Math 22, Fall 2009

Here are two more puzzles from Discrete Mathematics, by Ensley and Crawley, 2006:

All the inhabitants on an island are either liars who never tell the truth, or Truthtellers you always tell the truth.

(1) You meet two inhabitants.
A says, “If B is truthful, then so am I.”
B says, “At least one of us is lying.”

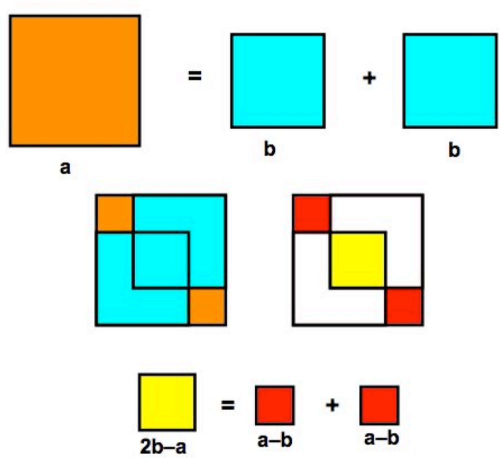
Who (if anyone) is telling the truth?

(2) A says, “If B is truthful, then so am I.”
B says, “A is lying.”
Is this enough information to figure out who is telling the truth?

(3) Here are those proofs of the irrationality of √2. The first is by Stanley Tennenbaum, from the 1950’s**.** Assume √2 is the rational number *a/b*, in lowest terms. Then squaring both sides gives:

$$\sqrt{2} = \frac{a}{b}$$
$$2 = \frac{a^2}{b^2}$$
$$2b^2 = b^2 + b^2 = a^2$$

Here *a* and *b* are the smallest such integers with this property, since a/b is assumed to be in lowest terms. Can you use the diagram below and explain why this leads to a contradiction?




(4) Here is another proof along the same lines, this one a paper folding proof by John Conway and Richard Guy, similar to a proof in an 1892 Geometry text by Russian

mathematician A.P. Kiselev. Again, assume we have a right isosceles triangle with integer sides *a* and hypotenuse *b*, such that *a/b* is in lowest terms. How does the diagram help explain the proof?