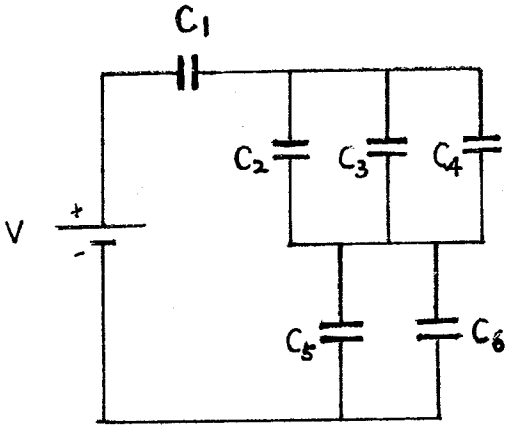
Physics 2B Exam II, Winter **Name:** Solution

1. (25 points) In the circuit, V = 12V, $C_1 = C_5 = C_6 = 6.0\ \mu F$, and $C_2 = C_3 = C_4 = 4.0\ \mu F$. Calculate (a) the equivalent capacitance Ceq, and (b) charge q_4 on the capacitor C_4 ?



a) C_2, C_3, C_4 in parallel

$C_{234} = C_2 + C_3 + C_4 = 3 \times 4\,\mu F = 12\,\mu F$ (5)

C_5, C_6 in parallel

$C_{56} = C_5 + C_6 = 2 \times 6\,\mu F = 12\,\mu F$ (5)

C_1, C_{234}, C_{56} in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{234}} + \frac{1}{C_{56}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{12} = \frac{1}{3}$$

$C_{eq} = \underline{3\,\mu F}$ (5)

b) $Q_1 = Q_{234} = Q_{56} = C_{eq} V = 3\,\mu F \cdot 12\,V$

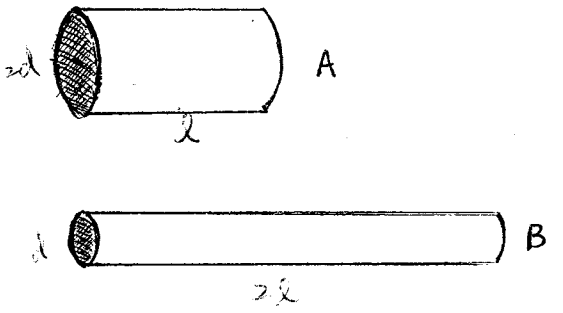
$= 36\,\mu C$ (5)

$C_2 = C_3 = C_4$

$Q_2 = Q_3 = Q_4 = \frac{Q_1}{3} = \frac{36\,\mu C}{3} = \underline{12\,\mu C}$

(5)

2. (25 points) Wire A and wire B are made from the same material. Wire A has twice the diameter and half the length of wire B and a resistance of 10.0 Ω. (a) What is the resistance of wire B? (b) If the wires have the equal currents, what is the ratio of their current-density magnitudes?



$R = \rho \frac{\ell}{A}$

a) $R_A = \rho \frac{\ell_A}{A_A} = \rho \frac{\ell}{\pi (\frac{2d}{2})^2} = \rho \frac{\ell}{\pi d^2}$ (5)

$R_B = \rho \frac{\ell_B}{A_B} = \rho \frac{2\ell}{\pi (\frac{d}{2})^2} = \rho \frac{2\ell}{\pi \frac{d^2}{4}} = \rho \frac{8\ell}{\pi d^2}$ (5)

$$\frac{R_A}{R_B} = \frac{\rho \frac{\ell}{\pi d^2}}{\rho \frac{8\ell}{\pi d^2}} = \frac{1}{8}$$

$\therefore R_B = 8 R_A = 8 \times 10\,\Omega = 80\,\Omega$ (5)

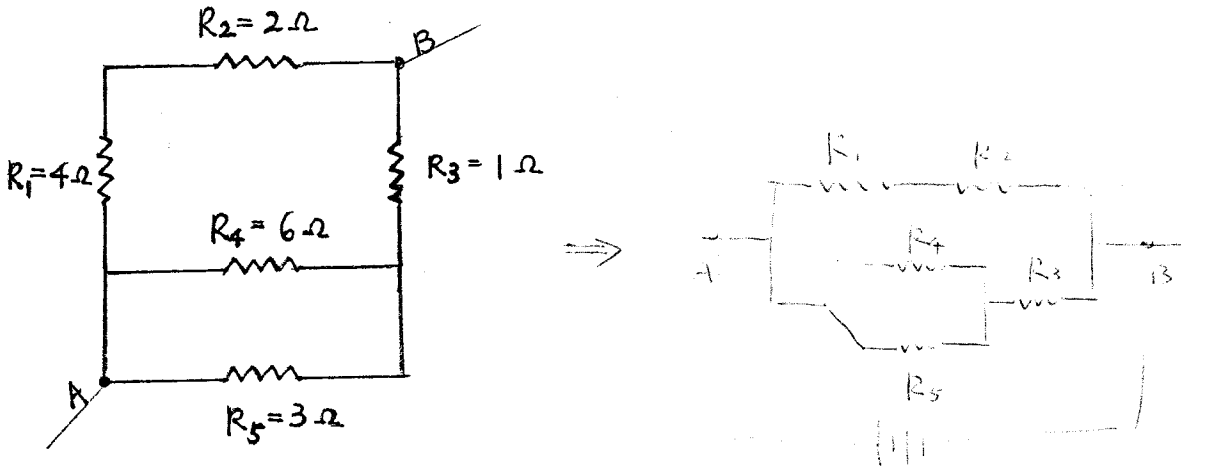
b) $J_A = \frac{I}{A_A} = \frac{I}{\pi d^2}$

$J_B = \frac{I}{A_B} = \frac{I}{\pi \cdot \frac{d^2}{4}} = \frac{4I}{\pi d^2}$ (5)

$$\frac{J_A}{J_B} = \frac{\frac{I}{\pi d^2}}{\frac{4I}{\pi d^2}} = \frac{1}{4}$$ (5)

-2/5

3. (25 points) In the following circuit diagram, (a) what is the equivalent resistance Req between points A and B? (b) If points A and B are connected to a battery emf $\varepsilon = 9$ V, what is the current I_5 in the resistor R_5?



a)

R_1, R_2 in series. $R_{12} = R_1 + R_2 = 4\Omega + 2\Omega = 6\Omega$ (5)

R_4, R_5 in parallel. $\frac{1}{R_{45}} = \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{6\Omega} + \frac{1}{3\Omega} = \frac{3}{6\Omega} = \frac{1}{2\Omega}$ $R_{45} = 2\Omega$

R_{45}, R_3 in series. $R_{345} = R_3 + R_{45} = 1\Omega + 2\Omega = 3\Omega$ (5)

R_{12}, R_{345} in parallel. $\frac{1}{R_{eq}} = \frac{1}{R_{12}} + \frac{1}{R_{345}} = \frac{1}{6\Omega} + \frac{1}{3\Omega} = \frac{1}{2\Omega}$

$R_{eq} = 2\Omega$. (5)

b) $I = \frac{\varepsilon}{R_{eq}} = \frac{9V}{2\Omega} = 4.5\,A$

$I_{12} = \frac{\varepsilon}{R_{12}} = \frac{9V}{6\Omega} = 1.5\,A$ (5)

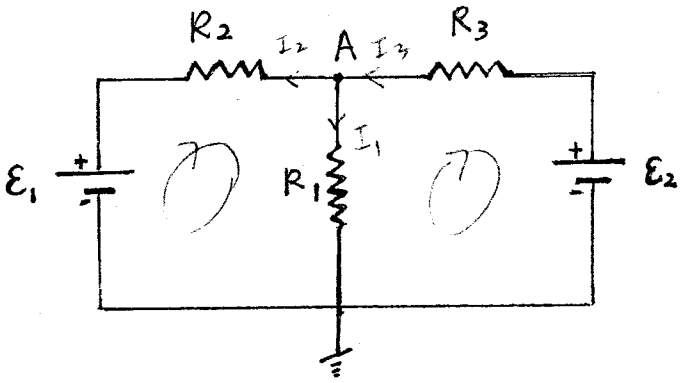
$I_3 = I - I_{12} = 4.5A - 1.5A = 3A$

$V_3 = I_3 R_3 = 3A\ 1\Omega = 3V$

$V_4 = V_5 = 9V - 3V = 6V$. (5)

$I_5 = \frac{V_5}{R_5} = \frac{6V}{3\Omega} = 2A$

4. (25 points) In the multi-loop circuit, ε_1=10V, ε_2=20V, R_1=1 Ω, R_2= R_3=2 Ω. One point of the circuit is grounded (V=0). What are the size and direction of the current through (a) R_1, R_2, and R_3? (b) What is the electric potential Va at point A?



at A $I_3 = I_1 + I_2$ —① (5)

loop $\varepsilon_1 + I_2R_2 - I_1R_1 = 0$ $10 + 2I_2 - I_1 = 0$ —② (5)

t loop $I_1R_1 + I_3R_3 - \varepsilon_2 = 0$ $I_1 + 2I_3 - 20 = 0$ —③ (5)

from ② $2I_2 = I_1 - 10$ $I_2 = \frac{1}{2}I_1 - 5$ —④

into ①

from ③ $2I_3 = 20 - I_1$ $I_3 = 10 - \frac{1}{2}I_1$ —⑤

$10 - \frac{1}{2}I_1 = I_1 + \frac{1}{2}I_1 - 5$

$10 = I_1 + I_1 - 5$

$\therefore I_1 = 15$

$I_1 = 7.5$A direction ↓

④ $I_2 = \frac{1}{2}I_1 - 5 = \frac{1}{2} \times 7.5 - 5 = -1.25$A → (6)

⑤ $I_3 = 10 - \frac{1}{2}I_1 = 10 - \frac{1}{2} \times 7.5 = 6.25$A ←

b) $V_A = 0 + I_1R_1 = 7.5A \times 1\Omega = +7.5$V (5)