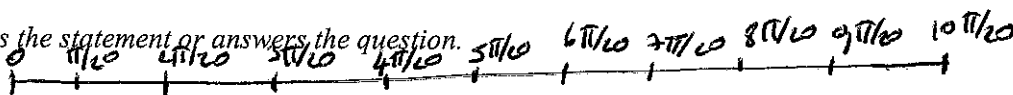


Quiz#1

Multiple Choice

Identify the choice that best completes the statement or answers the question.



- E 1. Approximate the area under the curve $y = \sin x$ from 0 to $\frac{\pi}{2}$ using ten approximating rectangles of equal widths and right endpoints. The choices are rounded to the nearest hundredth.

- a. 0.36
- b. 0.02
- c. 0.72
- d. 0.98
- e. 1.08

$$R_{10} = \sum_{i=1}^{10} f(x_i) \Delta x \rightarrow \Delta x = \frac{\pi/2 - 0}{10} = \frac{\pi}{20}$$

$$R_{10} = \left[\sin\left(\frac{\pi}{20}\right) + \sin\left(\frac{2\pi}{20}\right) + \sin\left(\frac{3\pi}{20}\right) + \dots + \sin\left(\frac{\pi}{2}\right) \right] \cdot \frac{\pi}{20} \approx 1.08$$

- D 2. Use the Midpoint Rule with $n = 10$ to approximate the integral.

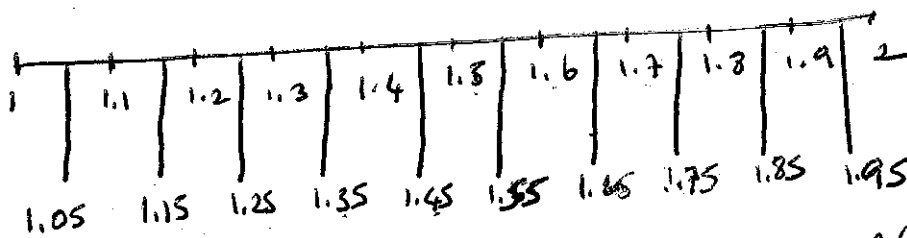
$$\int_1^2 \sqrt{4+t^2} dt$$

$$M_{10} = f(\bar{t}_i) \Delta t \rightarrow \Delta t = \frac{2-1}{10} = \frac{1}{10}$$

- a. 7.510716
- b. 1.510716
- c. 12.510716
- d. 2.510716
- e. 10.510716

$$\bar{t}_i = \frac{1}{2} (t_{i-1} + t_i)$$

$$\int_1^2 \sqrt{4+t^2} dt \approx \sum_{i=1}^{10} f(\bar{t}_i) \Delta t$$



$$\sum_{i=1}^{10} f(\bar{t}_i) \Delta t = \left[f(1.05) + f(1.15) + f(1.25) + \dots + f(1.95) \right] \cdot \frac{1}{10} \approx 2.510716$$

E

3. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1} dz$$

$$u = \sqrt{x} \rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$h'(x) = \frac{dh}{dx} = \frac{dh}{du} \cdot \frac{du}{dx}$$

a. $\frac{\sqrt{x+1}}{x^2+2}$

b. $\frac{\sqrt{x^2+1}}{2}$

c. none of these

d. $\frac{\sqrt{x}}{x^2+1}$

e. $\frac{\sqrt{x}}{2(x^2+1)}$

$$\Rightarrow h'(x) = \frac{d}{du} \left(\int_1^u \frac{z^2}{z^4+1} dz \right) \cdot \frac{du}{dx}$$

$$= \frac{u^2}{u^4+1} \cdot \frac{du}{dx}$$

$$\begin{aligned} \text{Since } u = \sqrt{x} & \\ \frac{(\sqrt{x})^2}{(\sqrt{x})^4+1} \cdot \frac{1}{2\sqrt{x}} &= \frac{x}{(x^2+1)2\sqrt{x}} \\ &= \frac{\sqrt{x}}{2(x^2+1)} \end{aligned}$$

E

4. An animal population is increasing at a rate of
- $16 + 51t$
- per year (where
- t
- is measured in years). By how much does the animal population increase between the fourth and tenth years?

a. 2248

b. 2288

c. 2338

d. 2258

e. 2238

$$\text{population} = n(t) \rightarrow \left(\text{rate of change in the population} \right) = n'(t)$$

(with respect to time t)

By the 'Net Change Theorem' the increase in the population between the 4th and tenth years:

$$\int_4^{10} 16 + 51t \, dt = 16t + \frac{51t^2}{2} \Big|_4^{10} = 2238$$

E

5. The velocity function (in meters per second) is given for a particle moving along a line. Find the distance traveled by the particle during the given time interval.

$$v(t) = 8t - 8, \quad 0 \leq t \leq 5$$

$$\text{distance traveled: } \int_0^5 |v(t)| \, dt$$

a. 36 m

b. 72 m

c. 100 m

d. 64 m

e. 68 m

$$|v(t)| = \begin{cases} -v, & \text{if } t < 1 \\ v, & \text{if } t \geq 1 \end{cases}$$

$$\Rightarrow \int_0^5 |v(t)| \, dt = \int_0^1 -v(t) \, dt + \int_1^5 v(t) \, dt$$

$$\begin{aligned} &= \int_0^1 (-8t + 8) \, dt + \int_1^5 (8t - 8) \, dt = \left(-\frac{8t^2}{2} + 8t \right) \Big|_0^1 + \left(\frac{8t^2}{2} - 8t \right) \Big|_1^5 \\ &= 68 \text{ meters.} \end{aligned}$$

Numeric Response

1. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find a lower estimate for the distance that she traveled during these three seconds.

$t(s)$	0	0.5	1.0	1.5	2.0	2.5	3.0
(ft/s)	0	2.8	3.5	6.9	8.2	12.2	16.3

2. Find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

$$f(x) = \sqrt{\tan x}, \quad 0 \leq x \leq \pi$$

3. Find an expression for the area under the graph of f as a limit. Do not evaluate the limit.

$$f(x) = x^2 + \sqrt{1+2x}, \quad 4 \leq x \leq 7$$

① since $u(t)$ is increasing L_6 will give us the lower estimate.

$$L_6 = (0.5) \cdot 0 + (0.5) \cdot 2.8 + (0.5) \cdot 3.5 + (0.5) \cdot 6.9 + (0.5) \cdot 8.2 + (0.5) \cdot 12.2 = 16.8$$

$$\Delta x = \frac{\pi - 0}{n} = \frac{\pi}{n}$$

$$x_i^* = 0 + i \frac{\pi}{n}$$

$$\int_0^\pi \sqrt{\tan x} \, dx = \lim_{n \rightarrow \infty} \left[\sum_{i=1}^n \sqrt{\tan\left(\frac{\pi i}{n}\right)} \cdot \frac{\pi}{n} \right]$$

where $\Delta x = \frac{b-a}{n}$
 $x_i^* = a + i \Delta x$

by using definition of a definite integral.

$$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x$$

③
$$\int_4^7 [x^2 + \sqrt{1+2x}] \, dx =$$

$$= \lim_{n \rightarrow \infty} \left[\sum_{i=1}^n \left(4 + \frac{3i}{n} \right)^2 + \sqrt{1 + 2 \left(4 + \frac{3i}{n} \right)} \right] \frac{3}{n}$$

$$\left| \begin{aligned} \Delta x &= \frac{7-4}{n} \\ &= \frac{3}{n} \\ x_i^* &= 4 + \frac{3i}{n} \end{aligned} \right.$$