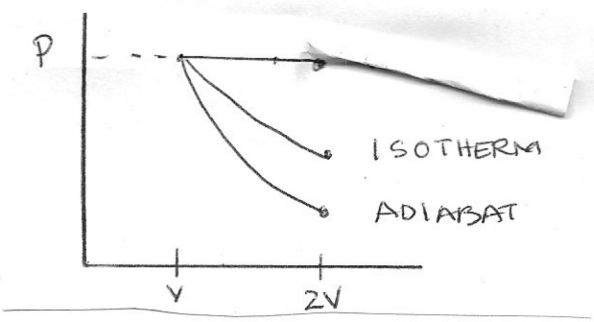
4C Homework Set 4 – The Second Law

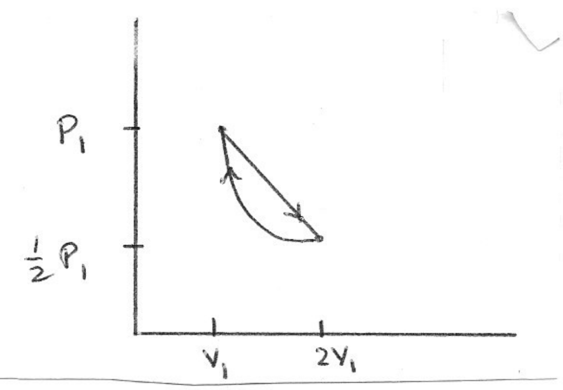
1. Using the equation of state of an ideal gas and the equation describing an adiabatic process for an ideal gas, show that the slope dp/dV on a p-V diagram can be written as $-\gamma p/V$ and of an isothermal can be written as $-p/V$. From these results prove that adiabatics are steeper curves than isothermals.

2. A mole of a monatomic ideal gas is taken from an initial state of pressure P and volume V to a final state of pressure $2P$ and volume $2V$ by two different processes: (1) It expands isothermally until its volume is doubled, and then its pressure is increased at constant volume to the final state; (2) It is compressed isothermally until its pressure is doubled, and then its volume is increased at constant pressure to the final state. Show the path of each process on a P-V diagram. For each process calculate in terms of p and V (a) the heat absorbed by the gas in each part of the process: (b) the work done by the gas in each part of the process (c) the change in internal energy of the gas $U_f - U_i$ (d) the change in entropy of the gas $S_f - S_i$.

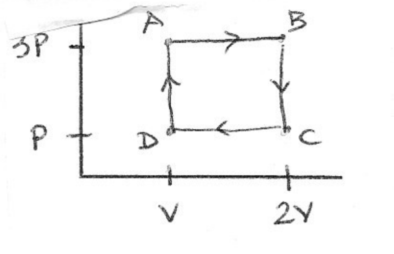
3. An ideal gas is expands to twice its original volume by three processes as shown. An isobaric expansion, an isothermal expansion, and an adiabatic expansion. Compute the change in entropy for each process.



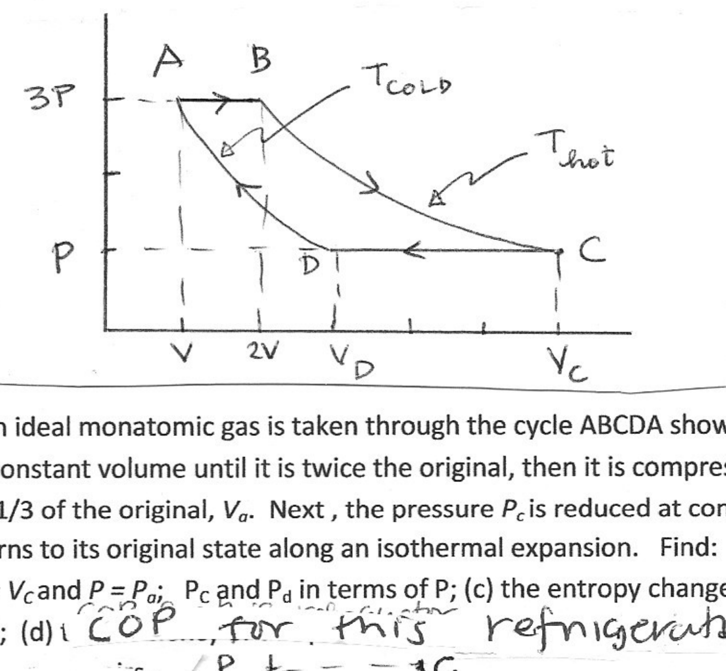
4. Consider the two-cycle reversible processes shown in the figure for an ideal monatomic gas: an expansion along a straight-line path from the initial state (p_1, V_1, T_1) to the *state* $(\frac{1}{2}p_1, 2V_1, T_1)$ followed by an isothermal compression back to the initial state. What is the highest temperature the gas reaches during this closed cycle? (b) Through what part of the straight-line path is heat entering the gas? (c) What is the thermal efficiency of this reversible cycle? (d) What would be the efficiency of a Carnot engine working between the same temperature extremes?



5. In the P-V diagram shown, one mole of an ideal monatomic gas is taken though the cycle ABCDA. (a) compute the entropy change for path AB and path CD. (b) compute the efficiency of this engine. (c) find the efficiency of a Carnot engine working between these temperature extremes.



6. One mole of an ideal monatomic gas is taken through the cycle ABCDA shown. First, the gas expands at constant pressure until the volume is doubled. Then, it is expanded along an isotherm until the pressure is 1/3 the original pressure, it is then compressed at that new pressure until the volume is V_D. Finally, it returns to the original state along an isothermal compression. In terms of the pressure $P = P_c$ along the *CD* process and the original volume $V = V_a$, find (a) the temperatures along the two isotherms T_{hot} and T_{cold}, (b) the volumes V_c and V_D, (c) the entropy change of the system from point A to point C; (d) the efficiency of this process; and (e) the efficiency of a Carnot engine working between these two reservoirs.



7. One mole of an ideal monatomic gas is taken through the cycle ABCDA shown. First the *pressure* P_a = *P* is increased at constant volume until it is twice the original, then it is compressed isothermally until the volume V_c is 1/3 of the original, V_a. Next, the pressure P_c is reduced at constant volume and $P_d = ½\, P_c$. Finally, it returns to its original state along an isothermal expansion. Find: (a) T_{hot} and T_{cold} in terms of the volume $V = V_c$ and $P = P_a$; P_C and P_d in terms of P; (c) the entropy change of the system from point A to point C; (d) COP, for this refrigerator