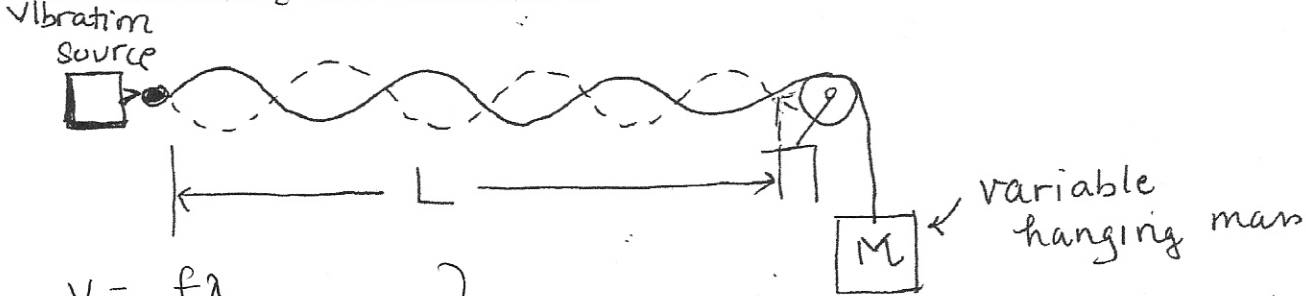
4C Spring 2019 Exam 2

Name SOLUTION

Show all your work for full credit. No calculators, note cards, electronic devices or scratch papers are allowed. You have one (1) hour.

1. In the arrangement shown, an object can be hung from a string with a linear mass density μ with passes over a light pulley. The string is connected to a vibration source of constant frequency f. The length between the pulley and the vibrating source is L and does not change. Standing waves are observed when the hanging mass is $M = 4m_o$ or when $M = 9m_o$, but no standing waves are observed for any masses in between those values. What is the largest mass for which standing waves can be observed?



$$\left.\begin{aligned} V &= f\lambda \\ V &= \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{Mg}{\mu}} \\ \lambda_n &= \frac{2L}{n} \end{aligned}\right\}$$

Here f is fixed and V and λ change. You are given the masses corresponding to successive resonances. Thus

$$\left. f = \frac{V_n}{\lambda_n} = \frac{V_{n+1}}{\lambda_{n+1}} \right\}$$

since $\lambda_n = \frac{2L}{n}$, $\lambda_{n+1} = \frac{2L}{n+1}$; $\lambda_n > \lambda_{n+1}$, $V_n > V_{n+1}$

$$V_n \cdot \lambda_{n+1} = V_{n+1} \cdot \lambda_n$$

$$\sqrt{\frac{9m_o g}{\cancel{\mu}}} \cdot \frac{\cancel{2L}}{n+1} = \sqrt{\frac{4m_o g}{\cancel{\mu}}} \cdot \frac{\cancel{2L}}{n}$$

$$\frac{3}{n+1} = \frac{2}{n}$$

$$3n = 2(n+1)$$

$$3n - 2n = 2$$

$$n = 2$$

Thus, given are the second & third harmonic.

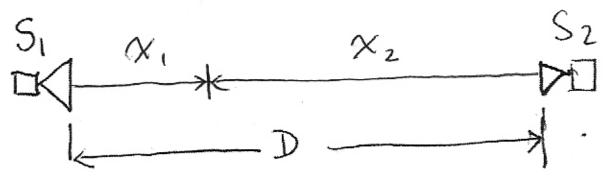
$$\frac{V_1}{\lambda_1} = \frac{\sqrt{\frac{M'g}{\mu}}}{2L} = \frac{V_2}{\lambda_2} = \frac{\sqrt{\frac{9m_o g}{\mu}}}{\frac{2L}{2}}$$

$$\sqrt{\frac{M'\cancel{g}}{\cancel{\mu}}} \cdot \frac{\cancel{2L}}{2} = \cancel{2L}\sqrt{\frac{9m_o \cancel{g}}{\cancel{\mu}}}$$

$$\left(\sqrt{M'} = 2 \cdot 3\sqrt{m_o}\right)^2$$

$$\boxed{M' = 36\, m_o}$$

2. Two sources of sound are separated by a distance *D*. They both emit sound at the same amplitude, *A*, and frequency, *f*, but are 180° out of phase. At what points along the line between them will the sound intensity be at a relative minimum due to destructive interference? (Recall that *sin a* + *sin b* = 2 *cos* ½ (*a* – *b*)*sin* ½ (*a* + *b*). You may assume that *D* >> λ).



Method 2:

Destructive interference occurs at D/2. When you move by amount x, the path changes by amount 2x. Minima occur when

$$2x = m\lambda \quad \text{or} \quad \boxed{x = \frac{m\lambda}{2}}$$

compare with Δx below

Method 1:

$$P_1 = P_0 \sin(kx_1 - \omega t)$$

$$P_2 = P_0 \sin(kx_2 - \omega t + \pi)$$

$$P = P_1 + P_2$$

$$= 2P_0 \cos\tfrac{1}{2}\left(kx_1 - \omega t - (kx_2 - \omega t + \pi)\right)\sin\tfrac{1}{2}\left(kx_1 - \omega t + kx_2 - \omega t + \pi\right)$$

$$= 2P_0 \underbrace{\cos\tfrac{1}{2}\left(k(x_1 - x_2) + \pi\right)}_{\text{Amplitude}} \underbrace{\sin\tfrac{1}{2}\left(k(x_1 + x_2) - 2\omega t + \pi\right)}_{\text{Time varying}}$$

$$\cos\tfrac{1}{2}\left(k(x_1 - x_2) + \pi\right) = 0$$

when

$$\left(k(x_1 - x_2) + \pi\right) = m\pi$$

$$\frac{2\cancel{\pi}}{\lambda}\Delta x = (m-1)\cancel{\pi}$$

$$\Delta x = \frac{(m-1)\lambda}{2}$$

But $x_2 = D - x_1$

$$\Delta x = x_1 - (D - x_1) = 2x_1 - D$$

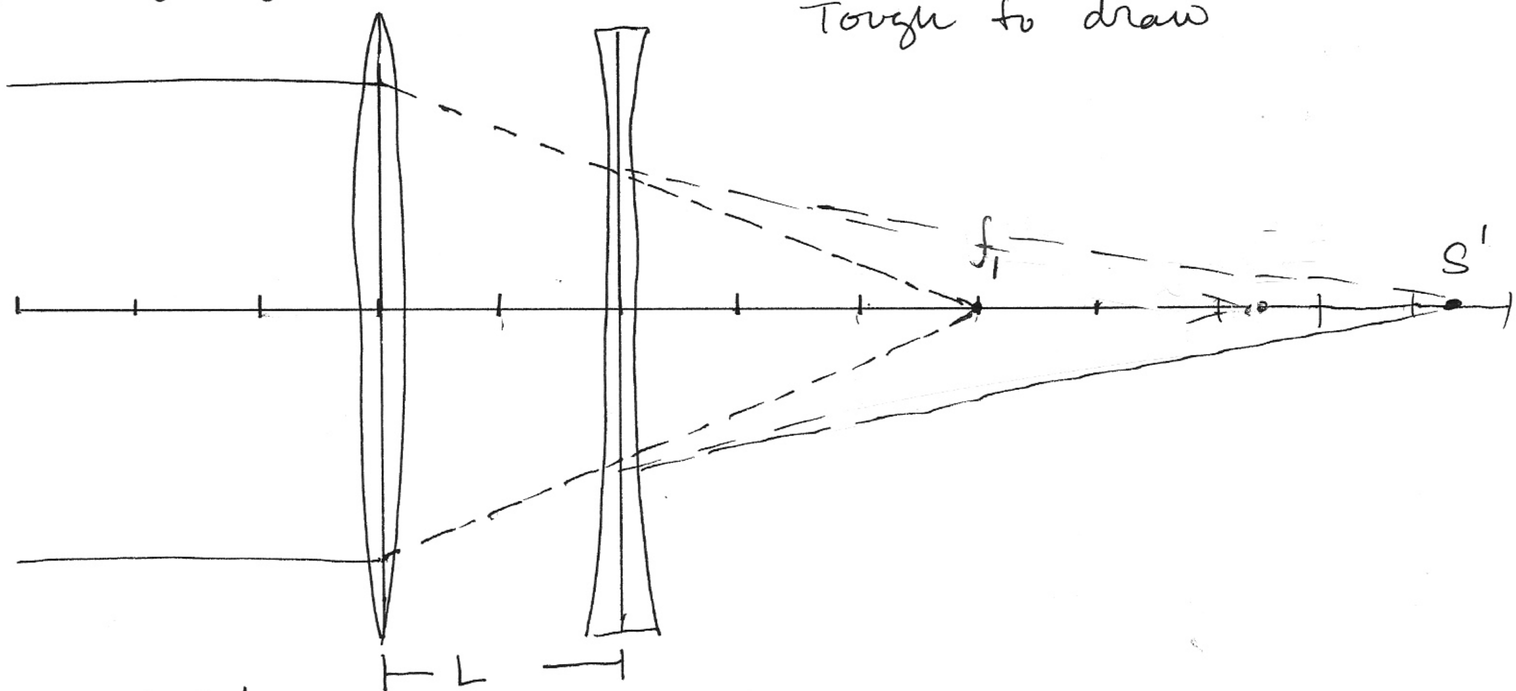
so when x_1

$$2x_1 - D = \frac{(m-1)\lambda}{2}$$

$$\boxed{x_1 = \frac{(m-1)\lambda}{4} + \frac{D}{2}}$$

3. Parallel light is incident on a two lens system, consisting of a converging lens of focal length f followed by a diverging lens of focal length, $-f$, separated by distance L, with $L < f$. Where is the light brought into focus?

Tough to draw



Lens 1

$$\frac{1}{S} + \frac{1}{S_1'} = \frac{1}{f_1}$$

$$S = \infty$$

$$\frac{1}{S_1'} = \frac{1}{f_1} \rightarrow S_1' = f$$

Lens 2

$$L - S_1' = S_2$$

$$L - f = S_2$$

$$\frac{1}{L-f} + \frac{1}{S'} = \frac{1}{-f}$$

$$\frac{1}{S'} = \frac{-1}{f} - \frac{1}{L-f}$$

$$\frac{1}{S'} = -\left(\frac{1}{f} + \frac{1}{L-f}\right)$$

$$\frac{1}{S'} = -\left(\frac{L - \cancel{f} + \cancel{f}}{f(L-f)}\right)$$

$$\boxed{S' = \frac{f(f-L)}{L}}$$

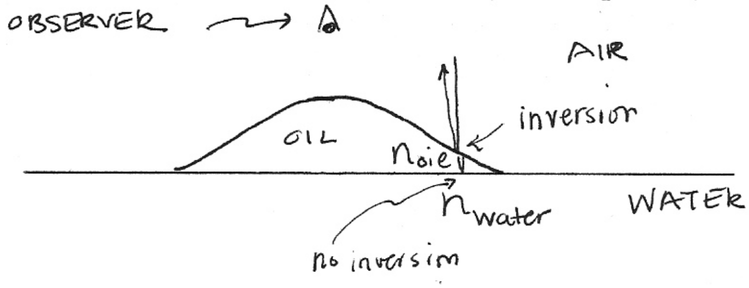
beyond Lens 2 (+)

here $L = 2d$, $f = 5d$

$$S' = \frac{5\cancel{d}(5d - 2d)}{2\cancel{d}}$$

$$S' = \frac{5(3d)}{2} = \frac{15d}{2}$$

4. An oil drop of index of refraction n_{oil} is floating on top of water with an index of refraction n_{water}, where $n_{water} < n_{oil}$. What is the thickness of the film when the third band of the blue light is observed as counted from the outside (thinnest portion) of the drop?



Condition for constructive with an odd # of phase shifts:

$$2t = \left(m + \tfrac{1}{2}\right)\lambda_n$$

1ST $\quad \left(\frac{1}{2}\right)\frac{\lambda_{blue}}{n} = 2t$

2ND $\quad \left(1 + \frac{1}{2}\right)\frac{\lambda_{blue}}{n} = 2t$

3RD $\quad \frac{2 + \frac{1}{2}\,\lambda_{blue}}{2n} = t \quad \rightarrow \quad \boxed{t = \frac{5\lambda}{4n}_{blue}}$

At the outermost (thinnest) edge of the drop, is the region dark or bright when viewed by reflection?

Since there are an odd number of phase changes, the outermost edge of the drop appears dark