Math 46 Study guide for first exam
Answers
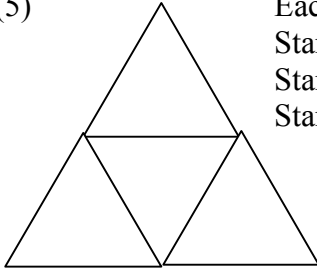
(1) (a) {1,3,5,6,7,8,9,10} (b) {2,4} (c) 5+4–0=9
(2) (a) and (b) 100=(7)(14)+2, so the first player wins by taking 2, and thereafter completing groups of 7. (c) $100 \equiv 2 \pmod 7$
(3) (100)(101)/2 = 5050
(4) Total 50 – those in neither set 10 = number in A∪B = 25 + 30 – number in both. So 40 = 55 – number in both, and this number must be 15.
(5)



Each of these triangles has sides of length 1 mile, so any two Starbucks within one triangle are 1 mile or less apart. Placing 5 Starbucks in 4 triangles means one triangle will have at least two Starbucks, so they are 1 mile or less apart.

(6) If they touch by 1 unit, perimeter $= 8 + 8 – 2 = 14$. If they touch by 2 units, perimeter $= 8 + 8 – 4 = 12$. If they touch by 3 units, perimeter $= 8 + 8 – 6 = 10$. These are all: 10,12,14.
(7)

| Pennies | 20 | 15 | 10 | 10 | 5 | 5 | 0 | 0 | 0 | | |
|---------|----|----|----|----|---|---|---|---|---|---|---|
| Nickels | 0 | 1 | 2 | 0 | 3 | 1 | 4 | 2 | 0 | | |
| Dimes | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 2 | | |

(8) (a) 250/7 rounds up to 36, so one bus holds at least 36.
(b) Pairs: 1,24  2,12  3,8  4,6
Five numbers must include one of these four pairs, by the pigeonhole principle
(9) (a) If you add two multiples of 5 you get another multiple of 5, so they are closed.
(b) If you divide, for example, 10 by 5, you don't get a multiple of 5, so False.
(c) True what's not in what's not in a set is the set itself!
(d) No, the third row is 1,2,1 (or some would say 1,3,3,1)
(10)



(11) We didn't get to this!
(12) Pattern repeats in 3's horizontally and in 2's vertically. Eyes are in columns congruent to 1, mod 3. Eyes are in odd rows. So eyes should be in box (313,359), use this to fill in the rest.
(13) We didn't get to this notation for Fibonacci numbers, but the next row would be
$(5)(21) = (8)(13) + 1$ or $(F_5)(F_8) = (F_6)(F_7) + 1$
General pattern is $(F_k)(F_{k+3}) = (F_{k+1})(F_{k+2}) + 1$ when k is odd.

(14) Columns indicate who might have taken the cards, row indicates the statement

|  | Arc | Barc | Carc |
|---|---|---|---|
| Arc says: | No | No | Yes |
| Barc says: | Yes | No | No |
| Carc says: | Yes | Yes | No |

Only if Arc took the cards is "at most one" lying (namely Arc). Otherwise, if Barc took the cards two would have to be lying, same for Carc.

(15)

|  | Alex | Bill | Cora | Dany |
|---|---|---|---|---|
| Monday | 20+20=40 | P, –20 |  | M, –20 |
| Tuesday | 40 | –20+30=10 | P,M, –30–30=–60 | –20+30=10 |
| Wednesday | 40 | M, 10–40=–30 | –60+40+40=20 | P,10–40=–30 |
| Thursday | P,M,40–50–50=–60 | –30+50=20 | 20 | –30+50=20 |

Alex lost $60, Bill and Cora and Dany each made $20.