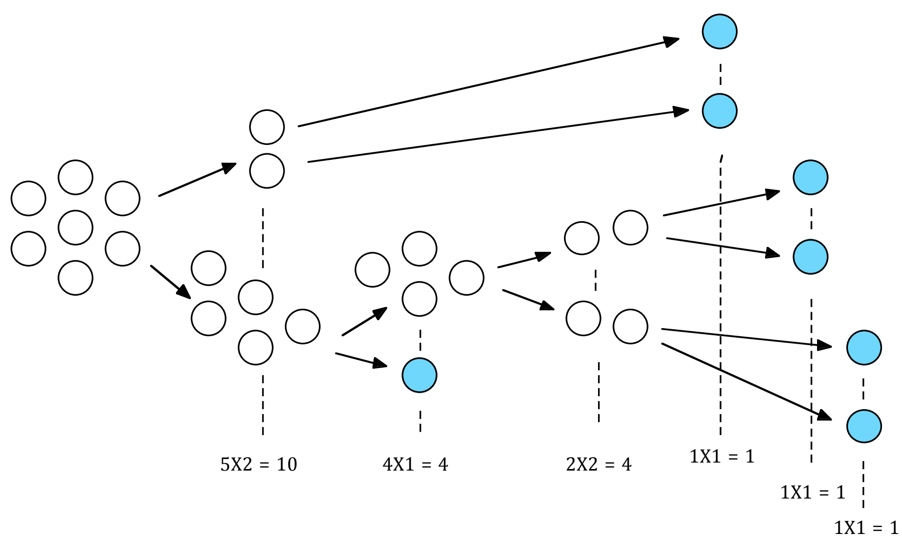
Pile Problem

Begin with a pile of 7 counters. Separate the pile into two smaller piles, for example with 5 and 2 counters, multiply 5 times 2, and write down the product 10. Continue separating piles into two smaller piles until you have 7 piles with one counter in each pile. Finally, add up all the products. Here is an example, with the final piles of size one shown in blue:



The sum of the products in this example is 10 + 4 + 4 + 1 + 1 + 1 = 21.
Try this with 7 counters, dividing the piles differently from how it's done above.
The amazing thing is that no matter how you divide the piles at each step, the final sum will always be the same. Can you explain?
Try this starting with another number of counters
What are the final sums?

Starting number	1	2	3	4	5	6	7	8	9	10
Final sum							**21**			

Given n counters, find a formula that shows what the final sum be.

Can you explain? This can be proved using "strong induction:"
If a statement P(n) says what the formula is when the starting pile has n counters, and
(1) P(1) is true
(2) P(1), P(2), P(3),..., P(k) together imply P(k+1)
Then the formula is true for all positive integers n.

However, there is a "visual proof;" can you find it?

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Source: *Resources for Teaching Discrete Mathematics*, ed. by Brian Hopkins, 2008, MAA.