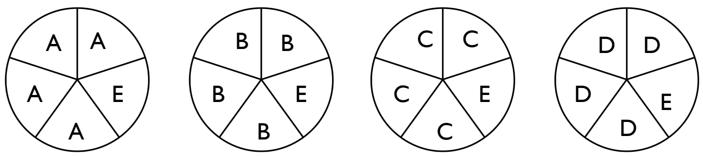
Egyptian Fractions Math 46 Name: ________________

How can we divide 4 pizzas among 5 students so everyone gets the same amount and the same size pieces?

Suppose the students are Allegra, Blair, Christian, Delaram, and Emily.
Since everyone gets 4/5 of a pie, we could divide each into 5 equal sections, and assign them as follows:



Unfortunately everyone except Emily gets contiguous slices; Emily might be unhappy! (Or perhaps the others would prefer one slice from each pizza!?)

We can use the ancient method of **Egyptian fractions** to divide the 4 pizzas so that everyone gets 4/5 of a pizza and *also* gets the same size pieces. The ancient Egyptians only used fractions with numerator 1 (except sometimes they used 2/3, but we will ignore that for now!). These are called ***unit fractions***. They represented all fractions in terms of unit fractions with different denominators.

What is a large unit fraction that is less than 4/5? Let's try 1/2.

Step 1. $\frac{4}{5}=\frac{1}{2}+\frac{?}{?}$

The missing fraction is found by doing a fraction subtraction (do not convert to decimals!)

$$\frac{?}{?}=\frac{4}{5}-\frac{1}{2}=\frac{8}{10}-\frac{5}{10}=\frac{3}{10}$$

Therefore $\frac{4}{5}=\frac{1}{2}+\frac{3}{10}$

We have not converted 4/5 entirely to unit fractions, but notice that 3/10 does have a smaller numerator than 4/5.
Now what is a large unit fraction that is less than 3/10? We might try 1/5, as we did in class:

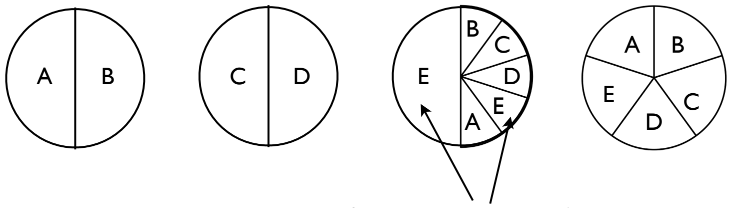
Step 2. $\frac{4}{5}=\frac{1}{2}+\frac{3}{10}=\frac{1}{2}+(\frac{1}{5}+\frac{?}{?})$

Again the missing fraction is found by doing a fraction subtraction of 3/10 – 1/5:

$$\frac{?}{?}=\frac{3}{10}-\frac{1}{5}=\frac{3}{10}-\frac{2}{10}=\frac{1}{10}$$

Therefore $\frac{4}{5}=\frac{1}{2}+\frac{1}{5}+\frac{1}{10}$

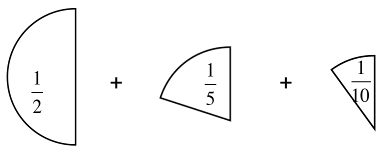
How does this help? Well, notice that giving each person 1/2 +1/5 +1/10 of a pizza insures that he or she gets a total of 4/5. But the sections are also all the same size:



Arranged so that Emily's two slices are not contiguous.

This diagram shows that since $\frac{4}{5} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10}$ therefore also $4 = 5(\frac{1}{2} + \frac{1}{5} + \frac{1}{10})$

So everyone gets pieces that are size 1/2, 1/5, and 1/10, which add up to 4/5.



You might have noticed that in the previous discussion, at step 2, we looked for a unit fraction less than 3/10, and decided to use 1/5. However, there is a unit fraction that is larger than 1/5 and also less than 3/10. Use this fraction to redo this problem. You will find a different solution.

(1) What is the largest unit fraction (numerator equal to 1) that is less than 3/10? _________
(Your answer should NOT have been 1/5!) In the following equation fill in this denominator under the 1.

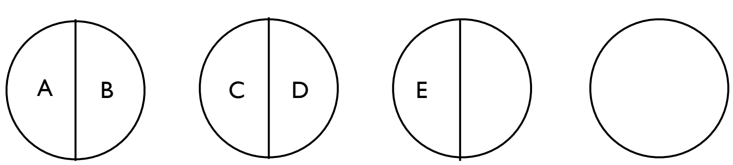
(2) We then need to find the third unit fraction, indicated by the question marks:

$$\frac{3}{10} = \frac{1}{\ } + \frac{?}{?} \quad \text{or} \quad \frac{?}{?} = \frac{3}{10} - \frac{1}{\ } = \frac{\ }{\ }$$

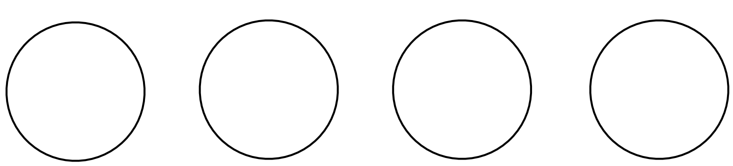
Fill in the first blank space above with the denominator you found in problem (1), and figure out the values of the question marks by doing the subtraction indicated.

$$\text{Therefore } \frac{4}{5} = \frac{1}{2} + \frac{1}{\ } + \frac{1}{\ } \text{ (fill in the denominators you just found.)}$$

Now draw the divisions of the circle that correspond to the denominators you found in problem (2) - we've filled in the halves for you:

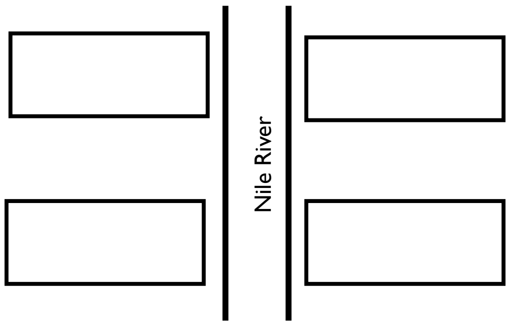


(3) Now try this problem: four pizzas are to be divided equally among nine people. Use the method of Egyptian fractions to find a way to do this so everyone gets the same amount and the same size pieces. Show your division in the following circles and fill in the denominators (this problem can be solved with two unit fractions instead of three.)

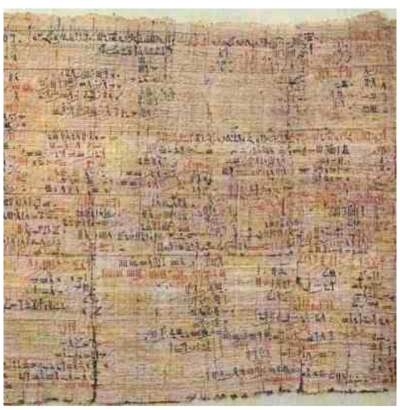


$$\frac{4}{9} = \frac{1}{\ } + \frac{1}{\ }$$

(4) Why did the ancient Egyptians insist on using fractions with numerators equal to 1? Historians do not agree, but here is a possible explanation. Every year the Nile flooded (the need to predict this gave rise to the mathematics involved in creating a calendar) and after the flood waters receded the land next to the river became the most fertile and was farmed. Suppose a family wished to divide it's 4 equal plots of land among the family's five children. How might this have been accomplished so that each child received the same amount of land and the same size pieces? Redraw the divisions below using either the division found on page 1 above, or that you found on page 2:



(5) The method of finding the largest unit fraction at each step was devised by the most famous 12th century CE European mathematician, who was someone we have studied. That mathematician was ________________



The Rhind Papyrus, dating back to 1650 BC, shows unit fraction decompositions of all fractions of the form 2/n, for n from 3 up to 101.

(6) Why are Egyptian fractions important? Fair division problems abound in contemporary society, from inheritance division to figuring out how to divide voters among distircts to deciding how to allocate funds to different competing programs

Recreational mathematics also has many unsolved problems relating to the Egyptian fractions. For example, no one knows whether it is always possible to decompose any fraction into 3 or fewer unit fractions, but it is suspected to be true.

$$\frac{a}{b} = \frac{1}{?} + \frac{1}{?} + \frac{1}{?}$$

It is also unknown whether any odd numerator fraction can be represented with only odd denominator unit fractions. For example, fill in the last denominator with the odd number that makes the equation true:

$$\frac{3}{5} = \frac{1}{3} + \frac{1}{5} + \frac{1}{}$$

In problems (2) and (3) you found decompositions of 4/5 and 4/9 into three or fewer unit fractions. Fill in the denominators below to decompose fractions with numerator 4 into three or fewer unit fractions:

$$\frac{4}{7} = \frac{1}{} + \frac{1}{} \qquad \frac{4}{11} = \frac{1}{} + \frac{1}{} \qquad \frac{4}{13} = \frac{1}{} + \frac{1}{} + \frac{1}{}$$