Name: Friday May, 23rd

Id:

EXAM #2

MATH1B

1) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curves $y = 4(x-2)^2$, $y = x^2 - 4x + 7$, about the y-axis. Sketch the region and a typical shell. (20 Marks)

$4(x-2)^2 = x^2 - 4x + 7$

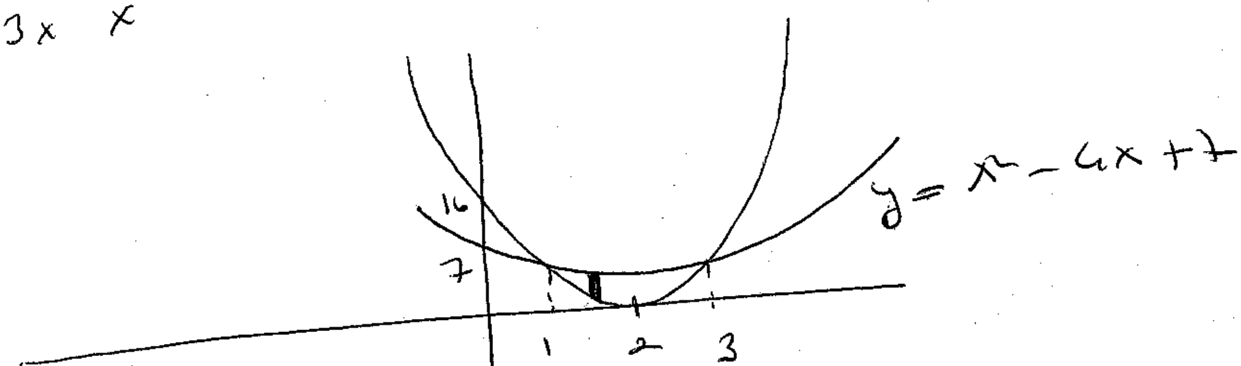
$4(x^2 - 4x + 4) = x^2 - 4x + 7$

$4x^2 - 16x + 16 = x^2 - 4x + 7$

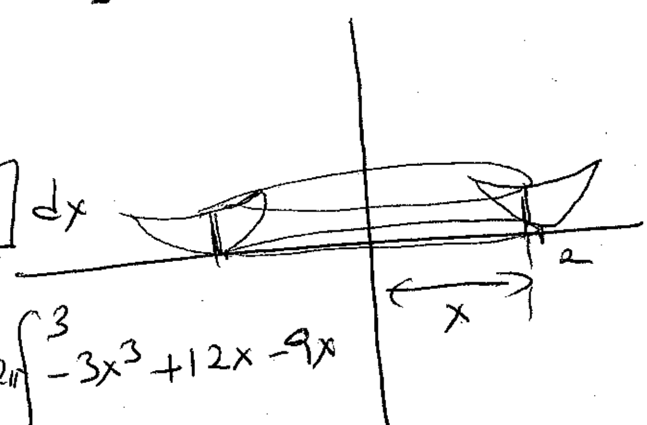
$3x^2 - 12x + 9 = 0 \quad (3x-3)(x-3) = 0$

$3x \quad x \qquad -3 \quad -3$

$\boxed{x = 1} \quad \boxed{x = 3}$



$$V = 2\pi \int_1^3 x\left[(x^2 - 4x + 7) - 4(x-2)^2\right] dx$$



$$= 2\pi \int_1^3 x(-3x^2 + 12x - 9)\, dx = 2\pi \int_1^3 -3x^3 + 12x - 9x$$

$$= 2\pi(-3) \int_1^3 (x^3 - 4x^2 + 3x)\, dx = -6\pi \left[\tfrac{1}{4}x^4 - \tfrac{4}{3}x^3 + \tfrac{3}{2}x^2\right]_1^3$$

$$= 16\pi$$

2) $\int \sec^4\theta \tan^4\theta d\theta$ (20 Marks)

$(\tan^2\theta+1)=\sec^2\theta$

$u=\tan\theta$

$du=\sec^2\theta d\theta$

$$\int (\tan^2\theta+1)^{\cancel{4}}\tan^4\theta \sec^2\theta d\theta$$

$$=\int (u^2+1)u^4\,du = \int (u^6+u^4)\,du$$

$$=\frac{1}{7}u^7+\frac{1}{5}u^5+C$$

$$=\frac{1}{7}\tan^7 x+\frac{1}{5}\tan^5 x+C$$

3) $\int \ln(2x+1)dx$ (20 Marks)

$u=\ln 2x+1$ | $dv=dx$

$du=\frac{2}{2x+1}dx$ | $v=x$

$$\int \ln(2x+1)dx = x\ln(2x+1)-\int\frac{2x\,dx}{2x+1}$$

$$=x\ln(2x+1)-\int\frac{2x+1-1}{2x+1}dx$$

$$=x\ln(2x+1)-\int\left(1-\frac{1}{2x+1}\right)dx$$

$$=x\ln(2x+1|-x+\frac{1}{2}\ln|2x+1|+C$$

4) $\int \frac{1}{(x+5)^2(x-1)}dx = \int\left(\frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{x-1}\right)dx$ (20 Marks)

$\Rightarrow \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{x-1} = \frac{1}{(x+5)^2(x-1)}$

$\Rightarrow A(x+5)(x-1) + B(x-1) + C(x+5)^2 = 1$

For $x = 1$ $36C = 1 \Rightarrow C = \frac{1}{36}$

For $x = 5$ $-6B = 1 \Rightarrow B = -1/6$

For $x = -2$ $1 = 9A - 3B + 9C$

$\Rightarrow A = -\frac{1}{36}$

$= \int\left[-\frac{1}{36}\frac{1}{x+5} - \frac{1}{6}\frac{1}{(x+5)^2} + \frac{1}{36}\frac{1}{x-1}\right]dx$

$= -\frac{1}{36}\ln|x+5| + \frac{1}{6}\frac{1}{x+5} + \frac{1}{36}\ln|x-1| + C$

5) $\int \frac{t^5}{\sqrt{t^2+4}}dt$ Bonus: (20 Marks)

$t = a\tan\theta$, $t = 2\tan\theta$ | $dt = 2\sec^2\theta\, d\theta$, $t^5 = 32\tan^5\theta$ | $\sqrt{t^2+4} = \sqrt{4\tan^2\theta+4}$

$= 2|\sec\theta| = 2\sec\theta$ since $\theta \in [-\pi/2, \pi/2]$

$\int \frac{t^5}{\sqrt{t^2+4}}dt = \int \frac{32\tan^5\theta}{\cancel{2}\cancel{\sec\theta}}\cancel{2}\,\overset{\sec\theta}{\cancel{\sec^2\theta}}\,d\theta$

$= 32\int \tan^4\theta \sec\theta\tan\theta\, d\theta$

since $\tan^2\theta + 1 = \sec^2\theta$

$= 32\int(\sec^2\theta - 1)^2\sec\theta\tan\theta\, d\theta$

let $u = \sec\theta \Rightarrow du = \sec\theta\tan\theta\, d\theta$

$= 32\int(u^2-1)^2\, du$

$= 32\int(u^4 - 2u^2 + \ )\, du$

$= 32\left[\frac{u^5}{5} - \frac{2}{3}u^3 + u\right] + C$

$= 32\left[\frac{1}{5}\sec^5\theta - \frac{2}{3}\sec^3\theta + \sec\theta\right] + C$

$\Rightarrow$ since $\sec\theta = \frac{\sqrt{t^2+4}}{2}$

$= 32\left[\frac{1}{5}\left(\frac{\sqrt{t^2+4}}{2}\right)^5 - \frac{2}{3}\left(\frac{\sqrt{t^2+4}}{2}\right)^3 + \left(\frac{\sqrt{t^2+4}}{2}\right)\right] + C$

$= \frac{1}{5}\left(\sqrt{t^2+4}\right)^5 - \frac{8}{3}\left(\sqrt{t^2+4}\right)^3 + 16\sqrt{t^2+4}$

$+ C$