Math 46, Winter 2009, Take-home exam Name: _____

Read the following instructions before you begin, and again before you turn in your exam to see if you have followed directions!!

(a) Due at the start of class, 3:45 PM, on Tuesday, March 20, 2009
(b) You are encouraged to work together; however if your work is identical to that of someone else in the class, neither will be counted.
(c) Only work appearing on these official pages will receive credit - do not use extra sheets. Make your answers fit in the space provided, one problem per page (you may use the back if absolutely necessary!)
(d) If asked to explain why your result is correct, do not merely state an answer, give a full explanation in complete sentences.
(e) **For each problem**, list who you worked with and give details as to who did what. For example: "Jane found many examples and spotted the pattern, I solved the problem in general, then we both figured out how to write down an accurate proof." No credit without this information.
(f) You may discuss these problems with others in this class, but not those not in the class.
(g) Do not hand in this cover sheet!

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(1) Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution:

How many different ways are there to replace the stars in the expression

$$* 1 * 3 * 5 * 7 * 9 * 11 * 13 * 15 * 17 * 19$$

with either a plus sign or a minus sign so that the resulting sum equals 60?

(a) (2 pts.) Number of ways: ____

List of ways:

(b) (5 pts.) Explanation that you have found them all:

(c) (3 pts.) What other non-negative sums, other than 60, are possible by some choice of plus or minus signs?

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(2) Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution:

In this problem you will each be given a set of numbers, A, at the class site, with which to play a game like the take-away game we played in class. In the game, two players start with a pile of counters, and take turns choosing a number from set A (repeats are allowed) to remove from the pile. The first person to leave 0 counters wins. For example, if A =- {1,2,3,4}, we saw that when starting with a number that is not divisible by 5, the second player can force a win by always bringing the running total to a multiple of 5. Hint: suppose the numbers in your set were A={1,3}. Then work "backwards" from the lowest numbers to see what are the winning and losing positions. For example, in this case, 1 would be a winning number, since that player could remove 1 counter. 2 is a losing number, since the player with that number can only remove 1, leaving a winning number for her opponent. 3 is a winning number, since that player can remove 3. And 4 is a losing number, since that player can remove 1 or 3, leaving a winning number for her opponent. If you continue with this example, you will begin to see that odd numbers are losing numbers, and even numbers are winning numbers. Your problem will most likely have a more complicated analysis than this though!

List the four numbers in your set A:_____
(a) Find and explain the winning strategy if the game begins with 15 counters in the pile.

(b) Find and explain the winning strategy if the game begins with 35 counters in the pile.

(c) Find and explain the winning strategy if the game begins with any number counters in the pile.

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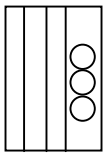
(3) Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution:



There are 200 cells in a prison, labeled 1 to 200, and all in a row. Each cell door has 3 locks on it.

Feeling in a good mood one day, the warden walks through and opens the first lock on every cell's door. The next day he walks through and opens the second lock on every second cell, starting with cell number 2, but leaves the odd-numbered cells alone. The third day he walks through and opens the next lock on every third cell, beginning with cell number 3. (That is, if only the first lock is open, then he opens the second lock. If both locks 1 and 2 are open, he opens the third lock.) The fourth day he does the same for every fourth cell, beginning with cell four, opening the next lock in sequence; however if he finds all three locks already open, then he closes them all!

Then he announces that he is going to continue this practice until day 200, opening the next unopened lock on every nth cell door, beginning with cell n, on day n, or else closing all three locks if he finds all three open, and that any prisoner who can tell him which doors will be open at the end of the 200th day may go free. Which doors will be open on the 200^{th} day? Note that if he returns to a cell after closing all its locks, then he begins again, opening the first lock, then the next time he comes to that cell opening the 2^{nd} lock, and so on.

Explain your answer.

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(4) Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution:

(a) You have unmarked 4 gallon, 10 gallon, and 18 gallon containers and an unlimited supply of water. Using only the containers, which of the amounts from 1 to 18 gallons can you measure out exactly? Explain how to do this for any amounts you believe are possible. Carefully label and explain any charts or notation you use!

(b) Which amounts would you not be able to measure out exactly? Explain.

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(5) Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution:

At the class web site you will be assigned 3 fractions, labeled set B. In the expression x * y * z replace the first * by either • or ÷, and replace the second * by either + or –. You may insert parentheses wherever you like. You must use each of your 3 fractions exactly once (in any order). For example, if your fractions were $\frac{1}{5}, \frac{2}{5}, \text{ and } \frac{3}{5}$, you might answer question (a) by writing $\frac{1}{5}(\frac{3}{5} - \frac{2}{5}) = \frac{1}{25}$.

List your three fractions: _____

(a) Find the smallest positive result: _____

(b) Find the largest positive result: _____

(c) Find the result closest to 1: _____

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For the purposes of questions 6-8, let $a_1, a_2, a_3, \ldots, a_8$ be the digits of your student ID number. **If any digits are 0, replace them with the digit 7.**
Write your ID number, with replacements for zeros, here: _____
(Note that this is NOT your social security number!! Ask the instructor immediately if you do not have yours!!)

(6) Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution:

(a) Let m be the four digit number given by $a_1a_2a_3a_4$, and let n be the three digit number given by $a_5a_6a_7a_8$. Show how to use the Euclidean algorithm to find GCD(m,n)
GCD = _____

(b) Suppose $a_1a_2a_3 \ldots a_8a_1$ are the first nine digits of the ISBN number of a book. Show how to find the check digit:
Check digit = _____

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(7) Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution:

(a) Convert m in problem 5 to base 2: _____

(b) Convert n in problem 5 to base 5: _____

(c) (4) Show how to use "Russian Peasant Multiplication" to multiply $a_5a_6a_7a_8$ by $a_1a_2a_3a_4$

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(8) Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution:

A palindrome is a number that reads the same forwards as backwards. Give the next three palindrome numbers after $a_4a_5a_6a_7$ that are also divisible by 3, and explain your method for finding them. (For example, the *next* palindrome number after 119 is 121, and the next after that is 131, etc. Do not count leading 0's as part of a number.)

Next three: _____

How did you find them?