

Pop Quiz #3

$$\textcircled{1} \int \tan^2 x \sec x \, dx$$

$$= \int (\sec^2 x - 1) \sec x \, dx$$

$$= \int (\sec^3 x - \sec x) \, dx$$

$$I = \int \sec^3 x \, dx = \int \underbrace{\sec x}_u \cdot \underbrace{\sec^2 x}_{du} \, dx$$

$$u = \sec x$$

$$du = \sec x \tan x \, dx$$

$$du = \sec^2 x \, dx$$

$$u = \tan x$$

$$I = \sec x \tan x - \int \tan x \cdot \sec x \tan x \, dx$$

$$I = \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$I = \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$$

$$I = \sec x \tan x - \underbrace{\int \sec^3 x \, dx}_I + \int \sec x \, dx$$

$$2I = \sec x \tan x + \ln |\sec x + \tan x|$$

$$I = \frac{1}{2} (\sec x \tan x + \ln |\sec x + \tan x|) + C$$

Therefore,

$$\begin{aligned} \int (\sec^3 x - \sec x) \, dx &= \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C_1 \\ &\quad - \ln |\sec x + \tan x| + C_2 \quad \text{let } K = C_1 + C_2 \\ &= \frac{1}{2} \sec x \tan x - \frac{1}{2} \ln |\sec x + \tan x| + K \end{aligned}$$