Some Linear Algebra vocabulary

Not all words we've used are included here. Some are defined here a bit "informally;" complete descriptions have been given in class or in the texts. All vector spaces are assumed to be finite-dimensional. All vector spaces are assumed to be "over the field" R, the Real Numbers.

Matrix: an m by n array of numbers, together with a component-wise addition operation and a "row-by-column" multiplication operation. We may symbolize the matrix with the number $a_{i,j}$ in the ith row and jth column by $[a_{i,j}]$.

Vector: an m by 1 matrix; an element in an abstract vector space; an element of R^n having both length and direction.

Scalar: a member a of the set of numbers used in conjunction with vectors; in our class this will always be the real numbers. The scalar a may be combined with vector v via "scalar multiplication" that produces another vector av.

Linear combination: a linear combination of vectors $v_1, v_2, ..., v_n$ is a sum of the form $a_1v_1 + a_2v_2 + ... + a_nv_n$, where the a_i's are scalars.

Transpose: the transpose of matrix $A=[a_{i,j}]$ is the matrix $A^t=[a_{j,i}]$. The i,j-th entry of A becomes the j,i-th entry of A^t.

Symmetric: A matrix is symmetric if it equals its transpose.

Identity: The n by n matrix with 1's down the main diagonal, and zeroes elsewhere, written I_n.

Inverse: The inverse of (square) matrix A is the matrix B such that AB = BA = I.

Singular or Non-Invertible: A square matrix that does not have an inverse.

Non-singular or Invertible: A square matrix that has an inverse.

Vector Space: A set of vectors together with vector addition and scalar multiplication, with the properties described in class.

Subspace: a subset of a vector space that also has all the properties of a vector space.

Linear independence: $v_1, v_2, ..., v_n$ are linearly independent if $0 = a_1v_1 + a_2v_2 + ... + a_nv_n$ has only the solution $a_1 = a_2 = ... = a_n = 0$.

Linearly dependence: $v_1, v_2, ..., v_n$ are linearly dependent if $0 = a_1v_1 + a_2v_2 + ... + a_nv_n$ has a solution in which not all the a_i's are equal to 0.

Nullspace or kernel: the nullspace or kernel of matrix A is the set of solutions of Ax=0.

Span: the span of a set of vectors is the set of all linear combinations of those vectors.

Basis: A linearly independent set of vectors that spans a given vector space.

Dimension: for finite vector spaces, the number of elements in a basis for the vector space.

Elementary Row Operations:

(1) Interchange two rows
(2) Multiply a row by a non-zero number
(3) Add a multiple of one row to another
(4)

Reduced Row Echelon Form: for a matrix A, the result of performing elementary row operations that result in

(1) all zero rows at the bottom
(2) the first non-zero entry in a non-zero row, taken from left to right, is 1.
(3) each leading 1 is the right of any leading 1's above it.
(4) Any leading 1 has only 0's above and below it.

Row Space: the vector subspace spanned by the rows of matrix A.

Column Space: the vector subspace spanned by the columns of matrix A.

Rank of matrix A: the dimension of the row space (or column space) of A (they are equal!).

Nullity of a matrix A: the dimension of the null space of A.

Homogenous System: A system of equations Ax=0.