Name: Solutions Class: ________________ Date: __________ ID: A

Quiz#1

Multiple Choice
Identify the choice that best completes the statement or answers the question.

$0 \quad \pi/20 \quad 2\pi/20 \quad 3\pi/20 \quad 4\pi/20 \quad 5\pi/20 \quad 6\pi/20 \quad 7\pi/20 \quad 8\pi/20 \quad 9\pi/20 \quad 10\pi/20$

E 1. Approximate the area under the curve $y = \sin x$ from 0 to $\frac{\pi}{2}$ using ten approximating rectangles of equal widths and right endpoints. The choices are rounded to the nearest hundredth.

a. 0.36
b. 0.02
c. 0.72
d. 0.98
e. 1.08

$$R_{10} = \sum_{i=1}^{10} f(x_i)\Delta x \longrightarrow \Delta x = \frac{\pi/2 - 0}{10} = \frac{\pi}{20}$$

$$R_{10} = \left[\sin\left(\frac{\pi}{20}\right) + \sin\left(\pi/10\right) + \sin\left(\frac{3\pi}{20}\right) + \cdots + \sin\left(\frac{\pi}{2}\right)\right] \cdot \frac{\pi}{20}$$

$$\approx 1.08$$

D 2. Use the Midpoint Rule with $n = 10$ to approximate the integral.

$$\int_1^2 \sqrt{4+t^2}\,dt$$

a. 7.510716
b. 1.510716
c. 12.510716
d. 2.510716
e. 10.510716

$$M_{10} = f(\bar{t}_i)\Delta t \rightarrow \Delta t = \frac{2-1}{10} = \frac{1}{10}$$

$$\bar{t}_i = \frac{1}{2}(t_{i-1} + t_i)$$

$$\int_1^2 \sqrt{4+t^2}\,dt \approx \sum_{i=1}^{10} f(\bar{t}_i)\Delta t$$

1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2

1.05 1.15 1.25 1.35 1.45 1.55 1.65 1.75 1.85 1.95

$$\sum_{i=1}^{10} f(\bar{t}_i)\Delta t = \left[f(1.05) + f(1.15) + f(1.25) + \cdots + f(1.95)\right] \cdot \frac{1}{10}$$

$$\approx 2.510716$$

Name: ____________________ ID: A

E 3. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\,dz$$

$u = \sqrt{x} \rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$h'(x) = \frac{dh}{dx} = \frac{dh}{du} \cdot \frac{du}{dx}$

a. $\frac{\sqrt{x+1}}{x^2+2}$

b. $\frac{\sqrt{x^2+1}}{2}$

$\Rightarrow h'(x) = \frac{d}{du}\left(\int_1^u \frac{z^2}{z^4+1}\,dz\right) \cdot \frac{du}{dx}$

c. none of these

d. $\frac{\sqrt{x}}{x^2+1}$

$= \frac{u^2}{u^4+1} \cdot \frac{du}{dx}$

e. $\frac{\sqrt{x}}{2(x^2+1)}$

Since $u = \sqrt{x}$

$= \frac{(\sqrt{x})^2}{(\sqrt{x})^4+1} \cdot \frac{1}{2\sqrt{x}} = \frac{x}{(x^2+1)\,2\sqrt{x}}$

$= \frac{\sqrt{x}}{2(x^2+1)}$

E 4. An animal population is increasing at a rate of $16 + 51t$ per year (where t is measured in years). By how much does the animal population increase between the fourth and tenth years?

Population $= n(t) \rightarrow$ (rate of change in the population with respect to time t) $= n'(t)$

By the "Net Change Theorem" the increase in the population between the 4th and tenth years:

$\int_4^{10} 16 + 51t = 16t + 51\frac{t^2}{2}\Big|_4^{10} = 2238$

a. 2248
b. 2288
c. 2338
d. 2258
e. 2238

E 5. The velocity function (in meters per second) is given for a particle moving along a line. Find the distance traveled by the particle during the given time interval.

$v(t) = 8t - 8,\ 0 \le t \le 5$

distance traveled: $\int_0^5 |v(t)|\,dt$

a. 36 m
b. 72 m
c. 100 m
d. 64 m
e. 68 m

$|v(t)| = \begin{cases} -v, & \text{if } t < 1 \\ v, & \text{if } t \ge 1 \end{cases}$

$\Rightarrow \int_0^5 |v(t)|\,dt = \int_0^1 -v(t)\,dt + \int_1^5 v(t)\,dt$

$= \int_0^1 (-8t+8)\,dt + \int_1^5 (8t-8)\,dt = \left(-\frac{8t^2}{2}+8t\right)\Big|_0^1 + \left(\frac{8t^2}{2}-8t\right)\Big|_1^5$

$= 68$ meters.

Name: ______________________ ID: A

Numeric Response

1. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find a lower estimate for the distance that she traveled during these three seconds.

t(s)	0	0.5	1.0	1.5	2.0	2.5	3.0
(ft/s)	0	2.8	3.5	6.9	8.2	12.2	16.3

2. Find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

$f(x) = \sqrt{\tan x},\ 0 \le x \le \pi$

3. Find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

$f(x) = x^2 + \sqrt{1+2x},\ \ 4 \le x \le 7$

① since $v(t)$ is increasing L_6 will give us the lower estimate.

$$L_6 = (0.5)\cdot 0 + (0.5)\cdot 2.8 + (0.5)\cdot 3.5 + (0.5)\,6.9 + (0.5)\,8.2 + (0.5)\cdot 12.2 = 16.8$$

$\Delta x = \frac{\pi - 0}{n} = \frac{\pi}{n}$

$x_i^* = 0 + i\frac{\pi}{n}$

② $$\int_0^{\pi} \sqrt{\tan x}\,dx = \lim_{n\to\infty}\left[\sum_{i=1}^{n} \sqrt{\tan\left(\frac{\pi i}{n}\right)}\right]\cdot\frac{\pi}{n}$$

by using definition of a definite integral.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$$ where $\Delta x = \frac{b-a}{n}$, $x_i^* = a + i\Delta x$

③ $$\int_4^7 \left[x^2 + \sqrt{1+2x}\right]dx$$

$$= \lim_{n\to\infty}\left[\sum_{i=1}^{n}\left(4+\frac{3i}{n}\right)^2 + \sqrt{1+2\left(4+\frac{3i}{n}\right)}\right]\frac{3}{n}$$

$\Delta x = \frac{7-4}{n} = \frac{3}{n}$

$x_i^* = 4 + \frac{3i}{n}$