Some vocabulary:

Rank of matrix A = dimension of **column space** of A = dimension of range of A = dimension of **row space** of A.

Nullity = dimension of null space of A = number of independent parameters in solution to AX=0.

Null space of A = set of all solutions to AX=0 = "**kernel**" of A.

Changing from one basis to another:

Let S = $\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\}$ be the so-called "standard basis," and let B = $\{\begin{pmatrix}1\\-1\end{pmatrix},\begin{pmatrix}-2\\1\end{pmatrix}\}$ be a second basis for $\mathbf{R}^2$. How do we change from one basis to another? Let $\begin{pmatrix}x\\y\end{pmatrix}_S$ represent the vector $\begin{pmatrix}x\\y\end{pmatrix}$ in the standard basis, and let $\begin{pmatrix}x\\y\end{pmatrix}_B$ represent the same vector in basis B.

For example, $\begin{pmatrix}1\\1\end{pmatrix}_S$ represents $(1)\begin{pmatrix}1\\0\end{pmatrix}+(1)\begin{pmatrix}0\\1\end{pmatrix}$, while $\begin{pmatrix}-3\\-2\end{pmatrix}_B$ represents $(-3)\begin{pmatrix}1\\-1\end{pmatrix}+(-2)\begin{pmatrix}-2\\1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}_S$.

Note that the matrix $\begin{pmatrix}1&-2\\-1&1\end{pmatrix}$ allows us to change from basis B to S, since

$$\begin{pmatrix}x\\y\end{pmatrix}_B = x\begin{pmatrix}1\\-1\end{pmatrix}+y\begin{pmatrix}-2\\1\end{pmatrix}=\begin{pmatrix}1&-2\\-1&1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x-2y\\-x+y\end{pmatrix}_S.$$

To indicate that it allows us to make a transition from basis B to S, we sometimes write this matrix as

$$\begin{pmatrix}1&-2\\-1&1\end{pmatrix}_{\text{B to S}}$$

Homework (1): What are the entries in $\begin{pmatrix}?&?\\?&?\end{pmatrix}_{\text{S to B}}$?

Homework (2): Let [T] = $\begin{pmatrix}2&3\\0&4\end{pmatrix}$ be the matrix representation for the linear transformation T, represented using the standard basis S. That is [T] X takes vector X, expressed in standard basis S, and gives result of applying transformation T, with the result also expressed in standard basis S. Sometimes we write

$[T]_{\text{S to S}}=\begin{pmatrix}2&3\\0&4\end{pmatrix}_{\text{S to S}}$. Find the entries in $[T]_{\text{B to B}}=\begin{pmatrix}?&?\\?&?\end{pmatrix}_{\text{S to S}}$.