Math 1D, Fall 2008, Exam 2 Sample Test

(1) Ch. 15.1 (#7): List the critical ponts and classify them as local maxima, local minima, saddle points, or none of these:

$$f(x,y) = x^3 + y^3 - 6y^2 - 3x + 9$$

(2) Ch. 15.2 (#21) Two products are manufactured in quantities q_1 and q_2 and sold at prices of p_1 and p_2 respectively. The cost of producing them is given by
$C = 2\,q_1^2 + 2\,q_2^2 + 10$.
(a) Find the maximum profit that can be made, assuming the prices are fixed.
(b) Find the rate of change of that maximum profit as p_1 increases.
(Remember that revenue R = (price)(quantity), and profit P = R – C.)

(3) Ch. 15.3: (a) Minimize $x^2 + y^2$ subject to $x^2y^2 = 4$ in the first quadrant.
(b) Maximize x^2y^2 subject to $x + y = 4$ in the first quadrant.

(4) Ch. 16.1 (#4): The table give s values of z = f(x,y) on the rectangle R with $0 \le x \le 6$ and $0 \le y \le 8$.
(a) Estimate $\int_R f(x,y)dA$ as accurately as possible.
(b) Estimate the average value of f(x,y) on R.

		x		
		0	3	6
y	0	100	90	81
	4	85	79	68
	8	65	61	55

(5) Ch. 16.2: #22 (this was a homework problem which we went over in class.)
(6) Ch. 16.3, #18: Find the volume of the region bounded by z = x + y, z = 10, and the planes x = 0 and y = 0.
(7) Ch. 16.4:

Convert the integral $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} e^{-(x^2+y^2)}\,dy\,dx$ to polar coordinates and hence evaluate it exactly. Sketch the region R over which the integration is being performed.

(8) Ch. 16.5: Each of the following, (a) – (f), represents a point, a curve, a surface, or a solid region in cylindrical or spherical coordinates. Decide which it represents and describe the region in words.

(a) $0 \le \rho < \infty,\ \theta = \pi,\ 0 \le \varphi \le \pi$
(b) $r = 3,\ 0 \le \theta \le 2\pi,\ -\infty < z < \infty$
(c) $r = 3,\ \theta = \pi/2,\ -\infty < z < \infty$
(d) $1 \le r \le 4,\ 0 \le \theta \le 2\pi,\ -5 \le z \le 2$
(e) $1 \le \rho \le 4,\ 0 \le \theta \le 2\pi,\ 0 \le \varphi \le \pi$
(f) $\rho = 1,\ \theta = 3,\ \varphi = 2$

(9)Ch.16.7:

Consider the change of variables $x = s + 3t,\ y = s - 2t$.

(a) Find the absolute value of the Jacobian $\left|\frac{\partial(x,y)}{\partial(s,t)}\right|$.
(b) Let R be the region bounded by the lines $2x + 3y = 1, 2x + 3y = 4, x - y = -3, x - y = 2$. Find the region T in the st-plane that corresponds to region R.
(c) Use the change of variables to evaluate $\int_R 2x + 3y dA$.