HW problems in logic, Math 22, due Monday, Nov. 5.

In class we saw the NAND and NOR operator | and ↓, which have truth tables:

		NAND	NOR
P	**Q**	**P|Q**	**P↓Q**
T	T	F	F
T	F	T	F
F	T	T	F
F	F	T	T

(1) Show that P|Q is logically equivalent to ~(P∧Q).
(2) Show that P↓Q is equivalent to ~(P∨Q).
(3) Show that P↓P is equivalent to ~P.
(4) Find a statement using only ↓ (you may use it multiple times) that is logically equivalent to P∨Q.
(5) Find a statement using only ↓ (you may use it multiple times) that is logically equivalent to P∧Q.
(6) Find a statement using only ↓ (you may use it multiple times) that is logically equivalent to P→Q.
(7) Repeat the above process for |. That is, find statements using only | that are equivalent to ~P, P∧Q, P∨Q, and P→Q.
(8) Show that | is not "associative," that is, P|(Q|R) is not the same as (P|Q)|R.
(9) The following sentence is taken from the specifications of a telephone syhstem: "IF the directory database is opened, then the monitor is put in a closed state, if the system in not in its initial state." This is difficult to understand since it uses two implications. Find an equivalent and more easily understood way to say it using disjunctions and negations. (From Rosen, *Discrete Mathematics and its Applications*.)

The following are from Wikipedia (you can look, but they only have the answer to the first one!)
John and Bill are residents of the island of knights (who always lie) and knaves (who always tell the truth).
(10) John says: We are both knaves.
Who is who?
(11) John: If Bill is a knave then I'm a knight.
Bill: We are different.
Who is who?
(12) Logician: Are you both knights? John answers either Yes or No, but the Logician does not have enough information to solve the problem. Logician: Are you both knaves? John answers either Yes or No, and the Logician can now solve the problem.
Who is who?
(13) Here is a rendition of perhaps the most famous of this type of puzzle:
John and Bill are standing at a fork in the road. You know that one of them is a knight and the other a knave, but you don't know which. You also know that one road leads to Someplaceorother, and the other leads to Nowheresville.
(a) By asking one yes/no question, can you determine the road to Someplaceorother?
(b) By asking one yes/no question, can you determine whether John is a knight?
This version of the puzzle was further popularised by a scene in the 1980's fantasy film, Labyrinth, in which Sarah (Jennifer Connelly) finds herself faced with two doors each guided by a two-headed knight. One door leads to the castle at the centre of the labyrinth, and one to certain doom.