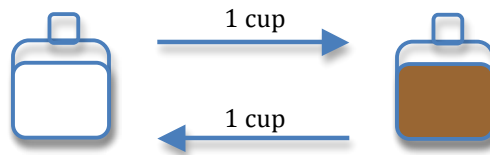


### Prediction Card Trick and Mixing Milk

This handout details two related puzzles.

**(1) Milk and Coffee.** Suppose you have a gallon of milk and a gallon of coffee. You pour one cup of milk into the coffee, mix it, then pour one cup of the mixture back into the milk. Is there now more milk in the coffee or coffee in the milk? Explain.



**(2) Prediction Trick.** The magician shows the audience 6 red and 6 black cards. While someone in the audience shuffles the cards, the magician tells the audience that she is writing a prediction on a sheet of paper, which she then folds and places within the audience's sight. She then has the shuffler deal the cards into two face down piles, one with 4 cards, the other with 8 cards.

She then shows the audience the prediction, which reads "There are two more red cards in the large pile than black cards in the small pile." The audience member turns over the cards and counts them, finding the magician's prediction is correct.

**Question:** how did the magician know what to write down as a prediction?

**Complete** the following chart showing the possible numbers of red and black cards in each pile. Write each number under red in the small pile in the form  $4 - b$ , where  $b$  is the number of black cards in the large pile. Also write the number under Black in the large pile in the form  $6 - b$ , as shown in the first row. Complete the algebraic expressions in the last row also, where  $b =$  the number black in the 4 card pile.

4 card pile		8 card pile	
# black	# red	# Black	# Red
4	$4 - 4 = 0$	$6 - 4 = 2$	$8 - (6 - 4) = 6$
3			
2			
1			
0			
$b$	$4 - b$		

(1) Why is the magician's prediction correct, that the number red in the large pile always equals 2 more than the number black in the small pile?

- (2) Why is the last numerical row of the table that in which number red in the 8 card pile = 2?
- (3) What would the magician write for a prediction about how many more black cards there are in the 8-card pile than red in the 4-card pile? Why?
- (4) What would your prediction be, if as the magician, you separate the 12 cards in the original problem into two stacks of 6 and 6? Why? Try this with actual cards, either to find out the answer, or to see if you are correct.
- (5) What would your prediction be if you separate the 12 cards into two stacks of 7 and 5 cards? Why? Try this!
- (6) What would your prediction be if you start with 9 red cards and 9 black cards, have an audience member shuffle them, and separate them into two piles of 12 and 6 cards? Why? Try this!
- (7) The magician starts with 9 black and 9 red cards. The magician's prediction is that the number of black cards in the large pile is 4 more than the number of red in the small pile. How many were in the large pile and how many in the small pile? Why? Try this!
- (8) The magician's prediction is that the number of black cards in the large pile is 4 more than the number of red in the small pile. Assuming the number of red and black cards are equal, what might those numbers have been, and what might the numbers in the large and small piles have been?
- (9) Solve number 8 algebraically, using variables R for Red in the large pile, B for number in the large pile, and n for number of either red or black cards used.
- (10) How are these questions related to the Milk and Coffee problem? Can you use the cards to model the Milk and Coffee problem?