

Name:

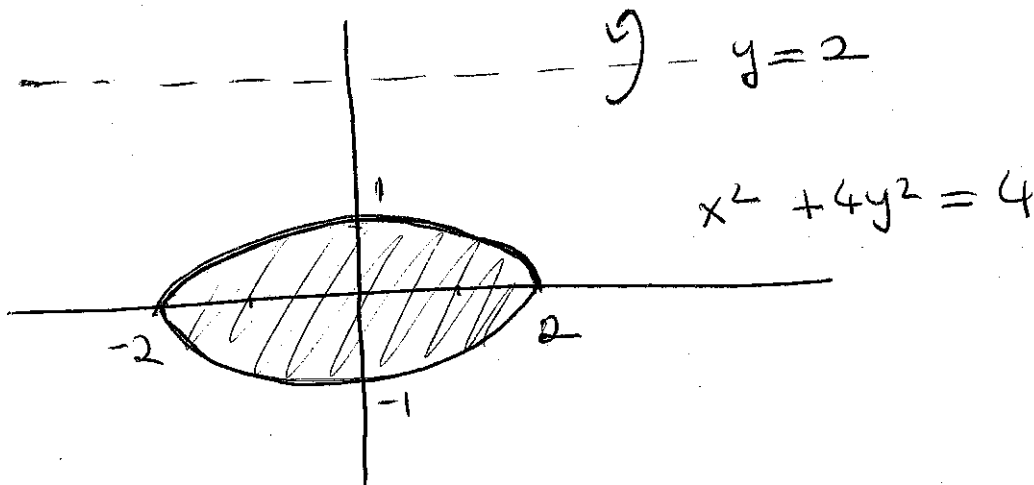
May-09-14

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Quiz#3

Math1B

- 1) Set up an integral for the volume of the solid obtained by rotating the region bounded by the curve  $x^2 + 4y^2 = 4$  about  $y=2$



$$\begin{aligned}x^2 + 4y^2 = 4 &\Rightarrow 4y^2 = 4 - x^2 \\&\Rightarrow y^2 = 1 - \frac{x^2}{4} \Rightarrow y = \pm \sqrt{1 - \frac{x^2}{4}}\end{aligned}$$

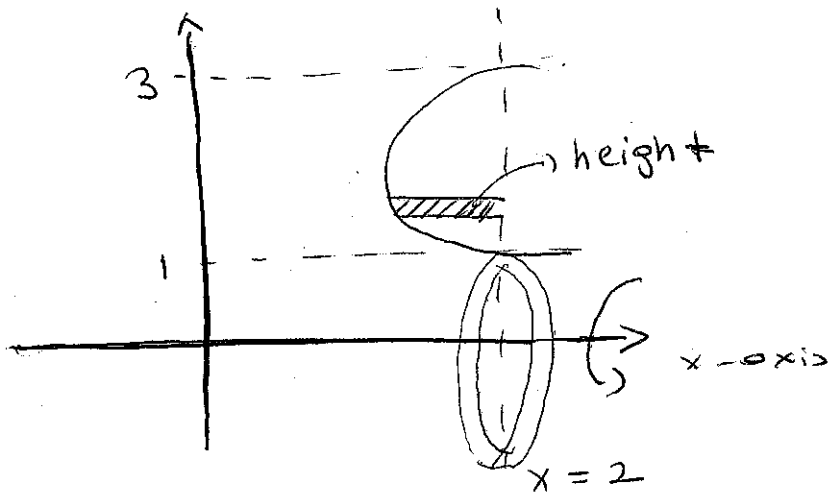
$$V = \int_{-2}^2 \pi \left[ \left( 2 - \left( -\sqrt{1 - \frac{x^2}{4}} \right) \right)^2 - \left( 2 - \sqrt{1 - \frac{x^2}{4}} \right)^2 \right] dx$$

(even function)

$$= 2\pi \int_0^2 8\sqrt{1 - \frac{x^2}{4}} dx = 78.95684$$

- 2) Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curve  $x = 1 + (y-2)^2$  ~~about~~  $x=2$

around  $x$ -axis



Intersection points

$$\begin{aligned}
 1 + (y-2)^2 &= 2 \\
 y^2 - 4y + 4 + 1 - 2 &= 0 \\
 y^2 - 4y + 3 &= 0 \\
 (y-3)(y-1) &= 0 \\
 y &= 3 \quad y = 1
 \end{aligned}$$

The height of the shell is:

$$\begin{aligned}
 2 - [1 + (y-2)^2] &= 1 - (y-2)^2 \\
 &= 1 - (y^2 - 4y + 4) \\
 &= -y^2 + 4y - 3
 \end{aligned}$$

$$\begin{aligned}
 V &= 2\pi \int_1^3 y (-y^2 + 4y - 3) dy \\
 &= 2\pi \int_1^3 (-y^3 + 4y^2 - 3y) dy \\
 &= 2\pi \left[ -\frac{1}{4}y^4 + \frac{4}{3}y^3 - \frac{3}{2}y^2 \right]_1^3 \\
 &= 2\pi \left( \frac{8}{3} \right) = \frac{16\pi}{3}
 \end{aligned}$$