Some study questions for review
Math 44
Spring 2009
These are some questions that will help you review what you have learned in the course.
(1) Give an example of a Hamiltonian circuit problem that you have to solve in your life. Your description should include an explanation of what are the vertices and what are the edges.
(2) (a) Does this graph have an Euler circuit? If yes, list an Euler cycle beginning with vertex A, (or else label the edges with the numbers $1,2,3, \ldots$ ), and if no explain why not:

(b) List a Hamiltonian cycle beginning at vertex A: $\qquad$
(3) Draw a graph with vertex degrees of $2,3,3,4$, and 4 :

0

(4). What symmetries does the object below have?
(All triangles are equilateral.) Draw all lines of reflection symmetry.

(5) Define a function $B(n)$, called the Blooper Function, by

$$
B(n)=\left\{\begin{array}{l}
n / 3 \text { if } n \text { is divisible by } 3 \\
2 n \text { if } n \text { is } 1 \text { more than a multiple of } 3 \\
n+1 \text { if } n \text { is } 2 \text { more than a multiple of } 3
\end{array}\right.
$$

Investigate the behavior of the blooper function for $\mathrm{n}=1$ through 10 (Draw arrows to show the sequences, and add any additional boxes that you need.):


Make a conjecture about the overall behavior of the Blooper Function:
(6) (a) If this pattern continues, which object will fall in each of the boxes shown? (Draw the object in the box!)

(b) Explain your reasoning:
(7) (a) Find one number $x$ such that $7 x \equiv 5(\bmod 4)$ is a true equation:
$\mathrm{x}=$ $\qquad$
(b) Find one number $x$ such that $35 \equiv 23(\bmod x)$ is a true equation:
$x=$ $\qquad$
(c) Fill in the spaces in this mod 4 addition table with the numbers $0,1,2$ or $3($ For example, $2+3=5$, and $5 \equiv 1(\bmod 4)$.)

| $\mathbf{X}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | 0 | 1 | 2 | 3 |
| $\mathbf{1}$ | 1 |  |  |  |
| $\mathbf{2}$ | 2 |  |  | 1 |
| $\mathbf{3}$ | 3 |  |  |  |

(8). Here are the first few steps in the creation of a certain fractal:


Look at the final shape on the right. If we magnify this final shape $\qquad$ times we would find that we have $\qquad$ full-size copies of the fractal. Use the formula
fractal dimension $=\frac{\log (\text { number of copies })}{\log (\text { magnification factor })}$
to calculate the fractal dimension of this fractal: $\qquad$
(9) How do we "generate" the Fibonacci numbers? Be specific, explaining the rule and showing the first ten numbers in the sequence under the proper term:
$\begin{array}{llllllllll}\mathrm{F}_{1} & \mathrm{~F}_{2} & \mathrm{~F} 3 & \mathrm{~F} 4 & \mathrm{~F}_{5} & \mathrm{~F}_{6} & \mathrm{~F}_{7} & \mathrm{~F}_{8} & \mathrm{~F} 9 & \mathrm{~F} 10\end{array}$
(b) Do the calculation in each row and look for the overall pattern:
$\mathrm{F}_{2}=$
$\mathrm{F}_{2}+\mathrm{F}_{4}=$
$\mathrm{F}_{2}+\mathrm{F}_{4}+\mathrm{F}_{6}=$
$\mathrm{F}_{2}+\mathrm{F}_{4}+\mathrm{F}_{6}+\mathrm{F}_{8}=$
$\mathrm{F}_{2}+\mathrm{F}_{4}+\mathrm{F}_{6}+\mathrm{F}_{8}+\mathrm{F}_{10}=$
(c) The overall pattern is (express the pattern using symbols such as F and appropriate subscripts):
$\mathrm{F}_{2}+\mathrm{F}_{4}+\mathrm{F}_{6}+\mathrm{F}_{8}+\mathrm{F}_{10}+\ldots+\mathrm{F}_{2 \mathrm{n}}=$
(10) Which mathematician did you study? $\qquad$
(a) How did this mathematician change your perceptions of who mathematicians are and what mathematics is? How did this person reconfirm some of your beliefs about mathematicians and mathematics?
(b) One of the common myths about mathematics is that some people "have a mind for mathematics and do not have to work hard to understand it, and some people do not." How did the mathematician you studied confirm or contradict this statement, and how did she or he compare with the others you heard reported on in class? Be specific.
(11) Construct the first three rows of the "multiplication table" for the symmetries of the equilateral triangle
 below. the six possible symmetry operations are:

I = the "identity" operation: no motion.
$r_{1}=$ a clockwise rotation of 120 degrees ( $1 / 3$ of a turn to the right).
$r_{2}=$ a clockwise rotation of 240 degrees ( $2 / 3$ of a turn to the right).
$\mathrm{V}=\mathrm{a}$ reflection across the vertical line, as shown in the diagram.
$\mathrm{R}=$ a reflection across the line labelled R in the diagram.
$\mathrm{L}=$ a reflection across the line labelled L in the diagram.

For example, $\mathrm{r}_{2}$ followed by V is the same as a reflection across the line R .
You might want to use the equilateral triangle provided to help figure out the table.


## Second operation

First operation


Is $r_{2} V=V r_{2}$ ? $\qquad$

