CHAPTER FIVE

Solutions for Section 5.1

1. (a) The velocity is 30 miles/hour for the first 2 hours, 40 miles/hour for the next $1/2$ hour, and 20 miles/hour for the last 4 hours. The entire trip lasts $2 + 1/2 + 4 = 6.5$ hours, so we need a scale on our horizontal (time) axis running from 0 to 6.5. Between $t = 0$ and $t = 2$, the velocity is constant at 30 miles/hour, so the velocity graph is a horizontal line at 30. Likewise, between $t = 2$ and $t = 2.5$, the velocity graph is a horizontal line at 40, and between $t = 2.5$ and $t = 6.5$, the velocity graph is a horizontal line at 20. The graph is shown in Figure 5.1.

(b) How can we visualize distance traveled on the velocity graph given in Figure 5.1? The velocity graph looks like the top edges of three rectangles. The distance traveled on the first leg of the journey is (30 miles/hour)(2 hours), which is the height times the width of the first rectangle in the velocity graph. The distance traveled on the first leg of the trip is equal to the area of the first rectangle. Likewise, the distances traveled during the second and third legs of the trip are equal to the areas of the second and third rectangles in the velocity graph. It appears that distance traveled is equal to the area under the velocity graph.

In Figure 5.2, the area under the velocity graph in Figure 5.1 is shaded. Since this area is three rectangles and the area of each rectangle is given by Height $\times$ Width, we have

$$\begin{aligned}\text{Total area} &= (30)(2) + (40)(1/2) + (20)(4)\\ &= 60 + 20 + 80 = 160.\end{aligned}$$

The area under the velocity graph is equal to distance traveled.

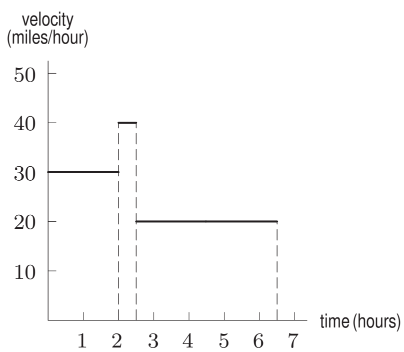


Figure 5.1: Velocity graph

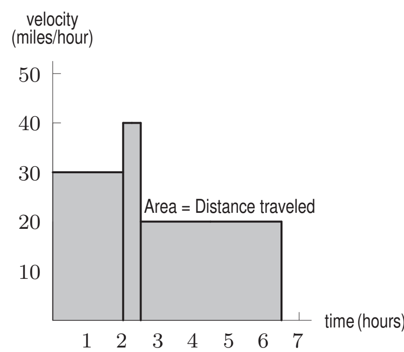


Figure 5.2: The area under the velocity graph gives distance traveled

2. Using the data in Table 5.3 of Example 4, we construct Figure 5.3.

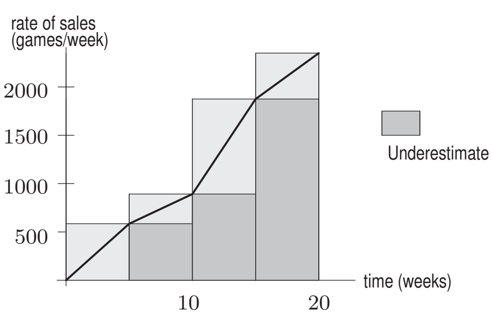
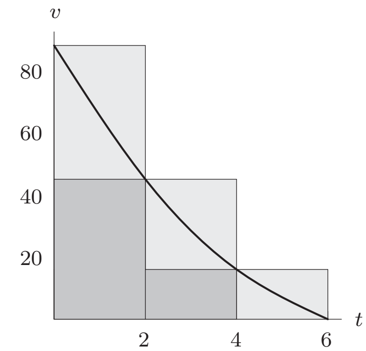


Figure 5.3

3. **(a)** Lower estimate $= (45)(2) + (16)(2) + (0)(2) = 122$ feet.
Upper estimate $= (88)(2) + (45)(2) + (16)(2) = 298$ feet.

(b)



4. We use Distance = Rate × Time on each subinterval with $\Delta t = 3$.

$$\text{Underestimate} = 0 \cdot 3 + 10 \cdot 3 + 25 \cdot 3 + 45 \cdot 3 = 240,$$
$$\text{Overestimate} = 10 \cdot 3 + 25 \cdot 3 + 45 \cdot 3 + 75 \cdot 3 = 465.$$

We know that

$$240 \leq \text{Distance traveled} \leq 465.$$

A better estimate is the average. We have

$$\text{Distance traveled} \approx \frac{240 + 465}{2} = 352.5.$$

The car travels about 352.5 feet during these 12 seconds.

5. Figure 5.4 shows the graph of $f(t)$. The region under the graph of $f(t)$ from $t = 0$ to $t = 10$ is a triangle of base 10 seconds and height 50 meter/sec. Then

$$\text{Distance traveled } = \text{Area of triangle} = \frac{1}{2} \cdot 10 \cdot 50 = 250 \text{ meters}.$$

Thus the distance traveled is 250 meters.

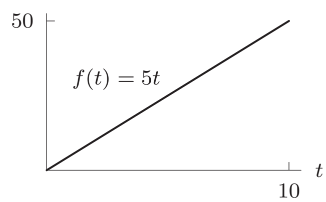


Figure 5.4

6. Just counting the squares (each of which has area 10), and allowing for the broken squares, we can see that the area under the curve from 0 to 6 is between 140 and 150. Hence the distance traveled is between 140 and 150 meters.

7. By counting squares and fractions of squares, we find that the area under the graph appears to be around 310 (miles/hour) sec, within about 10. So the distance traveled was about $310 \left(\frac{5280}{3600}\right) \approx 455$ feet, within about $10 \left(\frac{5280}{3600}\right) \approx 15$ feet. (Note that 455 feet is about 0.086 miles)

8. The table gives the rate of oil consumption in billions of barrels per year. To find the total consumption, we use left-hand and right-hand Riemann sums. We have

$$\text{Left-hand sum } = (22.3)(5) + (21.3)(5) + (23.9)(5) + (24.9)(5) + (27.0)(5) = 597.0 \text{ bn barrels}.$$
$$\text{Right-hand sum } = (21.3)(5) + (23.9)(5) + (24.9)(5) + (27.0)(5) + (29.3)(5) = 632.0 \text{ bn barrels}.$$
$$\text{Average of left- and right-hand sums} = \frac{597.0 + 632.0}{2} = 614.5 \text{ bn barrels}.$$

The consumption of oil between 1980 and 2005 is about 614.5 billion barrels.

9. **(a)** Let's begin by graphing the data given in the table; see Figure 5.5. The total amount of pollution entering the lake during the 30-day period is equal to the shaded area. The shaded area is roughly 40% of the rectangle measuring 30 units by 35 units. Therefore, the shaded area measures about $(0.40)(30)(35) = 420$ units. Since the units are kilograms, we estimate that 420 kg of pollution have entered the lake.

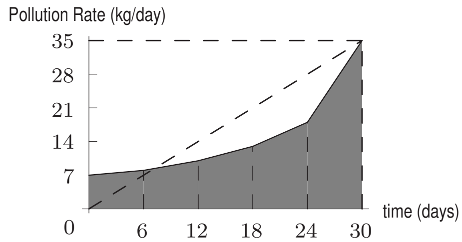


Figure 5.5

(b) Using left and right sums, we have

$$\text{Underestimate} = (7)(6) + (8)(6) + (10)(6) + (13)(6) + (18)(6) = 336 \text{ kg.}$$

$$\text{Overestimate} = (8)(6) + (10)(6) + (13)(6) + (18)(6) + (35)(6) = 504 \text{ kg.}$$

10. **(a)** Based on the data, we will calculate the underestimate and the overestimate of the total change. A good estimate will be the average of both results.
Underestimate of total change

$$= 37 \cdot 10 + 41 \cdot 10 + 77 \cdot 10 + 77 \cdot 10 + 79 \cdot 10 = 3110.$$

77 was considered twice since we needed to calculate the area under the graph.
Overestimate of total change

$$= 41 \cdot 10 + 78 \cdot 10 + 78 \cdot 10 + 86 \cdot 10 + 86 \cdot 10 = 3690.$$

78 and 86 were considered twice since we needed to calculate the area over the graph.
The average is: $(3110 + 3690)/2 = 3400$ million people.

(b) The total change in the world's population between 1950 and 2000 is given by the difference between the populations in those two years. That is, the change in population equals

$$6085 \text{ (population in 2000) } - 2555 \text{ (population in 1950) } = 3530 \text{ million people.}$$

Our estimate of 3400 million people and the actual difference of 3530 million people are close to each other, suggesting our estimate was a good one.

11. Suppose $f(t)$ is the flow rate in m^3/hr at time t. We are only given two values of the flow rate, so in making our estimates of the flow, we use one subinterval, with $\Delta t = 3/1 = 3$:

$$\text{Left estimate} = 3[f(6 \text{ am})] = 3 \cdot 100 = 300 \text{ m}^3 \quad \text{(an underestimate)}$$
$$\text{Right estimate} = 3[f(9 \text{ am})] = 3 \cdot 280 = 840 \text{ m}^3 \quad \text{(an overestimate).}$$

The best estimate is the average of these two estimates,

$$\text{Best estimate} = \frac{\text{Left} + \text{Right}}{2} = \frac{300 + 840}{2} = 570 \text{ m}^3.$$

12. **(a)** Note that 15 minutes equals 0.25 hours. Lower estimate $= 11(0.25) + 10(0.25) = 5.25$ miles. Upper estimate $= 12(0.25) + 11(0.25) = 5.75$ miles.

(b) Lower estimate $= 11(0.25) + 10(0.25) + 10(0.25) + 8(0.25) + 7(0.25) + 0(0.25) = 11.5$ miles. Upper estimate $= 12(0.25) + 11(0.25) + 10(0.25) + 10(0.25) + 8(0.25) + 7(0.25) = 14.5$ miles.

13. **(a)** See Figure 5.6.
(b) The distance traveled is the area under the graph of the velocity in Figure 5.6. The region is a triangle of base 5 seconds and altitude 50 ft/sec, so the distance traveled is $(1/2)5 \cdot 50 = 125$ feet.
(c) The slope of the graph of the velocity function is the same, so the triangular region under it has twice the altitude and twice the base (it takes twice as long to stop). See Figure 5.7. Thus, the area is 4 times as large and the car travels 4 times as far.

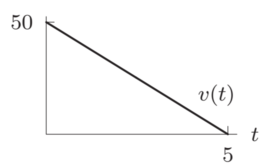


Figure 5.6

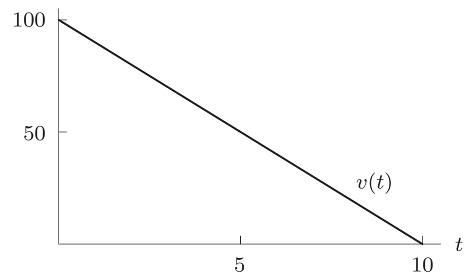


Figure 5.7

14. The rate at which the fish population grows varies between 10 fish per month and about 22 fish per month. If the rate of change were constant at the lower bound of 10 during the entire 12-month period, we obtain an underestimate for the total change in the fish population of $(10)(12) = 120$ fish. An overestimate is $(22)(12) = 264$ fish. The actual increase in the fish population is equal to the area under the curve. (Notice that the units of the area are height units times width units, or fish per month times months, as we want.) We estimate that the area is about 11 grid squares. The area of each grid square represents an increase of (5 fish per month) $\cdot$ (4 months) $= 20$ fish. We have

$$\text{Total change in fish population} = \text{Area under curve} \approx 11 \cdot 20 = 220.$$

We estimate that the fish population grew by 220 fish during this 12-month period.

15. **(a)** Car A has the largest maximum velocity because the peak of car A's velocity curve is higher than the peak of B's.
(b) Car A stops first because the curve representing its velocity hits zero (on the t-axis) first.
(c) Car B travels farther because the area under car B's velocity curve is the larger.

16. **(a)** Since car B starts at $t = 2$, the tick marks on the horizontal axis (which we assume are equally spaced) are 2 hours apart. Thus car B stops at $t = 6$ and travels for 4 hours.
Car A starts at $t = 0$ and stops at $t = 8$, so it travels for 8 hours.
(b) Car A's maximum velocity is approximately twice that of car B, that is 100 km/hr.
(c) The distance traveled is given by the area of under the velocity graph. Using the formula for the area of a triangle, the distances are given approximately by

$$\text{Car } A \text{ travels} = \frac{1}{2} \cdot \text{Base} \cdot \text{Height} = \frac{1}{2} \cdot 8 \cdot 100 = 400 \text{ km}$$
$$\text{Car } B \text{ travels} = \frac{1}{2} \cdot \text{Base} \cdot \text{Height} = \frac{1}{2} \cdot 4 \cdot 50 = 100 \text{ km}.$$

17. Sketch the graph of $v(t)$. See Figure 5.8. Adding up the areas using an overestimate with data every 1 second, we get $s \approx 2 + 5 + 10 + 17 + 26 = 60$ m. The actual distance traveled is less than 60 m.

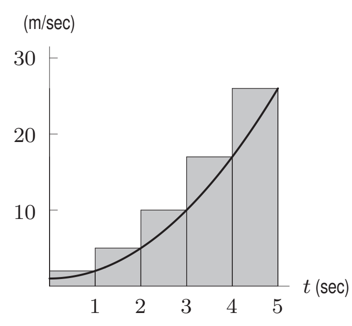


Figure 5.8

18. **(a)** Note that the rate $r(t)$ sometimes increases and sometimes decreases in the interval. We can calculate an upper estimate of the volume by choosing $\Delta t = 5$ and then choosing the highest value of $r(t)$ on each interval, and similarly a lower estimate by choosing the lowest value of $r(t)$ on each interval:

$$\text{Upper estimate} = 5[20 + 24 + 24] = 340 \text{ liters.}$$
$$\text{Lower estimate} = 5[12 + 20 + 16] = 240 \text{ liters.}$$

(b) A graph of $r(t)$ along with the areas represented by the choices of $r(t)$ in calculating the lower estimate is shown in Figure 5.9.

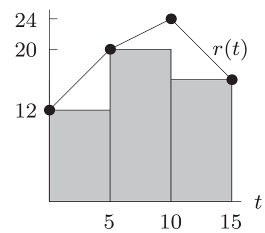


Figure 5.9

19. **(a)** See Table 5.1.

Table 5.1

t	0	2	4	6	8	10
R	500	541.64	586.76	635.62	688.56	745.91

(b) The total change is the rate of change, in dollars per year, times the number of years in the time interval, summed up over all time intervals. We find an underestimate and an overestimate and average the two:

$$\text{Underestimate} = 500 \cdot 2 + 541.64 \cdot 2 + 586.76 \cdot 2 + 635.62 \cdot 2 + 688.56 \cdot 2 = \$5905.16.$$
$$\text{Overestimate} = 541.64 \cdot 2 + 586.76 \cdot 2 + 635.62 \cdot 2 + 688.56 \cdot 2 + 745.91 \cdot 2 = \$6396.98.$$

A better estimate of the total change in the value of the fund is the average of the two:

$$\text{Total change in the value of the fund} \approx \frac{5905.16 + 6396.98}{2} = \$6151.$$

20. The car's speed increases by 60 mph in 1/2 hour, that is at a rate of $60/(1/2) = 120$ mph per hour, or $120/60 = 2$ mph per minute. Thus every 5 minutes the speed has increased by 10 mph. At the start of the first 5 minutes, the speed was 10 mph and at the end, the speed was 20 mph. To find the distance traveled, use Distance $=$ Speed $\times$ Time. Since 5 min $= 5/60$ hour, the distance traveled during the first 5 minutes was between

$$10 \cdot \frac{5}{60} \text{ mile} \qquad \text{and} \qquad 20 \cdot \frac{5}{60} \text{ mile.}$$

Since the speed was between 10 and 20 mph during this five minute period, the fuel efficiency during this period is between 15 mpg and 18 mpg. So the fuel used during this period is between

$$\frac{1}{18} \cdot 10 \cdot \frac{5}{60} \text{ gallons} \qquad \text{and} \qquad \frac{1}{15} \cdot 20 \cdot \frac{5}{60} \text{ gallons.}$$

Thus, an underestimate of the fuel used is

$$\text{Fuel} = \left(\frac{1}{18} \cdot 10 + \frac{1}{21} \cdot 20 + \frac{1}{23} \cdot 30 + \frac{1}{24} \cdot 40 + \frac{1}{25} \cdot 50 + \frac{1}{26} \cdot 60\right) \frac{5}{60} = 0.732 \text{ gallons.}$$

An overestimate of the fuel used is

$$\text{Fuel} = \left(\frac{1}{15} \cdot 20 + \frac{1}{18} \cdot 30 + \frac{1}{21} \cdot 40 + \frac{1}{23} \cdot 50 + \frac{1}{24} \cdot 60 + \frac{1}{25} \cdot 70\right) \frac{5}{60} = 1.032 \text{ gallons.}$$

We can get better bounds by using the actual distance traveled during each five minute period. For example, in the first five minutes the average speed is 15 mph (halfway between 10 and 20 mph because the speed is increasing at a constant rate). Since 5 minutes is 5/60 of an hour, in the first five minutes the car travels 15(5/60)=5/4 miles. Thus the fuel used during this period was between

$$\frac{5}{4} \cdot \frac{1}{18} \quad \text{and} \quad \frac{5}{4} \cdot \frac{1}{15}.$$

Using this method for each five minute period gives a lower estimate of 0.843 gallons and an upper estimate of 0.909 gallons.

Solutions for Section 5.2

1. Dividing the interval from 0 to 6 into 2 equal subintervals gives $\Delta x = 3$. Using $f(x) = 2^x$, we have

$$\begin{aligned} \text{Left-hand sum } &= f(0) \cdot \Delta x + f(3) \cdot \Delta x \\ &= 2^0 \cdot 3 + 2^3 \cdot 3 \\ &= 27. \end{aligned}$$

2. Dividing the interval from 0 to 12 into 3 equal subintervals gives $\Delta x = 4$. Using $f(x) = 1/(x+1)$, we have

$$\begin{aligned} \text{Left-hand sum } &= f(0) \cdot \Delta x + f(4) \cdot \Delta x + f(8) \cdot \Delta x \\ &= \frac{1}{0+1} \cdot 4 + \frac{1}{4+1} \cdot 4 + \frac{1}{8+1} \cdot 4 \\ &= 5.244. \end{aligned}$$

3. Calculating both the LHS and RHS and averaging the two, we get

$$\frac{1}{2}(5(100+82+69+60+53) + 5(82+69+60+53+49)) = 1692.5$$

4. Since we are given a table of values, we must use Riemann sums to approximate the integral. Values are given every 0.2 units, so $\Delta t = 0.2$ and $n = 5$. Our best estimate is obtained by calculating the left-hand and right-hand sums, and then averaging the two.

$$\begin{aligned} \text{Left-hand sum} &= 25(0.2) + 23(0.2) + 20(0.2) + 15(0.2) + 9(0.2) = 18.4 \\ \text{Right-hand sum} &= 23(0.2) + 20(0.2) + 15(0.2) + 9(0.2) + 2(0.2) = 13.8. \end{aligned}$$

We average the two sums to obtain our best estimate of the integral:

$$\int_3^4 W(t)dt \approx \frac{18.4 + 13.8}{2} = 16.1.$$

5.

$$\begin{aligned} \text{Left-hand sum} &= 50 \cdot 3 + 48 \cdot 3 + 44 \cdot 3 + 36 \cdot 3 + 24 \cdot 3 = 606 \\ \text{Right-hand sum} &= 48 \cdot 3 + 44 \cdot 3 + 36 \cdot 3 + 24 \cdot 3 + 8 \cdot 3 = 480 \\ \text{Average} &= \frac{606 + 480}{2} = 543 \end{aligned}$$

So we have $\displaystyle\int_0^{15} f(x)\,dx \approx 543.$

6. We estimate $\int_0^{40} f(x)dx$ using left- and right-hand sums:

$$\text{Left sum} = 350 \cdot 10 + 410 \cdot 10 + 435 \cdot 10 + 450 \cdot 10 = 16{,}450.$$

$$\text{Right sum} = 410 \cdot 10 + 435 \cdot 10 + 450 \cdot 10 + 460 \cdot 10 = 17{,}550.$$

We estimate that

$$\int_0^{40} f(x)dx \approx \frac{16450 + 17550}{2} = 17{,}000.$$

In this estimate, we used $n = 4$ and $\Delta x = 10$.

7.

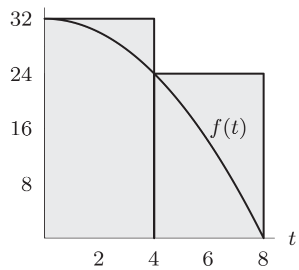


Figure 5.10: Left Sum, $\Delta t = 4$

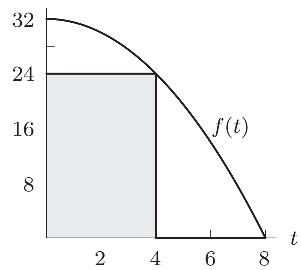


Figure 5.11: Right Sum, $\Delta t = 4$

(a) Left-hand sum $= 32 \cdot 4 + 24 \cdot 4 = 224$.
(b) Right-hand sum $= 24 \cdot 4 + 0 \cdot 4 = 96$.

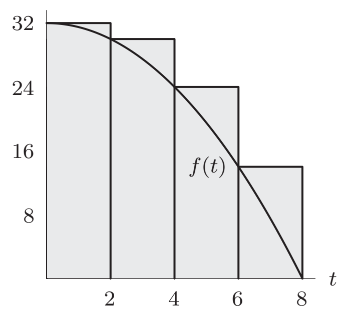


Figure 5.12: Left Sum, $\Delta t = 2$

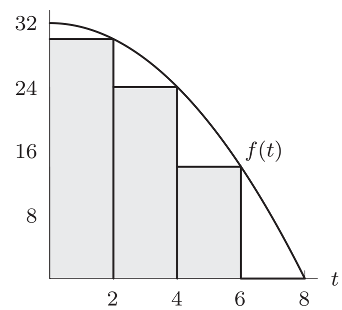


Figure 5.13: Right Sum, $\Delta t = 2$

(c) Left-hand sum $= 32 \cdot 2 + 30 \cdot 2 + 24 \cdot 2 + 14 \cdot 2 = 200$.
(d) Right-hand sum $= 30 \cdot 2 + 24 \cdot 2 + 14 \cdot 2 + 0 \cdot 2 = 136$.

8. $\displaystyle\int_0^{20} f(x)\,dx$ is equal to the area shaded. We can use Riemann sums to estimate the area, or we can count grid squares in Figure 5.14. There are about 15 grid squares and each grid square represents 4 square units, so the area shaded is about 60. We have $\displaystyle\int_0^{20} f(x)\,dx \approx 60$.

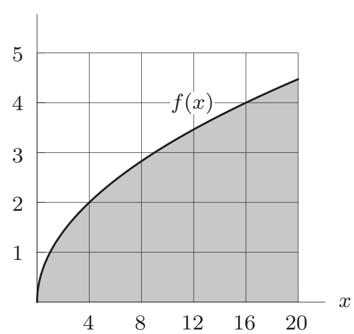


Figure 5.14

9. We know that
$$\int_{-10}^{15} f(x)dx = \text{Area under } f(x) \text{ between } x = -10 \text{ and } x = 15.$$
The area under the curve consists of approximately 14 boxes, and each box has area $(5)(5) = 25$. Thus, the area under the curve is about $(14)(25) = 350$, so
$$\int_{-10}^{15} f(x)dx \approx 350.$$

10. To estimate the integral, we count the rectangles under the curve and above the x-axis for the interval $[0, 3]$. There are approximately 17 of these rectangles, and each has area 1, so
$$\int_{0}^{3} f(x)dx \approx 17.$$

11. $\int_0^3 f(x)\,dx$ is equal to the area shaded. We can use Riemann sum to estimate this area, or we can count grid squares. These are 3 whole grid squares and about 4 half-grid squares, for a total of 5 grid squares. Since each grid square represent 4 square units, our estimated area is $5(4) = 20$. We have $\int_0^3 f(x)\,dx \approx 20$. See Figure 5.15.

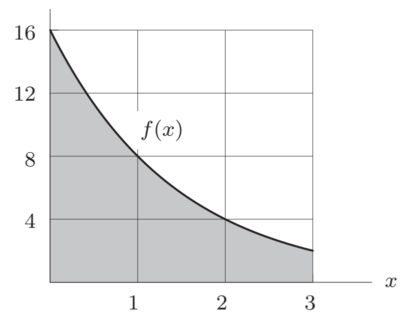


Figure 5.15

12. **(a)** See Figure 5.16.

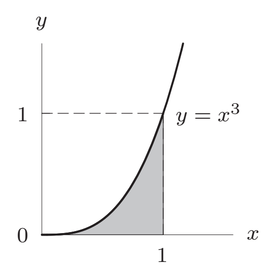


Figure 5.16

$\int_0^1 x^3\,dx =$ area shaded, which is less than 0.5. Rough estimate is about 0.3.

(b) $\int_0^1 x^3\,dx = 0.25$

13. (a) See Figure 5.17.

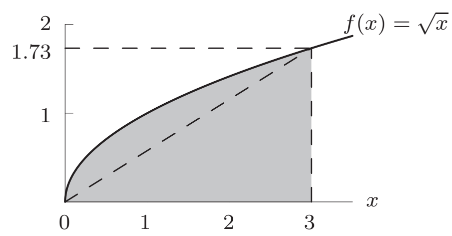


Figure 5.17

A rough estimate of the shaded area is 70% of the area of the rectangle 3 by 1.7. Thus,

$$\int_0^3 \sqrt{x}\,dx \approx 70\% \text{ of } 3 \cdot 1.7 = 0.70 \cdot 3 \cdot 1.7 = 3.6.$$

(b) $\int_0^3 \sqrt{x}\,dx = 3.4641$

14. (a) See Figure 5.18.

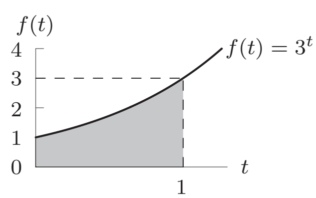


Figure 5.18

The shaded area is approximately 60% of the area of the rectangle 1 unit by 3 units. Therefore,

$$\int_0^1 3^t\,dt \approx 0.60 \cdot 1 \cdot 3 = 1.8.$$

(b) $\int_0^1 3^t\,dt = 1.8205.$

15. (a) See Figure 5.19.

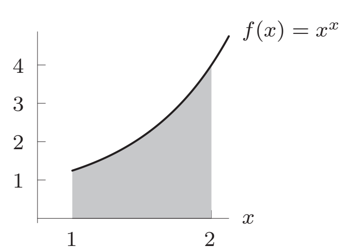


Figure 5.19

The shaded area appears to be approximately 2 units, and so $\int_1^2 x^x\,dx \approx 2.$

(b) $\int_1^2 x^x\,dx = 2.05045$

16. **(a)** If $\Delta t = 4$, then $n = 2$. We have:

$$\begin{aligned}\text{Underestimate of total change } &= f(0)\Delta t + f(4)\Delta t = 1 \cdot 4 + 17 \cdot 4 = 72.\\ \text{Overestimate of total change } &= f(4)\Delta t + f(8)\Delta t = 17 \cdot 4 + 65 \cdot 4 = 328.\end{aligned}$$

See Figure 5.20.

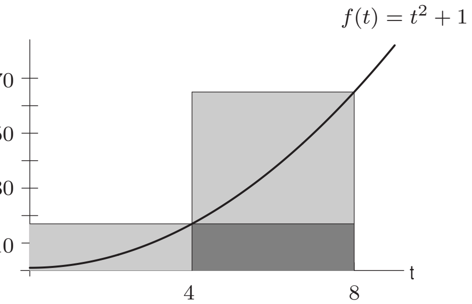


Figure 5.20

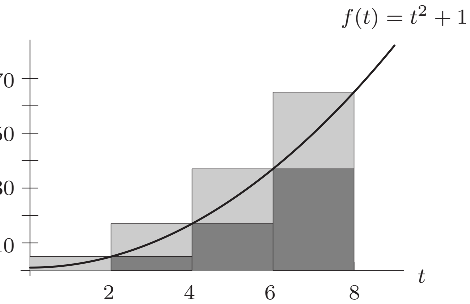


Figure 5.21

(b) If $\Delta t = 2$, then $n = 4$. We have:

$$\begin{aligned}\text{Underestimate of total change } &= f(0)\Delta t + f(2)\Delta t + f(4)\Delta t + f(6)\Delta t\\ &= 1 \cdot 2 + 5 \cdot 2 + 17 \cdot 2 + 37 \cdot 2 = 120.\\ \text{Overestimate of total change } &= f(2)\Delta t + f(4)\Delta t + f(6)\Delta t + f(8)\Delta t\\ &= 5 \cdot 2 + 17 \cdot 2 + 37 \cdot 2 + 65 \cdot 2 = 248.\end{aligned}$$

See Figure 5.21.

(c) If $\Delta t = 1$, then $n = 8$.
Underestimate of total change

$$\begin{aligned}&= f(0)\Delta t + f(1)\Delta t + f(2)\Delta t + f(3)\Delta t + f(4)\Delta t + f(5)\Delta t + f(6)\Delta t + f(7)\Delta t\\ &= 1 \cdot 1 + 2 \cdot 1 + 5 \cdot 1 + 10 \cdot 1 + 17 \cdot 1 + 26 \cdot 1 + 37 \cdot 1 + 50 \cdot 1 = 148.\end{aligned}$$

Overestimate of total change

$$\begin{aligned}&= f(1)\Delta t + f(2)\Delta t + f(3)\Delta t + f(4)\Delta t + f(5)\Delta t + f(6)\Delta t + f(7)\Delta t + f(8)\Delta t\\ &= 2 \cdot 1 + 5 \cdot 1 + 10 \cdot 1 + 17 \cdot 1 + 26 \cdot 1 + 37 \cdot 1 + 50 \cdot 1 + 65 \cdot 1 = 212.\end{aligned}$$

See Figure 5.22.

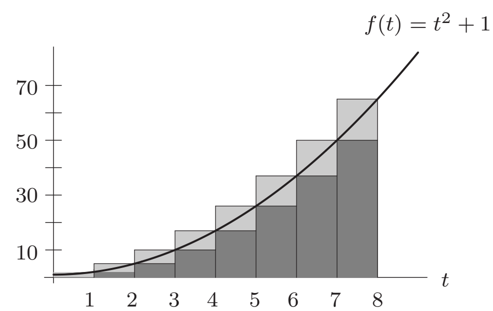
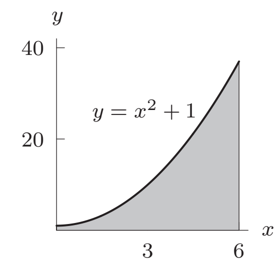


Figure 5.22

17. **(a)** $\int_0^6 (x^2+1)\,dx = 78$



(b) Using $n = 3$, we have

$$\text{Left-hand sum} = f(0)\cdot 2 + f(2)\cdot 2 + f(4)\cdot 2 = 1\cdot 2 + 5\cdot 2 + 17\cdot 2 = 46.$$

This sum is an underestimate. See Figure 5.23.

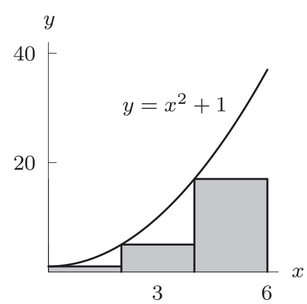


Figure 5.23

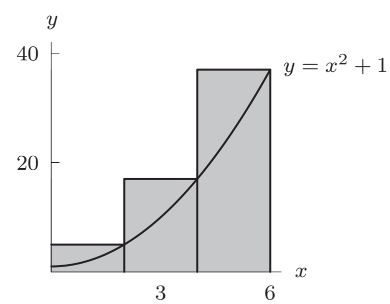


Figure 5.24

(c)

$$\text{Right-hand sum} = f(2)\cdot 2 + f(4)\cdot 2 + f(6)\cdot 2 = 5\cdot 2 + 17\cdot 2 + 37\cdot 2 = 118.$$

This sum is an overestimate. See Figure 5.24.

18. The integral represents the area below the graph of $f(x)$ but above the x-axis. Since each square has area 1, by counting squares and half-squares we find

$$\int_1^6 f(x)\,dx = 8.5.$$

19. The graph given shows that f is positive for $0 \le t \le 1$. Since the graph is contained within a rectangle of height 100 and length 1, the answers -98.35 and 100.12 are both either too small or too large to represent $\int_0^1 f(t)dt$. Since the graph of f is above the horizontal line $y = 80$ for $0 \le t \le 0.95$, the best estimate is 93.47 and not 71.84.

20. $\int_0^5 x^2\,dx = 41.7$

21. $\int_1^5 (3x+1)^2\,dx = 448.0$

22. $\int_1^4 \frac{1}{\sqrt{1+x^2}}\,dx = 1.2$

23. $\int_{-1}^1 \frac{1}{e^t}\,dt = 2.350$

24. $\int_{1.1}^{1.7} 10(0.85)^t\, dt = 4.8$

25. $\int_{1}^{2} 2^x\, dx = 2.9$

26. $\int_{1}^{2} (1.03)^t\, dt = 1.0$

27. The integral $\int_{1}^{3} \ln x dx \approx 1.30$

28. $\int_{1.1}^{1.7} e^t \ln t\, dt = 0.865$

29. $\int_{-3}^{3} e^{-t^2}\, dt \approx 2\int_{0}^{3} e^{-t^2}\, dt = 2(0.886) = 1.772$

30. **(a)** With $n = 4$, we have $\Delta t = 2$. Then

$$t_0 = 15, t_1 = 17, t_2 = 19, t_3 = 21, t_4 = 23 \quad \text{and} \quad f(t_0) = 10, f(t_1) = 13, f(t_2) = 18, f(t_3) = 20, f(t_4) = 30$$

(b)

$$\begin{aligned}\text{Left sum } &= (10)(2) + (13)(2) + (18)(2) + (20)(2) = 122\\ \text{Right sum } &= (13)(2) + (18)(2) + (20)(2) + (30)(2) = 162.\end{aligned}$$

(c) With $n = 2$,we have $\Delta t = 4$. Then

$$t_0 = 15, t_1 = 19, t_2 = 23 \quad \text{and} \quad f(t_0) = 10, f(t_1) = 18, f(t_2) = 30$$

(d)

$$\begin{aligned}\text{Left sum } &= (10)(4) + (18)(4) = 112\\ \text{Right sum } &= (18)(4) + (30)(4) = 192.\end{aligned}$$

31. **(a)** With $n = 4$, we have $\Delta t = 4$. Then

$$t_0 = 0, t_1 = 4, t_2 = 8, t_3 = 12, t_4 = 16 \quad \text{and} \quad f(t_0) = 25, f(t_1) = 23, f(t_2) = 22, f(t_3) = 20, f(t_4) = 17$$

(b)

$$\begin{aligned}\text{Left sum } &= (25)(4) + (23)(4) + (22)(4) + (20)(4) = 360\\ \text{Right sum } &= (23)(4) + (22)(4) + (20)(4) + (17)(4) = 328.\end{aligned}$$

(c) With $n = 2$,we have $\Delta t = 8$. Then

$$t_0 = 0, t_1 = 8, t_2 = 16 \quad \text{and} \quad f(t_0) = 25, f(t_1) = 22, f(t_2) = 17$$

(d)

$$\begin{aligned}\text{Left sum } &= (25)(8) + (22)(8) = 376\\ \text{Right sum } &= (22)(8) + (17)(8) = 312.\end{aligned}$$

Solutions for Section 5.3

1. The area $= \int_{0}^{2} (x^3 + 2)dx$. Using technology to evaluate the integral, we see that Area $= \int_{0}^{2} (x^3 + 2)dx = 8$. See Figure 5.25.

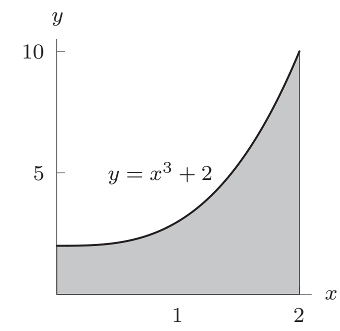


Figure 5.25

2. See Figure 5.26.

$$\text{Area} = \int_0^8 100(0.6)^t \, dt \approx 192.47$$

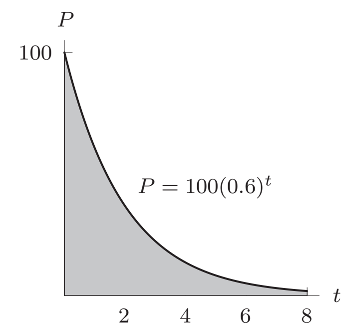


Figure 5.26

3. On the interval $0 \leq x \leq 2$, we see that $x+5$ is greater than $2x+1$. We have

$$\text{Area} = \int_0^2 ((x+5) - (2x+1))\, dx = 6.$$

4. The two functions intersect at $x = 0$ and $x = 3$. Between these values, $3x$ is greater than x^2. We have

$$\text{Area} = \int_0^3 (3x - x^2)\, dx = 4.5.$$

5. **(a)** The total area between $f(x)$ and the x-axis is the sum of the two given areas, so

$$\text{Area} = 7 + 6 = 13.$$

(b) To find the integral, we note that from $x = 3$ to $x = 5$, the function lies below the x-axis, and hence makes a negative contribution to the integral. So

$$\int_0^5 f(x)\, dx = \int_0^3 f(x)dx + \int_3^5 f(x)dx = 7 - 6 = 1.$$

6. The area below the x-axis is greater than the area above the x-axis, so the integral is negative.

7. The entire graph of the function lies above the x-axis, so the integral is positive.

8. The areas above and below the x-axis are approximately equal in size, so the integral is approximately zero.

9. The area above the x-axis is larger than the area below the x-axis, so the integral is positive.

10. (a) Counting the squares yields an estimate of 16.5, each with area $= 1$, so the total shaded area is approximately 16.5.

(b)

$$\int_0^8 f(x)dx = (\text{shaded area above } x\text{-axis}) - (\text{shaded area below } x\text{-axis})$$
$$\approx 6.5 - 10 = -3.5$$

(c) The answers in (a) and (b) are different because the shaded area below the x-axis is subtracted in order to find the value of the integral in (b).

11. We know that

$$\int_{-3}^{5} f(x)dx = \text{Area above the axis} - \text{Area below the axis.}$$

The area above the axis is about 3 boxes. Since each box has area $(1)(5) = 5$, the area above the axis is about $(3)(5) = 15$. The area below the axis is about 11 boxes, giving an area of about $(11)(5) = 55$. We have

$$\int_{-3}^{5} f(x)dx \approx 15 - 55 = -40.$$

12. (a) The area shaded between 0 and 1 is about the same as the area shaded between -1 and 0, so the area between 0 and 1 is about 0.25. Since this area lies below the x-axis, we estimate that

$$\int_0^1 f(x)\,dx = -0.25.$$

(b) Between -1 and 1, the area above the x-axis approximately equals the area below the x-axis, and so

$$\int_{-1}^{1} f(x)\,dx = (0.25) + (-0.25) = 0.$$

(c) We estimate

$$\text{Total area shaded} = 0.25 + 0.25 = 0.5.$$

13. The region shaded between $x = 0$ and $x = 2$ appears to have approximately the same area as the region shaded between $x = -2$ and $x = 0$, but it lies below the axis. Since $\int_{-2}^{0} f(x)dx = 4$, we have the following results:

(a) $\int_0^2 f(x)dx \approx -\int_{-2}^{0} f(x)dx = -4$.
(b) $\int_{-2}^{2} f(x)dx \approx 4 - 4 = 0$.
(c) The total area shaded is approximately $4 + 4 = 8$.

14. The area above the x-axis appears to be about half the area of the rectangle with area $10 \cdot 3 = 30$, so we estimate the area above the x-axis to be approximately 15. The area below the x-axis appears to be about half the area of the rectangle with area $5 \cdot 2 = 10$, so we estimate the area below the x-axis to be approximately 5. See Figure 5.27. The value of the integral is the area above the x-axis minus the area below the x-axis, so we estimate

$$\int_0^5 f(x)dx \approx 15 - 5 = 10.$$

The correct match for this function is IV.

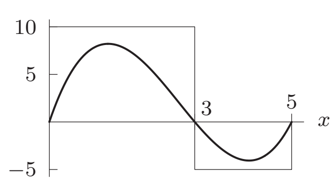


Figure 5.27

15. The area below the x-axis is bigger than the area above the x-axis, so the integral is negative. The area above the x-axis appears to be about half the area of the rectangle with area $20 \cdot 2 = 40$, so we estimate the area above the x-axis to be approximately 20. The area below the x-axis appears to be about half the area of the rectangle with area $15 \cdot 3 = 45$, so we estimate the area below the x-axis to be approximately 22.5. See Figure 5.28. The value of the integral is the area above the x-axis minus the area below the x-axis, so we estimate

$$\int_0^5 f(x)dx \approx 20 - 22.5 = -2.5.$$

The correct match for this function is II.

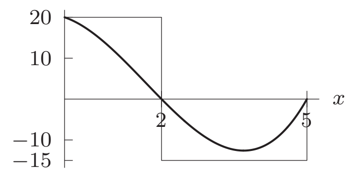


Figure 5.28

16. The area below the x-axis is bigger than the area above the x-axis, so the integral is negative. The area above the x-axis appears to be about half the area of the rectangle with area $5 \cdot 2 = 10$, so we estimate the area above the x-axis to be approximately 5. The area below the x-axis appears to be about half the area of the rectangle with area $10 \cdot 3 = 30$, so we estimate the area below the x-axis to be approximately 15. See Figure 5.29. The value of the integral is the area above the x-axis minus the area below the x-axis, so we estimate

$$\int_0^5 f(x)dx \approx 5 - 15 = -10.$$

The correct match for this function is I.

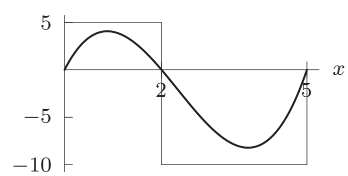


Figure 5.29

17. On the interval $0 \le x \le 5$, the entire graph lies above the x-axis, so the value of the integral is positive and equal to the area between the graph of the function and the x-axis. This area appears to be about half the area of the rectangle with area $2 \cdot 5 = 10$, so we estimate the area to be approximately 5. See Figure 5.30. Since $f(x)$ is positive on this interval, the value of the integral is equal to this area, so we have

$$\int_0^5 f(x)dx = \text{ Area } \approx 5.$$

The correct match for this function is III.

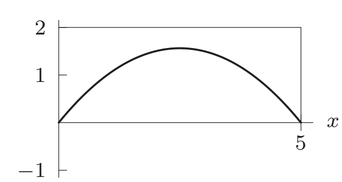


Figure 5.30

18. **(a)** The area between the graph of f and the x-axis between $x = a$ and $x = b$ is 13, so

$$\int_a^b f(x)\,dx = 13.$$

(b) Since the graph of $f(x)$ is below the x-axis for $b < x < c$,

$$\int_b^c f(x)\,dx = -2.$$

(c) Since the graph of $f(x)$ is above the x-axis for $a < x < b$ and below for $b < x < c$,

$$\int_a^c f(x)\,dx = 13 - 2 = 11.$$

(d) The graph of $|f(x)|$ is the same as the graph of $f(x)$ except that the part below the x-axis is reflected to be above it. See Figure 5.31. Thus

$$\int_a^c |f(x)|\,dx = 13 + 2 = 15.$$

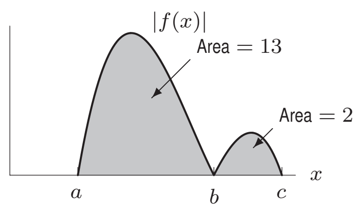


Figure 5.31

19. In Figure 5.32 the area A_1 is largest, A_2 is next, and A_3 is smallest. We have

$$\mathrm{I} = \int_a^b f(x)\,dx = A_1, \quad \mathrm{II} = \int_a^c f(x)\,dx = A_1 - A_2, \quad \mathrm{III} = \int_a^e f(x)\,dx = A_1 - A_2 + A_3,$$
$$\mathrm{IV} = \int_b^e f(x)\,dx = -A_2 + A_3, \quad \mathrm{V} = \int_b^c f(x)\,dx = -A_2.$$

The relative sizes of A_1, A_2, and A_3 mean that I is positive and largest, III is next largest (since $-A_2 + A_3$ is negative, but less negative than $-A_2$), II is next largest, but still positive (since A_1 is larger than A_2). The integrals IV and V are both negative, but V is more negative. Thus

$$\mathrm{V} < \mathrm{IV} < 0 < \mathrm{II} < \mathrm{III} < \mathrm{I}.$$

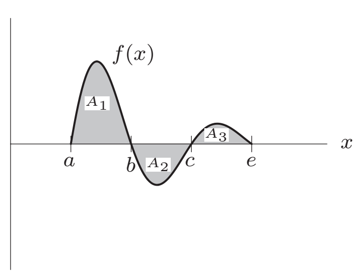
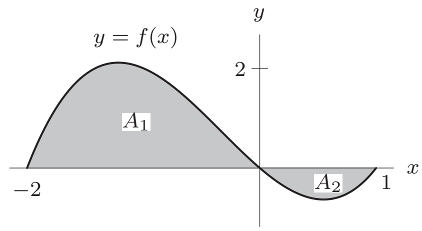


Figure 5.32

20. **(a)**



(b) $A_1 = \int_{-2}^{0} f(x)\,dx = 2.667.$

$A_2 = -\int_{0}^{1} f(x)\,dx = 0.417.$

So total area $= A_1 + A_2 \approx 3.084$. Note that while A_1 and A_2 are accurate to 3 decimal places, the quoted value for $A_1 + A_2$ is accurate only to 2 decimal places.

(c) $\int_{-2}^{1} f(x)\,dx = A_1 - A_2 = 2.250.$

21. **(a)** $\int_{-3}^{0} f(x)\,dx = -2.$

(b) $\int_{-3}^{4} f(x)\,dx = \int_{-3}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{4} f(x)\,dx = -2 + 2 - \frac{A}{2} = -\frac{A}{2}.$

22. It appears that $f(x)$ is positive on the whole interval $0 \le x \le 20$, so we have

$$\text{Area} = \int_{0}^{20} f(x)dx.$$

We estimate the value of the integral using left and right sums:

$$\text{Left-hand sum} = 15 \cdot 5 + 18 \cdot 5 + 20 \cdot 5 + 16 \cdot 5 = 345.$$
$$\text{Right-hand sum} = 18 \cdot 5 + 20 \cdot 5 + 16 \cdot 5 + 12 \cdot 5 = 330.$$

A better estimate of the area is the average of the two:

$$\text{Area} \approx \frac{345 + 330}{2} = 337.5.$$

23.

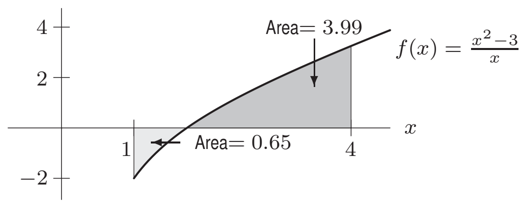


Figure 5.33

Using a calculator or computer, we see that

$$\int_{1}^{4} \frac{x^2 - 3}{x}\,dx \approx 3.34.$$

The graph of $f(x) = \dfrac{x^2 - 3}{x}$ is shown in Figure 5.33. The function is negative to the left of $x = \sqrt{3} \approx 1.73$ and positive to the right of $x = 1.73$. We compute

$$\int_{1}^{1.73} \frac{x^2 - 3}{x}\,dx = -\text{Area below axis} \approx -0.65$$

and

$$\int_{1.73}^{4} \frac{x^2-3}{x}\,dx = \text{Area above axis } \approx 3.99.$$

See Figure 5.33. Then

$$\int_{1}^{4} \frac{x^2-3}{x}\,dx = \text{Area above axis}-\text{Area below axis } = 3.99 - 0.65 = 3.34.$$

24. Using a calculator or computer, we find

$$\int_1^4 (x - 3\ln x)\,dx \approx -0.136.$$

The function $f(x) = x - 3\ln x$ crosses the x-axis at $x \approx 1.86$. See Figure 5.34.

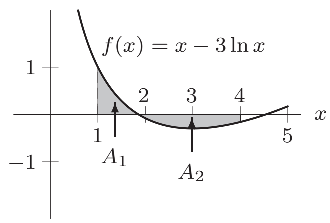


Figure 5.34

We find

$$\int_1^{1.86} (x - 3\ln x)\,dx = \text{Area above axis } \approx 0.347$$

$$\int_{1.86}^{4} (x - 3\ln x)\,dx = -\text{Area below axis } \approx -0.483.$$

Thus, $A_1 \approx 0.347$ and $A_2 \approx 0.483$, so

$$\int_1^4 (x - 3\ln x)\,dx = \text{Area above axis}-\text{Area below axis } \approx 0.347 - 0.483 = -0.136.$$

25. $\displaystyle\int_0^4 \cos\sqrt{x}\,dx = 0.80 =$ Area A_1 – Area A_2. See Figure 5.35.

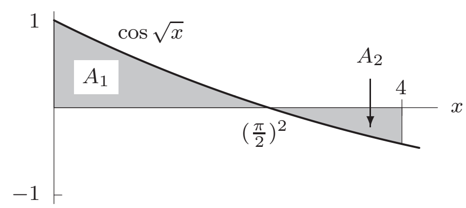


Figure 5.35

26. The graph of $y = x^2 - 2$ is shown in Figure 5.36, and the relevant area is shaded. If you compute the integral $\int_0^3 (x^2-2)dx$, you find that

$$\int_0^3 (x^2-2)dx = 3.0.$$

However, since part of the area lies below the x-axis and part of it lies above the x-axis, this computation does not help us at all. (In fact, it is clear from the graph that the shaded area is more than 3.) We have to find the area above the x-axis and the area below the x-axis separately. We find that the graph crosses the x-axis at $x = 1.414$, and we compute the two areas separately:

$$\int_0^{1.414} (x^2 - 2)dx = -1.886 \quad \text{and} \quad \int_{1.414}^{3} (x^2 - 2)dx = 4.886.$$

As we expect, we see that the integral between 0 and 1.414 is negative and the integral between 1.414 and 3 is positive. The total area shaded is the sum of the absolute values of the two integrals:

$$\text{Area shaded } = 1.886 + 4.886 = 6.772 \quad \text{square units.}$$

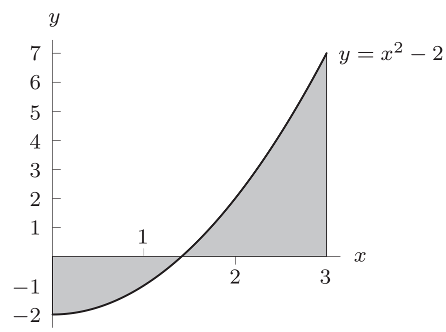


Figure 5.36

27. A graph of $y = 6x^3 - 2$ shows that this function is nonnegative on the interval $x = 5$ to $x = 10$. Thus,

$$\text{Area } = \int_5^{10} (6x^3 - 2)\, dx = 14{,}052.5.$$

The integral was evaluated on a calculator.

28. A graph of $y = 2\cos(t/10)$ shows that this function is nonnegative on the interval $t = 1$ to $t = 2$. Thus,

$$\text{Area } = \int_1^2 2\cos\frac{t}{10}\, dt = 1.977.$$

The integral was evaluated on a calculator.

29. The graph of $y = 5\ln(2x)$ is above the line $y = 3$ for $3 \le x \le 5$. See Figure 5.37. Therefore

$$\text{Area } = \int_3^5 (5\ln(2x) - 3)\, dx = 14.688.$$

The integral was evaluated on a calculator.

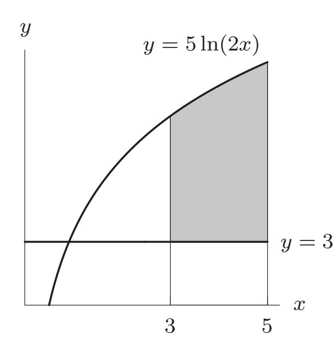


Figure 5.37

30. The graph of $y = \sin x + 2$ is above the line $y = 0.5$ for $6 \le x \le 10$. See Figure 5.38. Therefore

$$\text{Area } = \int_{6}^{10} \sin x + 2 - 0.5\, dx = 7.799.$$

The integral was evaluated on a calculator.

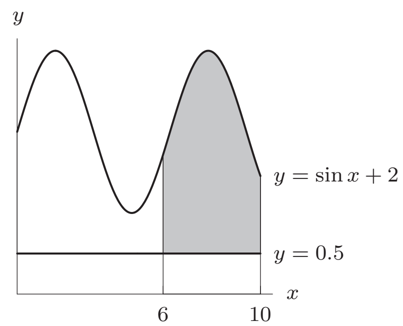


Figure 5.38

31. The graph of $y = \cos x + 7$ is above $y = \ln(x - 3)$ for $5 \le x \le 7$. See Figure 5.39. Therefore

$$\text{Area } = \int_{5}^{7} \cos x + 7 - \ln(x-3)\, dx = 13.457.$$

The integral was evaluated on a calculator.

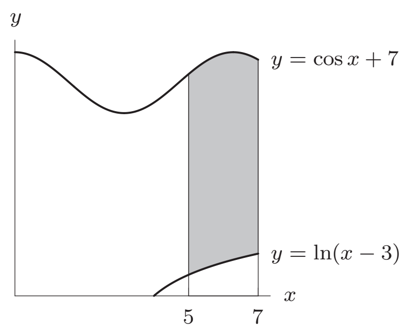


Figure 5.39

32. The graph of $y = x^4 - 8$ has intercepts $x = \pm\sqrt[4]{8}$. See Figure 5.40. Since the region is below the x-axis, the integral is negative, so

$$\text{Area } = -\int_{-\sqrt[4]{8}}^{\sqrt[4]{8}} (x^4 - 8)\, dx = 21.527.$$

The integral was evaluated on a calculator.

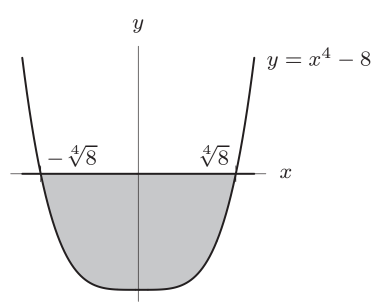


Figure 5.40

33. The graph of $y = -e^x + e^{2(x-1)}$ has intercepts where $e^x = e^{2(x-1)}$, or where $x = 2(x-1)$, so $x = 2$. See Figure 5.41. Since the region is below the x-axis, the integral is negative, so

$$\text{Area } = -\int_0^2 -e^x + e^{2(x-1)}\,dx = 2.762.$$

The integral was evaluated on a calculator.

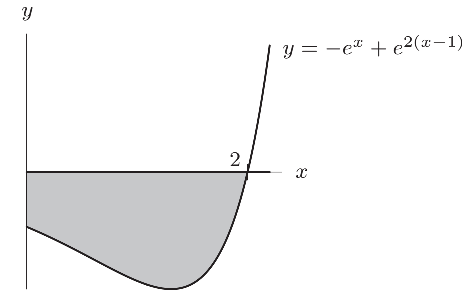


Figure 5.41

34. The graph of $y = \cos t$ is above the graph of $y = \sin t$ for $0 \le t \le \pi/4$ and $y = \cos t$ is below $y = \sin t$ for $\pi/4 < t < \pi$. See Figure 5.42. Therefore, we find the area in two pieces:

$$\text{Area } = \int_0^{\pi/4} (\cos t - \sin t)\,dt + \int_{\pi/4}^{\pi} (\sin t - \cos t)\,dt = 2.828.$$

The integral was evaluated on a calculator.

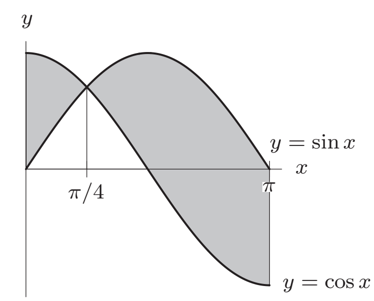


Figure 5.42

Solutions for Section 5.4

1. **(a)** The integral $\int_0^{30} f(t)dt$ represents the total emissions of nitrogen oxides, in millions of metric tons, during the period 1970 to 2000.

(b) We estimate the integral using left- and right-hand sums:

$$\text{Left sum} = (26.9)5 + (26.4)5 + (27.1)5 + (25.8)5 + (25.5)5 + (25.0)5 = 783.5.$$

$$\text{Right sum} = (26.4)5 + (27.1)5 + (25.8)5 + (25.5)5 + (25.0)5 + (22.6)5 = 762.0.$$

We average the left- and right-hand sums to find the best estimate of the integral:

$$\int_0^{30} f(t)dt \approx \frac{783.5 + 762.0}{2} = 772.8 \text{ million metric tons.}$$

Between 1970 and 2000, about 772.8 million metric tons of nitrogen oxides were emitted.

2. We use left- and right-hand sums to estimate the total amount of coal produced during this period:

$$\text{Left sum} = (10.82)5 + (13.06)5 + (14.61)5 + (14.99)5 + (18.60)5 + (19.33)5 = 457.05.$$

$$\text{Right sum} = (13.06)5 + (14.61)5 + (14.99)5 + (18.60)5 + (19.33)5 + (22.46)5 = 515.25.$$

We see that

$$\text{Total amount of coal produced} \approx \frac{457.05 + 515.25}{2} = 486.15 \text{ quadrillion BTU.}$$

The total amount of coal produced is the definite integral of the rate of coal production $r = f(t)$ given in the table. Since t is in years since 1960, the limits of integration are $t = 0$ and $t = 30$. We have

$$\text{Total amount of coal produced} = \int_0^{30} f(t)\,dt \text{ quadrillion BTU.}$$

3. The integral $\int_1^3 v(t)\,dt$ represents the change in position between time $t = 1$ and $t = 3$ seconds; it is measured in meters.

4. The integral $\int_0^6 a(t)\,dt$ represents the change in velocity between times $t = 0$ and $t = 6$ seconds; it is measured in km/hr.

5. The integral $\int_{2000}^{2004} f(t)\,dt$ represents the change in the world's population between the years 2000 and 2004. It is measured in billions of people.

6. The integral $\int_0^5 s(x)\,dx$ represents the change in salinity (salt concentration) in the first 5 cm of water; it is measured in gm/liter.

7. For any t, consider the interval $[t, t+\Delta t]$. During this interval, oil is leaking out at an approximately constant rate of $f(t)$ gallons/minute. Thus, the amount of oil which has leaked out during this interval can be expressed as

$$\text{Amount of oil leaked} = \text{Rate} \times \text{Time} = f(t)\,\Delta t$$

and the units of $f(t)\,\Delta t$ are gallons/minute $\times$ minutes $=$ gallons. The total amount of oil leaked is obtained by adding all these amounts between $t = 0$ and $t = 60$. (An hour is 60 minutes.) The sum of all these infinitesimal amounts is the integral

$$\begin{array}{c}\text{Total amount of} \\ \text{oil leaked, in gallons}\end{array} = \int_0^{60} f(t)\,dt.$$

8. If $H(t)$ is the temperature of the coffee at time t, by the Fundamental Theorem of Calculus

$$\text{Change in temperature} = H(10) - H(0) = \int_0^{10} H'(t)\,dt = \int_0^{10} -7(0.9^t)\,dt.$$

Therefore,

$$H(10) = H(0) + \int_0^{10} -7(0.9^t)\,dt \approx 90 - 44.2 = 45.8^\circ\text{C}.$$

9. The total amount of antibodies produced is

$$\text{Total antibodies} = \int_0^4 r(t)dt \approx 1.417 \text{ thousand antibodies}$$

10. **(a)** In 1990, when $t = 0$, gas consumption was 1770 millions of metric tons of oil equivalent. In 2010, when $t = 20$, gas consumption was $N = 1770 + 53(20) = 2830$ million metric tons of oil equivalent.

(b) We use an integral to approximate the sum giving the total amount consumed over the 20-year period:

$$\text{Total amount of gas consumed} = \int_0^{20} (1770 + 53t)\,dt = 46{,}000 \text{ million metric tons of oil equivalent.}$$

11. The quantity S is the rate of production, in megawatts per year. Use an integral to approximate the sum giving total production:

$$\text{Total PV production} = \int_0^{10} 277e^{0.368t}dt = 29{,}089.813 \text{ megawatts.}$$

12. (a) The distance traveled in the first 3 hours (from $t = 0$ to $t = 3$) is given by

$$\int_0^3 (40t - 10t^2)dt.$$

(b) The shaded area in Figure 5.43 represents the distance traveled.

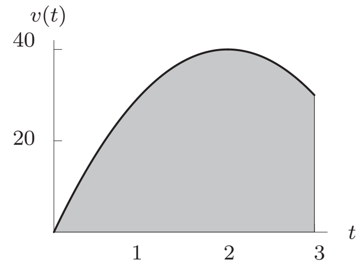


Figure 5.43

(c) Using a calculator, we get

$$\int_0^3 (40t - 10t^2)dt = 90.$$

So the total distance traveled is 90 miles.

13. Since $v(t) \geq 0$ for $0 \leq t \leq 3$, we can find the total distance traveled by integrating the velocity from $t = 0$ to $t = 3$:

$$\begin{aligned}\text{Distance} &= \int_0^3 \ln(t^2 + 1)\, dt \\ &= 3.4 \text{ ft, evaluating this integral by calculator.}\end{aligned}$$

14. Suppose $F(t)$ represents the total quantity of water in the water tower at time t, where t is in days since April 1. Then the graph shown in the problem is a graph of $F'(t)$. By the Fundamental Theorem,

$$F(30) - F(0) = \int_0^{30} F'(t)dt.$$

We can calculate the change in the quantity of water by calculating the area under the curve. If each box represents about 300 liters, there is about one box, or -300 liters, from $t = 0$ to $t = 12$, and 6 boxes, or about $+1800$ liters, from $t = 12$ to $t = 30$. Thus

$$\int_0^{30} F'(t)dt = 1800 - 300 = 1500,$$

so the final amount of water is given by

$$F(30) = F(0) + \int_0^{30} F'(t)dt = 12{,}000 + 1500 = 13{,}500 \text{ liters.}$$

15. (a) The weight growth rate is the derivative of the weight function. The fact that the weight growth rate is increasing means that the weight function has an increasing derivative. Thus the graph of the weight function is concave up.

(b) The total weight growth during the forty week gestation equals the integral of the growth rate over the time period. Thus the birth weight equals the area under the graph. The region is triangular, with base 28 weeks and height 0.22 kg/week. Thus,

$$\text{Total weight} = \frac{1}{2}28(0.22) = 3.08 \text{ kg} \approx 3.1 \text{ kg}.$$

16. **(a)** (i) The income curve shows the rate of change of the value of the fund due to inflow of money. The area under the curve,

$$\int_{2000}^{2015} I(t)\,dt,$$

represents the total change in the value of the fund that is due to income. It is the quantity of money, in billions of dollars, that is projected to flow into the fund between 2000 and 2015.

(ii) The expenditure curve shows the rate of change of the value of the fund due to outflow of money. The area under the curve,

$$\int_{2000}^{2015} E(t)\,dt,$$

represents the magnitude of the change in the value of the fund that is due to expenses. It is the quantity of money, in billions of dollars, that is projected to flow out of the fund between 2000 and 2015.

(iii) The area between the income and expenditure curves,

$$\int_{2000}^{2015} I(t) - E(t)\,dt,$$

represents the difference between total income and total expenses between 2000 and 2015. It is the projected change in value of the fund between 2000 and 2015.

(b) In the figure, we see that the value of the fund was about 1000 billion dollars in 2000 and is projected to be about 3500 billion dollars in 2015. The fund is projected to increase in value by about 2500 billion dollars, and that is the area between the income and expenditure curves on the graph.

17. **(a)** When income is greater than expenses, before 2023, the value of the fund increases. When the expenses are greater than income, after 2023, the value of the fund decreases. The value increases from 2000 through 2023, and then declines. The value is a maximum in 2023, where the income and expenditure curves cross.

(b) The value of the fund is represented by the height of the curve. Its highest point is in 2023, when the value is a maximum.

18. The rate of change of the value of the fund is the difference $I(t) - E(t)$ between the rate of increase due to income and the rate of decrease due to expenses. To find the total change in the value, integrate the rate of change:

$$\text{Change in value of fund} = \int_{2000}^{2030} I(t) - E(t)\,dt.$$

19. The change in the number of acres is the integral of the rate of change. We have

$$\text{Change in number of acres} = \int_{0}^{24} (8\sqrt{t})dt = 627.$$

The number of acres the fire covers after 24 hours is the original number of acres plus the change, so we have

$$\text{Acres covered after 24 hours} = 2000 + 627 = 2627 \text{ acres}.$$

20. The change in the amount of water is the integral of rate of change, so we have

$$\text{Number of liters pumped out} = \int_{0}^{60} (5 - 5e^{-0.12t})dt = 258.4 \text{ liters}.$$

Since the tank contained 1000 liters of water initially, we see that

$$\text{Amount in tank after one hour} = 1000 - 258.4 = 741.6 \text{ liters}.$$

21. Looking at the area under the graph we see that after 5 years Tree B is taller while Tree A is taller after 10 years.

22. The area under A's curve is greater than the area under B's curve on the interval from 0 to 6, so A had the most total sales in the first 6 months. On the interval from 0 to 12, the area under B's curve is greater than the area under A, so B had the most total sales in the first year. At approximately nine months, A and B appear to have sold equal amounts. Counting the squares yields a total of about 250 sales in the first year for B and 170 sales in first year for A.

23. **(a)** The black curve is for boys, the colored one for girls. The area under each curve represents the change in growth in centimeters. Since men are generally taller than women, the curve with the larger area under it is the height velocity of the boys.

(b) Each square below the height velocity curve has area $1\text{ cm/yr} \cdot 1\text{ yr} = 1$ cm. Counting squares lying below the black curve gives about 43 cm. Thus, on average, boys grow about 43 cm between ages 3 and 10.

(c) Counting squares lying below the black curve gives about 23 cm growth for boys during their growth spurt. Counting squares lying below the colored curve gives about 18 cm for girls during their growth spurt.

(d) We can measure the difference in growth by counting squares that lie between the two curves. Between ages 2 and 12.5, the average girl grows faster than the average boy. Counting squares yields about 5 cm between the colored and black curves for $2 \le x \le 12.5$. Counting squares between the curves for $12.5 \le x \le 18$ gives about 18 squares. Thus, there is a net increase of boys over girls by about $18 - 5 = 13$ cm.

24. **(a)** In the beginning, both birth and death rates are small; this is consistent with a very small population. Both rates begin climbing, the birth rate faster than the death rate, which is consistent with a growing population. The birth rate is then high, but it begins to decrease as the population increases.

(b)

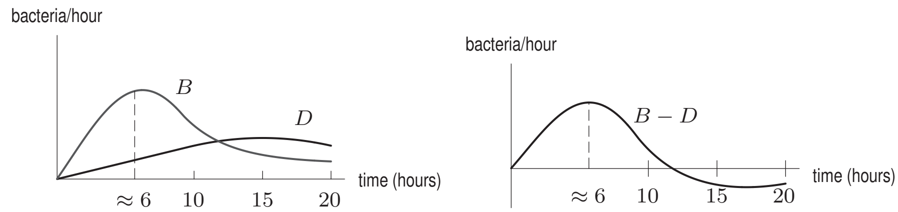


Figure 5.44: Difference between B and D is greatest at $t \approx 6$

The bacteria population is growing most quickly when $B - D$, the rate of change of population, is maximal; that happens when B is farthest above D, which is at a point where the slopes of both graphs are equal. That point is $t \approx 6$ hours.

(c) Total number born by time t is the area under the B graph from $t = 0$ up to time t. See Figure 5.45.

Total number alive at time t is the number born minus the number that have died, which is the area under the B graph minus the area under the D graph, up to time t. See Figure 5.46.

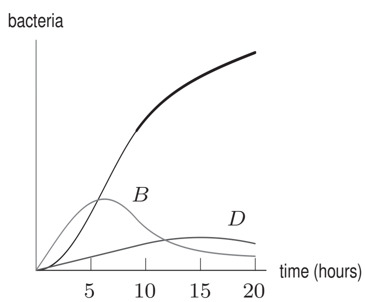


Figure 5.45: Number born by time t is $\int_0^t B(x)\,dx$

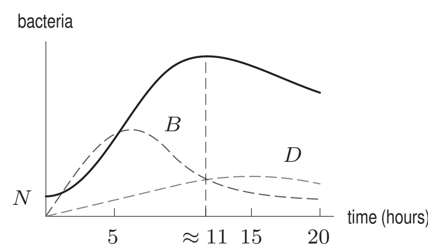


Figure 5.46: Number alive at time t is $\int_0^t (B(x) - D(x))\,dx$

From Figure 5.46, we see that the population is at a maximum when $B = D$, that is, after about 11 hours. This stands to reason, because $B - D$ is the rate of change of population, so population is maximized when $B - D = 0$, that is, when $B = D$.

25. The quantities consumed equal the areas under the graphs of the rates of consumption. There is more area under the graph of fat consumption rate, so more fat is consumed than protein.

26. **(a)** Figure 5.47 is the graph of a rate of blood flow versus time. The total quantity of blood pumped during the three hours is given by the area under the rate graph for the three-hour time interval. The area can be estimated by counting grid boxes under the graph.

Each grid rectangle has area 30 minutes $\times 1/2$ liter/minute $= 15$ liters, representing 15 liters of blood pumped. The grid boxes in the graph are stacked in six columns. Estimating the number of boxes in each column under the graph gives

$$\text{Number of boxes } = 10 + 7 + 3.75 + 3.5 + 3 + 1.5 = 28.75 \text{ boxes.}$$

Approximately

$$\text{Amount of blood pumped } = (28.75)(15) = 431.25 \text{ liters.}$$

Thus, about 431 liters of blood are pumped during the three hours leading to death.

(b) Since $f(t)$ is the pumping rate in liters/minute at time t hours, $60f(t)$ is the pumping rate in liters/hour. Thus $\int_0^3 60f(t)\,dt$ gives the total quantity of blood pumped in liters during the three hours.

(c) During three hours with no bleeding, the heart pumps 5 liters/minute for $3 \cdot 60 = 180$ minutes. Thus

$$\text{Total blood pumped } = 5 \cdot 180 = 900 \text{ liters.}$$

This is $900 - 431.25 = 468.75$ liters more than pumped with 2 liter bleeding. Thus, about 470 liters are pumped. This corresponds to the area on the graph between the 5 liters/minute line and the pumping rate for the 2 liter bleed. See Figure 5.48.

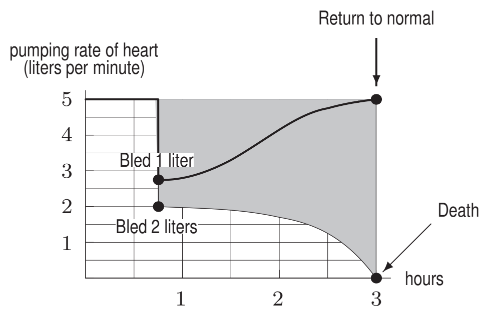


Figure 5.47

27. **(a)** Figure 5.48 is the graph of a rate of blood flow versus time. The total quantity of blood pumped during the three hours is given by the area under the rate graph for the three-hour time interval. The area can be estimated by counting grid boxes under the graph.

Each grid rectangle has area 30 minutes $\times 1/2$ liter/minute $= 15$ liters, corresponding to 15 liters of blood pumped. The grid boxes in the graph are stacked in six columns. Estimating the number of boxes in each column under the graph gives

$$\text{Number of boxes } = 10 + 7.75 + 6 + 7.5 + 9 + 9.75 = 50 \text{ boxes.}$$

Approximately

$$\text{Amount of blood pumped } = 50 \cdot 15 = 750 \text{ liters.}$$

Thus, about 750 liters of blood are pumped during the three hours leading to full recovery.

(b) Since $g(t)$ is the pumping rate in liters/minute at time t hours, $60g(t)$ is the pumping rate in liters/hour. Thus $\int_0^3 60g(t)\,dt$ gives the total quantity of blood pumped in liters during the three hours.

(c) During three hours with no bleeding, the heart pumps 5 liters/minute for $3 \cdot 60 = 180$ minutes. Thus,

$$\text{Total blood pumped } = 5 \cdot 180 = 900 \text{ liters.}$$

This is $900 - 750 = 150$ liters more than pumped with 1 liter bleeding. This corresponds to the area on the graph between the 5 liters/minute line and the pumping rate for the 1 liter bleed. See Figure 5.47.

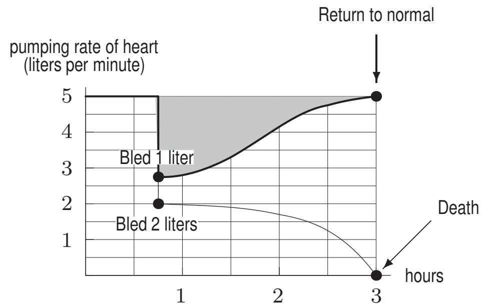


Figure 5.48

28. Since W is in tons per week and t is in weeks since January 1, 2005, the integral $\int_0^{52} W\,dt$ gives the amount of waste, in tons, produced during the year 2005.

$$\text{Total waste during the year} = \int_0^{52} 3.75e^{-0.008t}\,dt = 159.5249 \text{ tons}.$$

Since waste removal costs $15/ton, the cost of waste removal for the company is $159.5249 \cdot 15 = \$2392.87$.

29. The velocity is constant and negative, so the change in position is $-3 \cdot 5$ cm, that is 15 cm to the left.

30. From $t = 0$ to $t = 3$ the velocity is constant and positive, so the change in position is $2 \cdot 3$ cm, that is 6 cm to the right. From $t = 3$ to $t = 5$, the velocity is negative and constant, so the change in position is $-3 \cdot 2$ cm, that is 6 cm to the left. Thus the total change in position is 0. The particle moves 6 cm to the right, followed by 6 cm to the left, and returns to where it started.

31. From $t = 0$ to $t = 5$ the velocity is positive so the change in position is to the right. The area under the velocity graph gives the distance traveled. The region is a triangle, and so has area $(1/2)bh = (1/2)5 \cdot 10 = 25$. Thus the change in position is 25 cm to the right.

32. From $t = 0$ to $t = 4$ the velocity is positive so the change in position is to the right. The area under the velocity graph gives the distance traveled. The region is a triangle, and so has area $(1/2)bh = (1/2)4 \cdot 8 = 16$. Thus the change in position is 16 cm to the right for $t = 0$ to $t = 4$. From $t = 4$ to $t = 5$, the velocity is negative so the change in position is to the left. The distance traveled to the left is given by the area of the triangle, $(1/2)bh = (1/2)1 \cdot 2 = 1$. Thus the total change in position is $16 - 1 = 15$ cm to the right.

33. From $t = 0$ to $t = 3$, you are moving away from home ($v > 0$); thereafter you move back toward home. So you are the farthest from home at $t = 3$. To find how far you are then, we can measure the area under the v curve as about 9 squares, or $9 \cdot 10$ km/hr $\cdot$ 1 hr $= 90$ km. To find how far away from home you are at $t = 5$, we measure the area from $t = 3$ to $t = 5$ as about 25 km, except that this distance is directed toward home, giving a total distance from home during the trip of $90 - 25 = 65$ km.

34. **(a)** At $t = 20$ minutes, she stops moving toward the lake (with $v > 0$) and starts to move away from the lake (with $v < 0$). So at $t = 20$ minutes the cyclist turns around.

(b) The cyclist is going the fastest when v has the greatest magnitude, either positive or negative. Looking at the graph, we can see that this occurs at $t = 40$ minutes, when $v = -25$ and the cyclist is pedaling at 25 km/hr away from the lake.

(c) From $t = 0$ to $t = 20$ minutes, the cyclist comes closer to the lake, since $v > 0$; thereafter, $v < 0$ so the cyclist moves away from the lake. So at $t = 20$ minutes, the cyclist comes the closest to the lake. To find out how close she is, note that between $t = 0$ and $t = 20$ minutes the distance she has come closer is equal to the area under the graph of v. Each box represents 5/6 of a kilometer, and there are about 2.5 boxes under the graph, giving a distance of about 2 km. Since she was originally 5 km away, she then is about $5 - 2 = 3$ km from the lake.

(d) At $t = 20$ minutes she turns around , since v changes sign then. Since the area below the t-axis is greater than the area above, the farthest she is from the lake is at $t = 60$ minutes. Between $t = 20$ and $t = 60$ minutes, the area under the graph is about 10.8 km. (Since 13 boxes $\cdot 5/6 = 10.8$.) So at $t = 60$ she will be about $3 + 10.8 = 13.8$ km from the lake.

35. Looking at the figure in the problem, we note that Product B has a greater peak concentration than Product A; Product A peaks sooner than Product B; Product B has a greater overall bioavailability than Product A. Since we are looking for the product providing the faster response, Product A should be used as it peaks sooner.

36. Looking at the figure in the problem, we note that Product A has a greater peak concentration than Product B; Product A peaks much faster that Product B; Product B has greater overall bioavailability than Product A. Since Product B has greater bioavailability, it provides a greater amount of drug reaching the bloodstream. Therefore Product B provides relief for a longer time than Product A. On the other hand, Product A provides faster relief.

37.

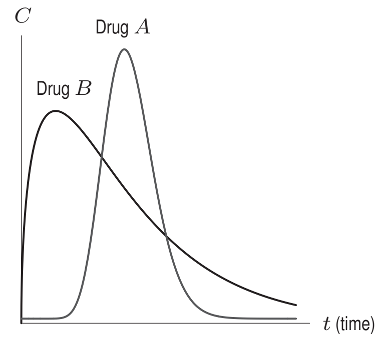


Figure 5.49

38. The total number of "worker-hours" is equal to the area under the curve. The total area is about 14.5 boxes. Since each box represents (10 workers)(8 hours) $=$ 80 worker-hours, the total area is 1160 worker-hours. At \$10 per hour, the total cost is \$11,600.

39. The time period 9am to 5pm is represented by the time $t = 0$ to $t = 8$ and $t = 24$ to $t = 32$. The area under the curve, or total number of worker-hours for these times, is about 9 boxes or $9(80) = 720$ worker-hours. The total cost for 9am to 5pm is $(720)(10) =$ \$7200. The area under the rest of the curve is about 5.5 boxes, or $5.5(80) = 440$ worker-hours. The total cost for this time period is $(440)(15) =$ \$6600. The total cost is about $7200 + 6600 =$ \$13,800.

40. **(a)** The amount when $t = 0$ is four times the maximum acceptable limit, or $4 \cdot 0.6 = 2.4$. Therefore the level of radiation is given by

$$R(t) = 2.4(0.996)^t.$$

We want to find the value of t making $R(t) = 0.6$. The graph of the function in Figure 5.50 shows that $R(t) = 0.6$ at about $t = 346$, so the level of radiation reaches an acceptable level after about 346 hours (or about 2 weeks).

Alternatively, using logarithms gives

$$\begin{aligned}\frac{0.6}{2.4} &= (0.996)^t \\ \ln(0.25) &= \ln(0.996)^t = t\ln(0.996) \\ t &= \frac{\ln(0.25)}{\ln(0.996)} = 346.\end{aligned}$$

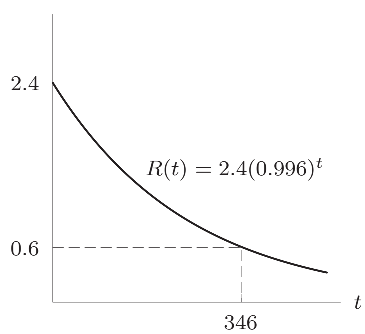


Figure 5.50

(b) Total radiation emitted between $t = 0$ and $t = 346$ is

$$\int_0^{346} 2.4(0.996)^t\,dt \approx 449 \text{ millirems.}$$

41. (a) The distance traveled is the integral of the velocity, so in T seconds you fall

$$\int_0^T 49(1 - 0.8187^t)\,dt.$$

(b) We want the number T for which

$$\int_0^T 49(1 - 0.8187^t)\,dt = 5000.$$

We can use a calculator or computer to experiment with different values for T, and we find $T \approx 107$ seconds.

42. (a) The acceleration is positive for $0 \le t < 40$ and for a tiny period before $t = 60$, since the slope is positive over these intervals. Just to the left of $t = 40$, it looks like the acceleration is approaching 0. Between $t = 40$ and a moment just before $t = 60$, the acceleration is negative.

(b) The maximum altitude was about 500 feet, when t was a little greater than 40 (here we are estimating the area under the graph for $0 \le t \le 42$).

(c) The total change in altitude for the Montgolfiers and their balloon is the definite integral of their velocity, or the total area under the given graph (counting the part after $t = 42$ as negative, of course). As mentioned before, the total area of the graph for $0 \le t \le 42$ is about 500. The area for $t > 42$ is about 220. So subtracting, we see that the balloon finished 280 feet or so higher than where it began.

Solutions for Section 5.5

1. The units for the integral $\int_{800}^{900} C'(q)dq$ are $\left(\frac{\text{dollars}}{\text{tons}}\right) \cdot (\text{tons}) = \text{dollars}$.

$\int_{800}^{900} C'(q)dq$ represents the cost of increasing production from 800 tons to 900 tons.

2. We are told that $C'(q) = q^2 - 50q + 700$ for $0 \le q \le 50$. We also know that

$$\int_0^{50} C'(q)dq = C(50) - C(0).$$

Since we are told that fixed costs are \$500 we know that

$$C(0) = \$500.$$

Thus

$$C(50) = \int_0^{50} C'(q)dq + \$500 \approx \$14{,}667.$$

3. (a)

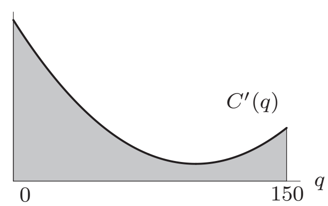


Figure 5.51

The total variable cost of producing 150 units is represented by the area under the graph of $C'(q)$ between 0 and 150, or

$$\int_0^{150} (0.005q^2 - q + 56)dq.$$

(b) An estimate of the total cost of producing 150 units is given by

$$20{,}000 + \int_0^{150} (0.005q^2 - q + 56)dq.$$

This represents the fixed cost ($20,000) plus the variable cost of producing 150 units, which is represented by the integral. Using a calculator, we see

$$\int_0^{150} (0.005q^2 - q + 56)dq \approx 2{,}775.$$

So the total cost is approximately

$$\$20{,}000 + \$2{,}775 = \$22{,}775.$$

(c) $C'(150) = 0.005(150)^2 - 150 + 56 = 18.5$. This means that the marginal cost of the 150th item is 18.5. In other words, the 151st item will cost approximately $18.50.

(d) $C(151)$ is the total cost of producing 151 items. This can be found by adding the total cost of producing 150 items (found in part (b)) and the additional cost of producing the 151st item ($C'(150)$, found in (c)). So we have

$$C(151) \approx 22{,}775 + 18.50 = \$22{,}793.50.$$

4. **(a)** There are approximately 5.5 squares under the curve of $C'(q)$ from 0 to 30. Each square represents $100, so the total variable cost to produce 30 units is around $550. To find the total cost, we had the fixed cost

$$\begin{aligned}\text{Total cost} &= \text{fixed cost} + \text{total variable cost}\\ &= 10,000 + 550 = \$10,550.\end{aligned}$$

(b) There are approximately 1.5 squares under the curve of $C'(q)$ from 30 to 40. Each square represents $100, so the additional cost of producing items 31 through 40 is around $150.

(c) Examination of the graph tells us that $C'(25) = 10$. This means that the cost of producing the 26th item is approximately $10.

5. The area between 1970 and 1990 is about 15.3 grid squares, each of which has area $0.1(5) = 0.5$ million people. So

$$\text{Change in population} = \int_{1970}^{1990} P'(t)\,dt \approx 15.3(0.5) \approx 7.65 \text{ million people.}$$

The population of Tokyo increased by about 8 million people between 1970 and 1990.

6. Since $C(0) = 500$, the fixed cost must be $500. The total *variable* cost to produce 20 units is

$$\int_0^{20} C'(q)\,dq = \int_0^{20} (q^2 - 16q + 70)\,dq = \$866.67 \text{ (using a calculator).}$$

The *total* cost to produce 20 units is the fixed cost plus the variable cost of producing 20 units. Thus,

$$\text{Total cost} = \$500 + \$866.67 = \$1{,}366.67.$$

7. **(a)** Total variable cost in producing 400 units is

$$\int_0^{400} C'(q)\,dq.$$

We estimate this integral:

$$\begin{aligned}\text{Left–hand sum} &= 25(100) + 20(100) + 18(100) + 22(100) = 8500;\\ \text{Right–hand sum} &= 20(100) + 18(100) + 22(100) + 28(100) = 8800;\end{aligned}$$

and so $\displaystyle\int_0^{400} C'(q)\,dq \approx \frac{8500 + 8800}{2} = \$8650.$

$$\begin{aligned}\text{Total cost} &= \text{Fixed cost} + \text{Variable cost}\\ &= \$10{,}000 + \$8650 = \$18{,}650.\end{aligned}$$

(b) $C'(400) = 28$, so we would expect that the 401st unit would cost an extra $28.

8. **(a)** Using the Fundamental Theorem we get that the cost of producing 30 bicycles is

$$C(30) = \int_0^{30} C'(q)dq + C(0)$$

or

$$\int_0^{30} \frac{600}{0.3q+5} dq + \$2000 \approx \$4059.24$$

(b) If the bikes are sold for \$200 each the total revenue from 30 bicycles is

$$30 \cdot 200 = \$6000,$$

and so the total profit is

$$\$6000 - \$4059.24 = \$1940.76.$$

(c) The marginal profit on the 31^{st} bicycle is the difference between the marginal cost of producing the 31^{st} bicycle and the marginal revenue, which is the price. Thus the marginal profit is

$$200 - C'(30) = 200 - \frac{600}{14} = \$157.14$$

9. **(a)**

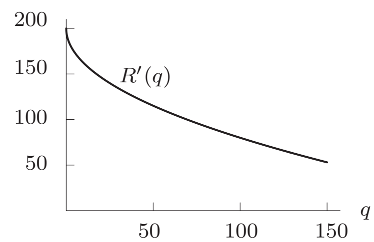


Figure 5.52

(b) By the Fundamental Theorem,

$$\int_0^{100} R'(q)dq = R(100) - R(0).$$

$R(0) = 0$ because no revenue is produced if no units are sold. Thus we get

$$R(100) = \int_0^{100} R'(q)dq \approx \$12{,}000.$$

(c) The marginal revenue in selling the 101st unit is given by $R'(100) = \$80$/unit. The total revenue in selling 101 units is:

$$R(100) + R'(100) = \$12{,}080.$$

10. **(a)** The area under the curve of $P'(t)$ from 0 to t gives the change in the value of the stock. Examination of the graph suggests that this area is greatest at $t = 5$, so we conclude that the stock is at its highest value at the end of the 5th week.

(Some may also conclude that the area is greatest at $t = 1.5$, making the stock most valuable in the middle of the second week. Both are valid answers.)

Since $P'(t) < 0$ from $t = 1.5$ to about $t = 3.8$, we know that the value of the stock decreases in this interval. This is the only interval in which the stock's value is decreasing, so the stock will reach its lowest value at the end of this interval, which is near the end of the fourth week.

(b) We know that $P(t) - P(0)$ is the area under the curve of $P'(t)$ from 0 to t, so examination of the graph leads us to conclude that

$$P(4) < P(3) \approx P(0) < P(1) \approx P(2) < P(5).$$

11. (a) The rate of ice formation is $\dfrac{dy}{dt} = \dfrac{\sqrt{t}}{2}$ inches/hour. So the amount of ice formed in 8 hours equals

$$y(8) - y(0) = \int_0^8 \frac{\sqrt{t}}{2} dt.$$

Now, $y(0) = 0$ because the ice starts forming at $t = 0$. Thus there are

$$\int_0^8 \frac{\sqrt{t}}{2} dt \approx 7.54 \text{ inches of ice in 8 hours.}$$

(b) At 8 hours, $\dfrac{dy}{dt} = \dfrac{\sqrt{8}}{2} = \sqrt{2}$ inches/hour. The ice is increasing at a rate of

$$\sqrt{2} = 1.41 \text{ inches/hour.}$$

12. The total change in the net worth of the company from 2005 ($t = 0$) to 2015 ($t = 10$) is found using the Fundamental Theorem:

$$\text{Change in net worth} = f(10) - f(0) = \int_0^{10} f'(t)\, dt = \int_0^{10} (2000 - 12t^2)\, dt = 16{,}000 \text{ dollars.}$$

The worth of the company in 2015 is the worth of the company in 2005 plus the change in worth between 2005 and 2015. Thus, in 2015,

$$\begin{aligned} \text{Net worth} = f(10) &= f(0) + \text{Change in worth} \\ &= \text{Worth in 2005} + \text{Change in worth between 2005 and 2015} \\ &= 40{,}000 + 16{,}000 \\ &= \$56{,}000. \end{aligned}$$

13. We find the changes in $f(x)$ between any two values of x by counting the area between the curve of $f'(x)$ and the x-axis. Since $f'(x)$ is linear throughout, this is quite easy to do. From $x = 0$ to $x = 1$, we see that $f'(x)$ outlines a triangle of area $1/2$ below the x-axis (the base is 1 and the height is 1). By the Fundamental Theorem,

$$\int_0^1 f'(x)\, dx = f(1) - f(0),$$

so

$$f(0) + \int_0^1 f'(x)\, dx = f(1)$$
$$f(1) = 2 - \frac{1}{2} = \frac{3}{2}$$

Similarly, between $x = 1$ and $x = 3$ we can see that $f'(x)$ outlines a rectangle below the x-axis with area -1, so $f(2) = 3/2 - 1 = 1/2$. Continuing with this procedure (note that at $x = 4$, $f'(x)$ becomes positive), we get the table below.

x	0	1	2	3	4	5	6
$f(x)$	2	$3/2$	$1/2$	$-1/2$	-1	$-1/2$	$1/2$

Solutions for Chapter 5 Review

1. (a) Since the velocity is decreasing, for an upper estimate, we use a left sum. With $n = 5$,we have $\Delta t = 2$. Then

$$\text{Upper estimate } = (44)(2) + (42)(2) + (41)(2) + (40)(2) + (37)(2) = 408.$$

(b) For a lower estimate, we use a right sum, so

$$\text{Lower estimate } = (42)(2) + (41)(2) + (40)(2) + (37)(2) + (35)(2) = 390.$$

2. **(a)** (i) Since the velocity is increasing, for an upper estimate we use a right sum. Using $n = 4$, we have $\Delta t = 3$, so

$$\text{Upper estimate } = (37)(3) + (38)(3) + (40)(3) + (45)(3) = 480.$$

(ii) Using $n = 2$, we have $\Delta t = 6$, so

$$\text{Upper estimate } = (38)(6) + (45)(6) = 498.$$

(b) The answer using $n = 4$ is more accurate as it uses the values of $v(t)$ when $t = 3$ and $t = 9$.

(c) Since the velocity is increasing, for a lower estimate we use a left sum. Using $n = 4$, we have $\Delta t = 3$, so

$$\text{Lower estimate } = (34)(3) + (37)(3) + (38)(3) + (40)(3) = 447.$$

3. We estimate the integral by finding left- and right-hand sums and averaging them:

$$\text{Left-hand sum} = 100 \cdot 4 + 88 \cdot 4 + 72 \cdot 4 + 50 \cdot 4 = 1240$$

$$\text{Right-hand sum} = 88 \cdot 4 + 72 \cdot 4 + 50 \cdot 4 + 28 \cdot 4 = 952.$$

We have

$$\int_{10}^{26} f(x)\,dx \approx \frac{1240 + 952}{2} = 1096.$$

4. $\int_a^b f(t)dt$ is measured in

$$\left(\frac{\text{miles}}{\text{hours}}\right) \cdot (\text{hours}) = \text{miles}.$$

5. The units of measurement are meters per second (which are units of velocity).

6. The units of measurement are dollars.

7. The units of measurement are foot-pounds (which are units of work).

8. **(a)** Using rectangles under the curve, we get

$$\text{Acres defaced } \approx (1)(0.2 + 0.4 + 1 + 2) = 3.6 \text{ acres}.$$

(b) Using rectangles above the curve, we get

$$\text{Acres defaced } \approx (1)(0.4 + 1 + 2 + 3.5) = 6.9 \text{ acres}.$$

(c) The number of acres defaced is between 3.6 and 6.9, so we estimate the average, 5.25 acres.

9. $\int_0^{10} 2^{-x}\,dx = 1.44$

10. $\int_1^5 (x^2 + 1)\,dx = 45.33$

11. $\displaystyle\int_0^1 \sqrt{1 + t^2}dt = 1.15.$

12. $\int_{-1}^1 \frac{x^2+1}{x^2-4}\,dx = -0.75.$

13. $\int_2^3 \frac{-1}{(r+1)^2}\,dr = -0.083$

14. $\int_1^3 \frac{z^2+1}{z}dz = 5.10$

15. Since $f(x)$ is positive along the interval from 0 to 6 the area is simply $\displaystyle\int_0^6 (x^2 + 2)dx = 84.$

16. Since $x^3 \leq x^2$ for $0 \leq x \leq 1$, we have

$$\text{Area } = \int_0^1 (x^2 - x^3)\,dx = 0.083.$$

The integral was evaluated on a calculator.

17. Since $x^{1/2} \leq x^{1/3}$ for $0 \leq x \leq 1$, we have

$$\text{Area} = \int_0^1 (x^{1/3} - x^{1/2})\, dx = 0.0833.$$

The integral was evaluated on a calculator.

18.

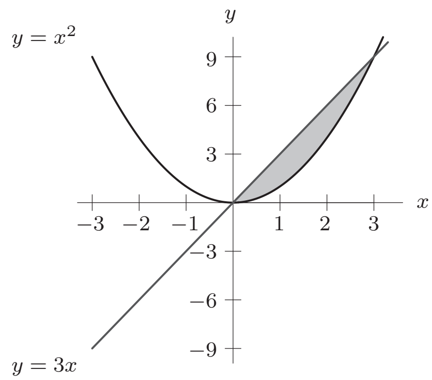


Figure 5.53

Inspection of the graph tells us that the curves intersect at $(0,0)$ and $(3,9)$, with $3x \geq x^2$ for $0 \leq x \leq 3$, so we can find the area by evaluating the integral

$$\int_0^3 (3x - x^2) dx.$$

Using technology to evaluate the integral, we see

$$\int_0^3 (3x - x^2) dx = 4.5.$$

So the area between the graphs is 4.5.

19. See Figure 5.54.

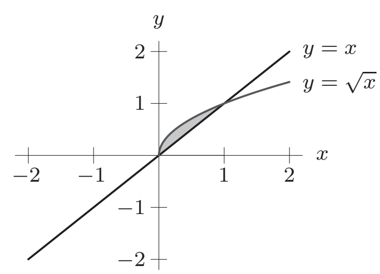


Figure 5.54

Inspection of the graph tell us that the curves intersect at $(0,0)$ and $(1,1)$, with $\sqrt{x} \geq x$ for $0 \leq x \leq 1$, so we can find the area by evaluating the integral

$$\int_0^1 (\sqrt{x} - x) dx.$$

Using technology to evaluate the integral, we see

$$\int_0^1 (\sqrt{x} - x) dx \approx 0.1667.$$

So the area between the graphs is about 0.1667.

20. **(a)** An overestimate is 7 tons. An underestimate is 5 tons.
(b) An overestimate is $7 + 8 + 10 + 13 + 16 + 20 = 74$ tons. An underestimate is $5 + 7 + 8 + 10 + 13 + 16 = 59$ tons.

21. To find the distance the car moved before stopping, we estimate the distance traveled for each two-second interval. Since speed decreases throughout, we know that the left-handed sum will be an overestimate to the distance traveled and the right-hand sum an underestimate. Applying the formulas for these sums with $\Delta t = 2$ gives:

$$\text{LEFT} = 2(100 + 80 + 50 + 25 + 10) = 530 \text{ ft.}$$
$$\text{RIGHT} = 2(80 + 50 + 25 + 10 + 0) = 330 \text{ ft.}$$

(a) The best estimate of the distance traveled will be the average of these two estimates, or

$$\text{Best estimate} = \frac{530 + 330}{2} = 430 \text{ ft.}$$

(b) All we can be sure of is that the distance traveled lies between the upper and lower estimates calculated above. In other words, all the black-box data tells us for sure is that the car traveled between 330 and 530 feet before stopping. So we can't be completely sure about whether it hit the skunk or not.

22. The change in position is calculated from the area between the velocity graph and the t-axis, with the region below the axis corresponding to negatives velocities and counting negatively.

Figure 5.55 shows the graph of $f(t)$. From $t = 0$ to $t = 3$ the velocity is positive. The region under the graph of $f(t)$ is a triangle with height 6 cm/sec and base 3 seconds. Thus, from $t = 0$ to $t = 3$, the particle moves

$$\text{Distance moved to right } = \frac{1}{2} \cdot 3 \cdot 6 = 9 \text{ centimeters.}$$

From $t = 3$ to $t = 4$, the velocity is negative. The region between the graph of $f(t)$ and the t-axis is a triangle with height 2 cm/sec and base 1 second, so in this interval the particle moves

$$\text{Distance moved to left } = \frac{1}{2} \cdot 1 \cdot 2 = 1 \text{ centimeter.}$$

Thus, the total change in position is $9 - 1 = 8$ centimeters to the right.

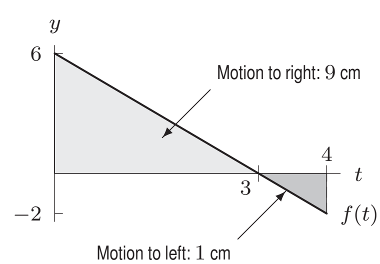


Figure 5.55

23. **(a)** See Figure 5.56.

(b) The peak of the flight is when the velocity is 0, namely $t = 3$. The height at $t = 3$ is given by the area under the graph of the velocity from $t = 0$ to $t = 3$; see Figure 5.56. The region is a triangle of base 3 seconds and altitude 96 ft/sec, so the height is $(1/2)3 \cdot 96 = 144$ feet.

(c) The velocity is negative from $t = 3$ to $t = 5$, so the motion is downward then. The distance traveled downward can be calculated by the area of the triangular region which has base of 2 seconds and altitude of -64 ft/sec. Thus, the baseball travels $(1/2)2 \cdot 64 = 64$ feet downward from its peak height of 144 feet at $t = 3$. Thus, the height at time $t = 5$ is the total change in position, $144 - 64 = 80$ feet.

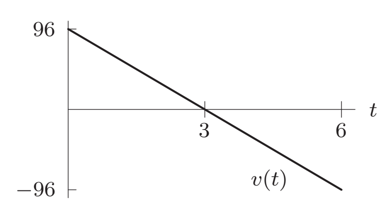


Figure 5.56

24. Change in income $= \int_0^{12} r(t)\,dt = \int_0^{12} 40(1.002)^t\,dt = \$485.80$

25. **(a)** The area under the curve is greater for species B for the first 5 years. Thus species B has a larger population after 5 years. After 10 years, the area under the graph for species B is still greater so species B has a greater population after 10 years as well.

(b) Unless something happens that we cannot predict now, species A will have a larger population after 20 years. It looks like species A will continue to quickly increase, while species B will add only a few new plants each year.

26. You start traveling toward home. After 1 hour, you have traveled about 5 miles. (The distance traveled is the area under the first bump of the curve. The area is about $1/2$ grid square; each grid square represents 10 miles.) You are now about $10 - 5 = 5$ miles from home. You then decide to turn around. You ride for an hour and a half away from home, in which time you travel around 13 miles. (The area under the next part of the curve is about 1.3 grid squares below the axis.) You are now $5 + 13 = 18$ miles from home and you decide to turn around again at $t = 2.5$. The area above the curve from $t = 2.5$ to $t = 3.5$ has area 18 miles. This means that you ride for another hour, at which time you reach your house ($t = 3.5$.) You still want to ride, so you keep going past your house for a half hour. You make about 8 miles in this time and you decide to stop. So, at the end of the ride, you end up 8 miles away from home.

27. **(a)** The length growth rate is the derivative of the length function. A graph of the length function has an inflection point when its derivative has a maximum.

(b) The total growth in length during the forty week gestation equals the integral of the growth rate over the time period. Thus, the birth length equals the area under the graph. The region consists of a triangle (between $t = 2$ and $t = 14$) of base 12 weeks and height 2.3 cm/week, another triangle (between $t = 14$ and $t = 40$) of base 26 weeks and height 1.8 cm/week, and a rectangle (between $t = 14$ and $t = 40$) of length 26 weeks and height 0.5 cm/week. Thus, approximately

$$\text{Total length} = \frac{1}{2}12(2.3) + \frac{1}{2}26(1.8) + 26(0.5) = 50.2 \text{ cm.}$$

28. Notice that the area of a square on the graph represents $10/6$ miles. At $t = 1/3$ hours, $v = 0$. The area between the curve v and the t-axis over the interval $0 \le t \le 1/3$ is $-\int_0^{1/3} v\,dt \approx 5/3$. Since v is negative here, she is moving toward the lake. At $t = 1/3$, she is about $5 - 5/3 = 10/3$ miles from the lake. Then, as she moves away from the lake, v is positive for $1/3 \le t \le 1$. At $t = 1$,

$$\int_0^1 v\,dt = \int_0^{1/3} v\,dt + \int_{1/3}^1 v\,dt \approx -\frac{5}{3} + 8 \cdot \frac{10}{6} = \frac{35}{3} = 11.667 \text{ miles,}$$

and the cyclist is about $5 + 35/3 = 50/3 = 16.667$ miles from the lake. Since, starting from the moment $t = 1/3$, she moves away from the lake, the cyclist will be farthest from the lake at $t = 1$. The maximal distance equals 16.667 miles.

29. **(a)** Negative, since $f \le 0$ everywhere on the interval $-5 \le x \le -4$.

(b) Positive, since $f \ge 0$ everywhere on the interval $-4 \le x \le 1$.

(c) Negative, since the graph of f has more area under the x-axis than above on the interval $1 \le x \le 3$.

(d) Positive, since the graph of f has more area above the x-axis than underneath on the interval $-5 \le x \le 3$.

30. The integral $\int_{-5}^{-3} f\,dx$ is the smallest, since it has a negative value on the interval $-5 \le x \le -4$ plus a small positive value on $-4 \le x \le -3$. The integral $\int_{-5}^{-1} f\,dx$ is larger than $\int_{-5}^{-3} f\,dx$, since $f > 0$ on the interval $-3 \le x \le -1$. The integral $\int_{-5}^{1} f\,dx$ is larger than $\int_{-5}^{-1} f\,dx$ for the same reason. However, since $f < 0$ during part of the interval $1 \le x \le 3$, the integral $\int_{-5}^{3} f\,dx$ is less than $\int_{-5}^{1} f\,dx$. But $\int_{-5}^{3} f\,dx > \int_{-5}^{-1} f\,dx$, since there is more area above the x-axis on the interval $-1 \le x \le 1$ than there is below the x-axis on the interval $1 \le x \le 3$. Thus in ascending order the integrals are $\int_{-5}^{-3} f\,dx < \int_{-5}^{-1} f\,dx < \int_{-5}^{3} f\,dx < \int_{-5}^{1} f\,dx$.

31.

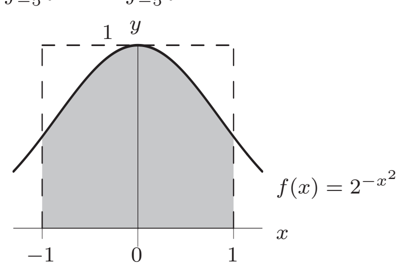


Figure 5.57

Since the shaded area is positive and less than the area of the rectangle measuring two units by one unit, we can say that

$$0 < \int_{-1}^{1} 2^{-x^2}\,dx < 2.$$

32. Notice that $f(x) = \sqrt{1+x^3}$ is increasing for $0 \le x \le 2$, since x^3 gets bigger as x increases. This means that $f(0) \le f(x) \le f(2)$. For this function, $f(0) = 1$ and $f(2) = 3$. Thus, the area under $f(x)$ lies between the area under the line $y = 1$ and the area under the line $y = 3$ on the interval $0 \le x \le 2$. See Figure 5.58. That is,

$$1(2-0) \;\le\; \int_0^2 \sqrt{1+x^3}\,dx \;\le\; 3(2-0).$$

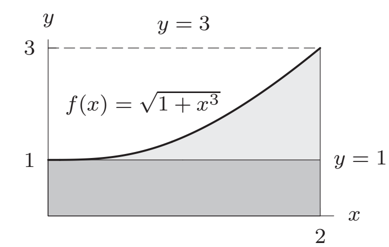


Figure 5.58

33. **(a)** Since velocity is the rate of change of distance, we have

$$\text{Distance traveled} = \int_0^5 (10 + 8t - t^2)\,dt.$$

This distance is the shaded area in Figure 5.59.

(b) A graph of this velocity function is given in Figure 5.59. Finding the distance traveled is equivalent to finding the area under this curve between $t = 0$ and $t = 5$. We estimate that this area is about 100 since the average height appears to be about 20 and the width is 5.

(c) We use a calculator or computer to calculate the definite integral

$$\text{Distance traveled} = \int_0^5 (10 + 8t - t^2)\,dt \approx 108.33 \quad \text{meters.}$$

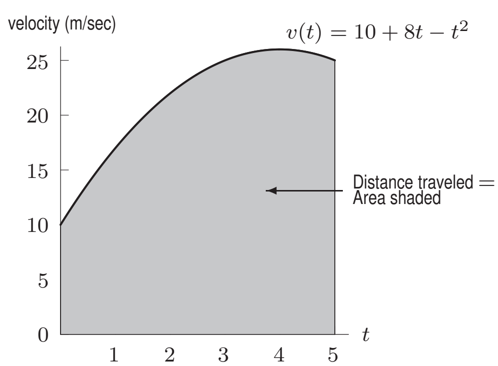


Figure 5.59: A velocity function

34. **(a)** Quantity used $= \int_0^{15} f(t)\,dt$.

(b) Using a left hand-sum, since $\Delta t = 3$, our approximation is

$$32(1.05)^0 \cdot 3 + 32(1.05)^3 \cdot 3 + 32(1.05)^6 \cdot 3 + 32(1.05)^9 \cdot 3 + 32(1.05)^{12} \cdot 3 \approx 657.11.$$

Since f is an increasing function, this represents an underestimate.

(c) Each term is a lower estimate of 3 years' consumption of oil.

35. **(a)** The equation $v = 6 - 2t$ implies that $v > 0$ (the car is moving forward) if $0 \leq t \leq 3$ and that $v < 0$ (the car is moving backward) if $t > 3$. When $t = 3$, $v = 0$, so the car is not moving at the instant $t = 3$. The car arrives back at its starting point when $t = 6$.

(b) The car moves forward on the interval $0 \leq t \leq 3$, so it is farthest forward when $t = 3$. For all $t > 3$, the car is moving backward. There is no upper bound on the car's distance behind its starting point since it is moving backward for all $t > 3$.

36. The fixed cost is $C(0) = 1{,}000{,}000$.

$$\text{Total variable cost} = \int_0^{500} C'(x)\,dx = \int_0^{500} (4000 + 10x)\,dx = 3{,}250{,}000.$$

Therefore,

$$\begin{aligned}\text{Total cost} &= \text{Fixed Cost} + \text{Total variable cost}\\ &= 4{,}250{,}000 \text{ riyals.}\end{aligned}$$

37. The area under the curve represents the number of cubic feet of storage times the number of days the storage was used. This area is given by

$$\begin{aligned}\text{Area under graph} &= \text{Area of rectangle} + \text{Area of triangle}\\ &= 30 \cdot 10{,}000 + \frac{1}{2} \cdot 30(30{,}000 - 10{,}000)\\ &= 600{,}000.\end{aligned}$$

Since the warehouse charges \$5 for every 10 cubic feet of storage used for a day, the company will have to pay $(5)(60{,}000) = \$300{,}000$.

38. **(a)**

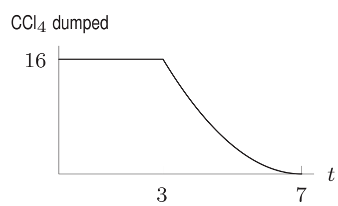


Figure 5.60

(b) 7 years, because $t^2 - 14t + 49 = (t - 7)^2$ indicates that the rate of flow was zero after 7 years.

(c)

$$\begin{aligned}\text{Area under the curve} &= 3(16) + \int_3^7 (t^2 - 14t + 49)\,dt\\ &= 69.3.\end{aligned}$$

So $69\frac{1}{3}$ cubic yards of CCl_4 entered the waters from the time the EPA first learned of the situation until the flow of CCl4 was stopped.

39. **(a)**

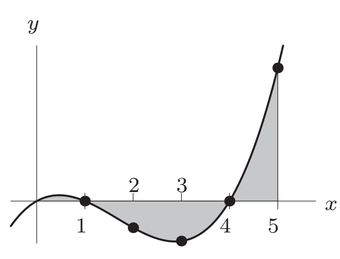


Figure 5.61: Graph of $y = x^3 - 5x^2 + 4x$

(b) $I_1 = \int_0^1 (x^3 - 5x^2 + 4x)dx$ is positive, since

$$x^3 - 5x^2 + 4x > 0 \quad \text{for} \quad 0 < x < 1.$$

$I_2 = \int_0^2 (x^3 - 5x^2 + 4x)dx$ seems to be negative, since the area under the x-axis is bigger than the area above the x-axis for $0 \le x \le 2$.

I_3 and I_4 will also be negative, since the area under the x-axis will also be larger than the area above the x-axis for $0 \le x \le 3$ and $0 \le x \le 4$. I_5 will also be negative, since the area above the x-axis will still be less than the area below the x-axis for $0 \le x \le 5$. From this we conclude that I_1 is the largest and I_4 is the smallest.

40. **(a)** The mouse changes direction (when its velocity is zero) at about times 17, 23, and 27.

(b) The mouse is moving most rapidly to the right at time 10 and most rapidly to the left at time 40.

(c) The mouse is farthest to the right when the integral of the velocity, $\int_0^t v(t)\,dt$, is most positive. Since the integral is the sum of the areas above the axis minus the areas below the axis, the integral is largest when the velocity is zero at about 17 seconds. The mouse is farthest to the left of center when the integral is most negative at 40 seconds.

(d) The mouse's speed decreases during seconds 10 to 17, from 20 to 23 seconds, and from 24 seconds to 27 seconds.

(e) The mouse is at the center of the tunnel at any time t for which the integral from 0 to t of the velocity is zero. This is true at time 0 and again somewhere around 35 seconds.

41. The graph of rate against time is the straight line shown in Figure 5.62. Since the shaded area is 270, we have

$$\frac{1}{2}(10 + 50) \cdot t = 270$$

$$t = \frac{270}{60} \cdot 2 = 9 \text{ years}$$

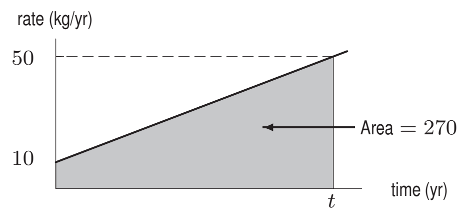


Figure 5.62

42. Using the Fundamental Theorem of Calculus with $f(x) = 4 - x^2$ and $a = 0$, we see that

$$F(b) - F(0) = \int_0^b F'(x)dx = \int_0^b (4 - x^2)dx.$$

We know that $F(0) = 0$, so

$$F(b) = \int_0^b (4 - x^2)dx.$$

Using Riemann sums to estimate the integral for values of b, we get

Table 5.2

b	0.0	0.5	1.0	1.5	2.0	2.5
$F(b)$	0	1.958	3.667	4.875	5.333	4.792

43. From the graph we see that $F'(x) < 0$ for $x < -2$ and $x > 2$, and that $F'(x) > 0$ for $-2 < x < 2$, so we conclude that $F(x)$ is decreasing for $x < -2$ and $x > 2$ and $F(x)$ is increasing for $-2 < x < 2$.

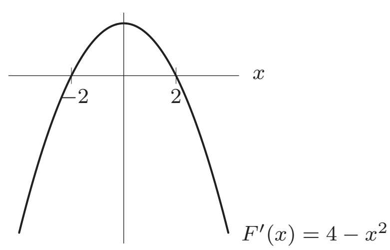


Figure 5.63

44. Since $F'(x)$ is positive for $0 < x < 2$ and $F'(x)$ is negative for $2 < x < 2.5$, $F(x)$ increases on $0 < x < 2$ and decreases on $2 < x < 2.5$. From this we conclude that $F(x)$ has a maximum at $x = 2$. From the process used in Problem 42 we see that the chart agrees with this assumption and that $F(2) = 5.333$.

CHECK YOUR UNDERSTANDING

1. True, as specified in the text.

2. True, as specified in the text.

3. False, since using the smaller velocity at the beginning of each subinterval gives an *underestimate* of the distance traveled.

4. False, since the width of the interval is 2 seconds and an underestimate for the distance traveled is given by $10 \cdot 2 = 20$ ft and an overestimate for the distance traveled is given by $20 \cdot 2 = 40$ ft. The distance traveled is between 20 and 40 feet during this 2-second interval.

5. True, since the width of the interval is 2 seconds and an underestimate for the distance traveled is given by $10 \cdot 2 = 20$ ft and an overestimate for the distance traveled is given by $20 \cdot 2 = 40$ ft. The distance traveled is between 20 and 40 feet during this 2-second interval.

6. True, since 20 gallons per minute times 10 minutes is 200 gallons and that is an underestimate.

7. False. The estimate of 50 gallons per minute times 10 minutes is 500 gallons but it is an overestimate not an underestimate.

8. False. Since the width of the interval is 2 days, an overestimate is $2 \cdot 1000 = \$2000$ and an underestimate is $2 \cdot 800 = \$1600$.

9. True, since the triangular area under the graph of $v(t) = 3t$ when $0 \leq t \leq 10$ is $(1/2)(30)(10) = 150$ feet.

10. False, since the rate of change has units quantity per time, while total change has units of quantity.

11. False, since the left-hand value on each subinterval is the least value f has on that subinterval, so the left-hand sum gives an *underestimate* of the integral.

12. False, since the left-hand sum of a decreasing function will be an overestimate whether the function is concave up or concave down.

13. False. Since the width of the interval is 2, the left-hand sum estimate is $2 \cdot 1000 = \$2000$.

14. True. Since the width of the interval is 2, the right-hand sum estimate is $2 \cdot 800 = 1600$.

15. True. If the number of rectangles increases, the width of each rectangle decreases.

16. False. If $n = 4$ on this interval, then $\Delta t = 0.5$.

17. False, since right-hand rectangles could be above, below, or neither. Only when f is increasing are the right-hand rectangles guaranteed to be above the graph.

18. True, as specified in the text.

19. True, since f is decreasing over the interval $-2 \leq t \leq -1$.

20. False, since $25 - 5 = 20$ which divided into five pieces gives $\Delta t = 4$.

21. False, since we don't know whether $f(x) \geq 0$ between $x = 0$ and $x = 2$.

22. False, since the graph of f could just have equal areas above and below the x-axis between $x = 0$ and $x = 2$.

23. False, since the graph of f could just have more area above than below the x-axis between $x = 0$ and $x = 2$.

24. True, since all of the terms in a left- or right-hand sum defining $\int_0^2 f(x)\,dx$ are positive.

25. True, since the graph of $f(x) = x^2 - 1$ is entirely on or below the x-axis between $x = -1$ and $x = 1$.

26. False, since $\int_0^2 f(x)\,dx$ is the sum of areas where $f(x) > 0$ and (-1) times the areas where $f(x) < 0$.

27. False, since the graph of $f(x) = 1 - x$ is not always above the x-axis between $x = 0$ and $x = 3$.

28. True, since $f(x) = e^x$ is always positive.

29. True, as specified in the text.

30. True, since $\int_1^{10} f(x)\,dx$ is the sum of areas above the x-axis and (-1) times the sum of areas below the x-axis.

31. True, since the units of the integral are the product of units of $r(t)$ and units of t.

32. False, the integral has units dollars per day times days, or dollars.

33. True, since the units of the integral are the product of the units of $a(t)$ and the units of t.

34. False, since the units of the integral are the product of the units of $w(t)$ and the units of t, giving worker-days.

35. True, since the units of the integral are the product of the units of $Ct)$ and the units of t, giving dollars.

36. True, since the integral of the rate of change, dollars per day, gives total change in the cost.

37. False. The integral gives the total *change* in the volume of water. We have to add the change to the starting volume at $t = 0$ to find the volume at $t = 30$.

38. True. The integral gives the total change in the volume of water. We can add the change to the starting volume at $t = 0$ to find the volume at $t = 30$.

39. True, as specified in the text.

40. False, since bioavailability depends on the area under the drug concentration curve, not its peak. So drug B might last longer in the bloodstream over time, while drug A might grow quickly to a high concentration, but then go quickly back to zero.

41. False, since the correct statement is if $F'(t)$ is continuous, then $\int_a^b F'(t)\,dt = F(b) - F(a)$.

42. True, as specified in the text.

43. False, since the correct statement is if $F'(t)$ is continuous, then $\int_a^b F'(t)\,dt = F(b) - F(a)$.

44. True, since by the Fundamental Theorem, $\int_{100}^{200} C'(q)\,dq = C(200) - C(100)$.

45. False, since by the Fundamental Theorem, $\int_0^{1000} C'(q)\,dq = C(1000) - C(0)$ which is the total *variable* costs, as we have subtracted the fixed costs of $C(0)$.

46. False, since the integral $\int_0^{1000} 0.03q + 0.1\,dq = 15{,}100$ gives only the variable costs. The total cost is the sum of the variable cost and the fixed cost, or $16,600.

47. False, since the integral of the marginal cost function only gives the change in cost, or variable costs.

48. False, since $\int_0^5 F'(t)\,dt = F(5) - F(0) = 0$ only says that $F(0) = F(5)$, not that F has the same value between $t = 0$ and $t = 5$.

49. True, since if the total change of F over the interval $0 \le t \le 5$ is negative, then $F(0)$ must be greater than $F(5)$.

50. False, since if $F'(t) = k$, then $\int_0^5 k\,dt = 5k$.

PROJECTS FOR CHAPTER FIVE

1. (a) CO_2 is being taken out of the water during the day and returned at night. The pond must therefore contain some plants. (The data is in fact from pond water containing both plants and animals.)

(b) Suppose t is the number of hours past dawn. The graph in Figure 5.76 of the text shows that the CO_2 content changes at a greater rate for the first 6 hours of daylight, $0 < t < 6$, than it does for the final 6 hours of daylight, $6 < t < 12$. It turns out that plants photosynthesize more vigorously in the morning than in the afternoon. Similarly, CO_2 content changes more rapidly in the first half of the night, $12 < t < 18$, than in the 6 hours just before dawn, $18 < t < 24$. The reason seems to be that at night plants quickly use up most of the sugar that they synthesized during the day, and then their respiration rate is inhibited. So the constant rate hypothesis is false, if we assume plants are the main cause of CO_2 changes in the pond.

(c) The question asks about the total quantity of CO_2 in the pond, rather than the rate at which it is changing. We will let $f(t)$ denote the CO_2 content of the pond water (in $\mathrm{mmol/l}$) at t hours past dawn. Then Figure 5.76 of the text is a graph of the derivative $f'(t)$. There are $2.600\ \mathrm{mmol/l}$ of CO_2 in the water at dawn, so $f(0) = 2.600$.

The CO_2 content $f(t)$ decreases during the 12 hours of daylight, $0 < t < 12$, when $f'(t) < 0$, and then $f(t)$ increases for the next 12 hours. Thus, $f(t)$ is at a minimum when $t = 12$, at dusk. By the Fundamental Theorem,

$$f(12) = f(0) + \int_0^{12} f'(t)\,dt = 2.600 + \int_0^{12} f'(t)\,dt.$$

We must approximate the definite integral by a Riemann sum. From the graph in Figure 5.76 of the text, we estimate the values of the function $f'(t)$ in Table 5.3.

Table 5.3 *Rate, $f'(t)$, at which CO_2 is entering or leaving water*

t	$f'(t)$	t	$f'(t)$	t	$f'(t)$	t	$f'(t)$	t	$f'(t)$	t	$f'(t)$
0	0.000	4	−0.039	8	−0.026	12	0.000	16	0.035	20	0.020
1	−0.042	5	−0.038	9	−0.023	13	0.054	17	0.030	21	0.015
2	−0.044	6	−0.035	10	−0.020	14	0.045	18	0.027	22	0.012
3	−0.041	7	−0.030	11	−0.008	15	0.040	19	0.023	23	0.005

The left Riemann sum with $n = 12$ terms, corresponding to $\Delta t = 1$, gives

$$\int_0^{12} f'(t)\,dt \approx (0.000)(1) + (-0.042)(1) + (-0.044)(1) + \cdots + (-0.008)(1) = -0.346.$$

At 12 hours past dawn, the CO_2 content of the pond water reaches its lowest level, which is approximately

$$2.600 - 0.346 = 2.254 \text{ mmol/l}.$$

(d) The increase in CO_2 during the 12 hours of darkness equals

$$f(24) - f(12) = \int_{12}^{24} f'(t)\,dt.$$

Using Riemann sums to estimate this integral, we find that about 0.306 mmol/l of CO_2 was released into the pond during the night. In part (c) we calculated that about 0.346 mmol/l of CO_2 was absorbed from the pond during the day. If the pond is in equilibrium, we would expect the daytime absorption to equal the nighttime release. These quantities are sufficiently close (0.346 and 0.306) that the difference could be due to measurement error, or to errors from the Riemann sum approximation.

If the pond is in equilibrium, the area between the rate curve in Figure 5.76 of the text and the t-axis for $0 \le t \le 12$ will equal the area between the rate curve and the t-axis for $12 \le t \le 24$. In this experiment the areas do look approximately equal.

(e) We must evaluate

$$f(b) = f(0) + \int_0^b f'(t)\,dt = 2.600 + \int_0^b f'(t)\,dt$$

for the values $b = 0, 3, 6, 9, 12, 15, 18, 21, 24$. Left Riemann sums with $\Delta t = 1$ give the values for the CO_2 content in Table 5.4. The graph is shown in Figure 5.64.

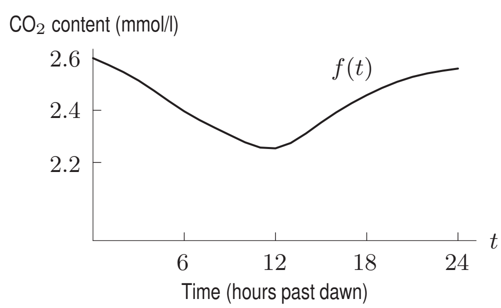


Figure 5.64: CO_2 content in pond water throughout the day

Table 5.4 CO_2 *content throughout the day*

b (hours after dawn)	0	3	6	9	12	15	18	21	24
$f(b)$ (CO_2 content)	2.600	2.514	2.396	2.305	2.254	2.353	2.458	2.528	2.560

2. (a) About 300 meter^3/sec.

(b) About 250 meter^3/sec.

(c) Looking at the graph, we can see that the 1996 flood reached its maximum just between March and April, for a high of about 1250 meter^3/sec. Similarly, the 1957 flood reached its maximum in mid-June, for a maximum flow rate of 3500 meter^3/sec.

(d) The 1996 flood lasted about $1/3$ of a month, or about 10 days. The 1957 flood lasted about 4 months.

(e) The area under the controlled flood graph is about $2/3$ box. Each box represents 500 meter^3/sec for one month. Since

$$\begin{aligned}1 \text{ month} &= 30\frac{\text{days}}{\text{month}} \cdot 24\frac{\text{hours}}{\text{day}} \cdot 60\frac{\text{minutes}}{\text{hour}} \cdot 60\frac{\text{seconds}}{\text{minute}}\\ &= 2.592 \cdot 10^6 \approx 2.6 \cdot 10^6 \text{seconds},\end{aligned}$$

each box represents

$$\text{Flow} \approx (500 \text{ meter}^3/\text{sec}) \cdot (2.6 \cdot 10^6 \text{ sec}) = 13 \cdot 10^8 \text{ meter}^3 \text{of water.}$$

So, for the artificial flood,

$$\text{Additional flow} \approx \frac{2}{3} \cdot 13 \cdot 10^8 = 8.7 \cdot 10^8 \text{ meter}^3 \approx 10^9 \text{ meter}^3.$$

(f) The 1957 flood released a volume of water represented by about 12 boxes above the 250 meter/sec baseline. Thus, for the natural flood,

$$\text{Additional flow} \approx 12 \cdot 13 \cdot 10^8 = 1.95 \cdot 10^{10} \approx 2 \cdot 10^{10} \text{ meter}^3.$$

So, the natural flood was nearly 20 times larger than the controlled flood and lasted much longer.

Solutions to Problems on the Second Fundamental Theorem of Calculus

1. By the Second Fundamental Theorem, $G'(x) = x^3$.

2. By the Second Fundamental Theorem, $G'(x) = 3^x$.

3. By the Second Fundamental Theorem, $G'(x) = xe^x$.

4. The variable of integration does not affect the value of the integral, so by the Fundamental Theorem of Calculus, $G'(x) = \ln x$.

5. (a) If $F(b) = \int_0^b 2^x\,dx$ then $F(0) = \int_0^0 2^x\,dx = 0$ since we are calculating the area under the graph of $f(x) = 2^x$ on the interval $0 \le x \le 0$, or on no interval at all.

(b) Since $f(x) = 2^x$ is always positive, the value of F will increase as b increases. That is, as b grows larger and larger, the area under $f(x)$ on the interval from 0 to b will also grow larger.

(c) Using a calculator or a computer, we get

$$\begin{aligned}F(1) &= \int_0^1 2^x\,dx \approx 1.4,\\ F(2) &= \int_0^2 2^x\,dx \approx 4.3,\\ F(3) &= \int_0^3 2^x\,dx \approx 10.1.\end{aligned}$$

6.

Table 5.5

x	0	0.5	1	1.5	2
$I(x)$	0	0.50	1.09	2.03	3.65

7. Using the Fundamental Theorem, we know that the change in F between $x = 0$ and $x = 0.5$ is given by

$$F(0.5) - F(0) = \int_0^{0.5} \sin t \cos t \, dt \approx 0.115.$$

Since $F(0) = 1.0$, we have $F(0.5) \approx 1.115$. The other values are found similarly, and are given in Table 5.6.

Table 5.6

b	0	0.5	1	1.5	2	2.5	3
$F(b)$	1	1.11492	1.35404	1.4975	1.41341	1.17908	1.00996

8. Note that $\int_a^b g(x)\,dx = \int_a^b g(t)\,dt$. Thus, we have

$$\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx = 8 + 2 = 10.$$

9. Note that $\int_a^b (g(x))^2\,dx = \int_a^b (g(t))^2\,dt$. Thus, we have

$$\int_a^b \left((f(x))^2 - (g(x))^2\right)\,dx = \int_a^b (f(x))^2\,dx - \int_a^b (g(x))^2\,dx = 12 - 3 = 9.$$

10. We have

$$\int_a^b (f(x))^2\,dx - \left(\int_a^b f(x)\,dx\right)^2 = 12 - 8^2 = -52.$$

11. Note that $\int_a^b f(z)\,dz = \int_a^b f(x)\,dx$. Thus, we have

$$\int_a^b cf(z)\,dz = c\int_a^b f(z)\,dz = 8c.$$