[\tfrac{1}{2}^{\gamma} - \tfrac{1}{2}\right]$$

$$W_{BC} = 0 \qquad \Delta V = 0 \Rightarrow W_{BC} = 0$$

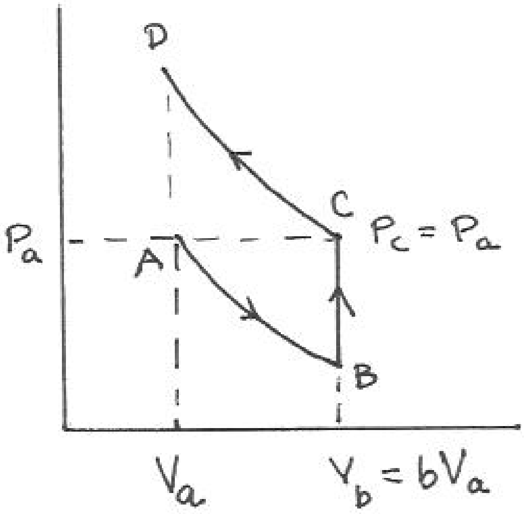
$$Q_{CA} = \cancel{\Delta U}_{CA}^{\,0} + W_{CA} \qquad \Delta T = 0 \Rightarrow \Delta U = 0$$

$$W_{CA} = \int_C^A P\,dV = nRT_a\int_{V_c}^{V_a}\frac{dV}{V} = \frac{\cancel{n}R P_aV_a}{\cancel{nR}}\left[\ln\frac{V_a}{V_b}\right]$$

$$W_{CA} = -P_aV_a \ln 2$$

$$\boxed{W_{TOT} = W_{AB} + W_{BC} + W_{CA} = \bar{3}P_aV_a\left(\frac{\ln 2}{3} + \tfrac{1}{2}^{\gamma} - \tfrac{1}{2}\right)}$$

4. One mole of an ideal monatomic gas undergoes the three processes ABCD as shown: Starting from pressure P_a and volume V_a, it expands isothermally to point B with the volume $V_b = b\,V_a$ (b is a positive constant), then it is heated at constant volume until the pressure is $P_c = P_a$. It then contracts isothermally, to volume $V_d = V_a$. Find the entropy change for this gas. Your answer may be in terms of b and R.



$$P_a V_a = nRT_a$$

$$P_B V_B = nRT_b = nRT_a = P_b \cdot bV_a \quad *$$

$$P_C V_C = nRT_c = P_a bV_a = nRT_c \quad **$$

$$P_D V_D = nRT_d \Rightarrow P_D V_a = nRT_c \quad ***$$

$$* \quad P_b bV_a = P_a V_a$$

$$P_b = \frac{P_a}{b}$$

$$** \quad \frac{nRT_c}{b} = P_a V_a = nRT_a$$

$$T_c = bT_a$$

$$*** \quad P_D V_a = nRbT_a$$

$$P_D = bP_a$$

$$\Delta S = \int dS = \int_A^D \frac{dQ}{T}$$

Any path will do: $dQ = dU + dW$

$$\Delta S = \int_A^D \frac{nC_v\, dT}{T} + \cancel{\int_A^D \frac{P\,dV}{T}}^{\,0}$$

$$\Delta S = \frac{3}{2} R \ln\left(\frac{T_D}{T_a}\right)$$

$$\Delta S = \frac{3}{2} R \ln\left(\frac{bT_a}{T_a}\right)$$

$$\boxed{\Delta S = \frac{3}{2} R \ln b}$$