Chapter Three

Chapter Three

ConcepTests for Section 3.1

1. If $f'(x) = g'(x)$, then $f(x) = g(x)$.
   (a) True
   (b) False

   ANSWER:
   (b). If $f'(x) = g'(x)$, then $f(x) = g(x) + C$, where C is some constant.
   COMMENT:
   You might point out that the graphs of f and g differ by a vertical shift. A student's instinct is to answer **True**. Have students suggest their own counterexamples.

2. If $y = \pi^5$, then $y' = 5\pi^4$.
   (a) True
   (b) False

   ANSWER:
   (b). Since π^5 is a constant, then $y' = 0$.
   COMMENT:
   This question seems remarkably obvious in class. However, on an exam students tend to miss this question.

3. If $y = (x+1)(x+2)(x+3)(x+4)$, then $\dfrac{d^5y}{dx^5} = 0$.
   (a) True
   (b) False

   ANSWER:
   (a). y is a polynomial of degree four.
   COMMENT:
   Make sure your students realize that they don't need to take a derivative to answer this question.

4. The graph of a function f is given in Figure 3.1. If f is a polynomial of degree 3, then the value of $f'''(0)$ is
   (a) Positive
   (b) Negative
   (c) Zero

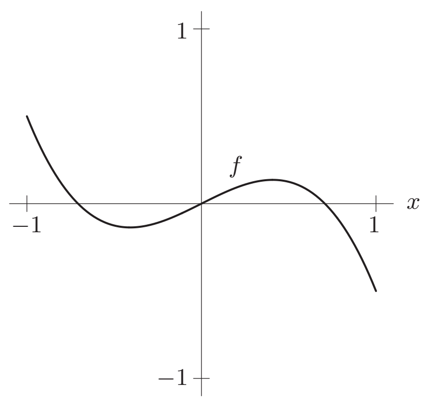

Figure 3.1

   ANSWER:
   (b). Because the graph of this polynomial of degree 3 is negative for large values of x, the coefficient of x^3 will be negative. (Recall the third derivative of a polynomial of degree 3 is a constant.)
   COMMENT:
   You could ask students why $f(x)$ could not become positive for $x > 1$.

5. The graph of a function f is given in Figure 3.2. If f is a polynomial of degree 3, then $f'''(0)$ is
   (a) Positive
   (b) Negative
   (c) Zero

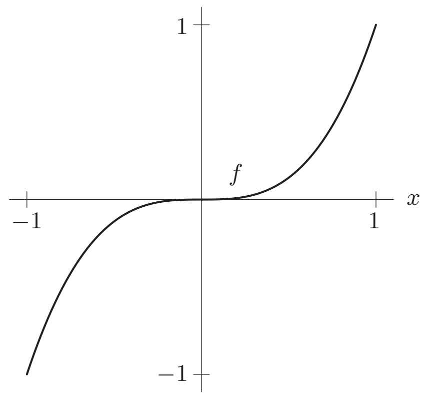

Figure 3.2

ANSWER:

(a). Because the graph of this polynomial of degree 3 is positive for large values of x, the coefficient of x^3 will be positive. (Recall the third derivative of a polynomial of degree 3 is a constant.)

COMMENT:

You could ask students if there could be other inflection points for f.

6. The graph of a function f is given in Figure 3.3. If f is a polynomial of degree 3, then the values of $f'(0)$, $f''(0)$, and $f'''(0)$ are (respectively)

(a) $0, 0, +$
(b) $0, 0, -$
(c) $0, +, -$
(d) $0, -, -$
(e) $+, -, +$
(f) $0, +, +$

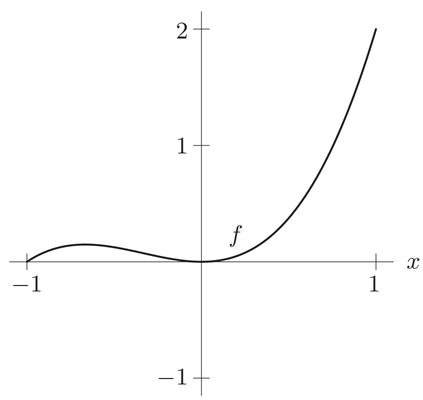

Figure 3.3

ANSWER:

(f). There is a horizontal tangent at the origin, so $f'(0) = 0$. The graph shows that f has horizontal intercepts at -1 and 0, with a double root at 0. Thus f has the form $f(x) = k(x + 1)x^2$. Because $f(x) > 0$ for $x > 0$, then $k > 0$. So $f'(x) = k(3x^2 + 2x)$, $f''(x) = k(6x + 2)$, and $f'''(x) = 6k$.

COMMENT:

You could ask students why a double root at zero means that f has a factor of x^2.

7. The graph of a function f is given in Figure 3.4. If f is a polynomial of degree 3, then the values of $f'(0)$, $f''(0)$, and $f'''(0)$ are (respectively)

(a) $+, 0, +$ (b) $-, 0, -$ (c) $+, 0, -$
(d) $+, -, -$ (e) $+, -, +$ (f) $+, +, +$

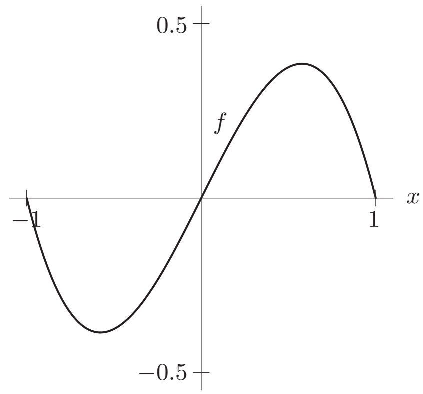

Figure 3.4

ANSWER:

(c). The graph shows that f has horizontal intercepts at $x = -1, 0$, and 1. Thus f has the form $f(x) = kx(x+1)(x-1)$. Because $f(x) > 0$ for $0 < x < 1$, we have $k < 0$. Then $f'(x) = k(3x^2 - 1)$, $f''(x) = k(6x)$, and $f'''(x) = 6k$.

COMMENT:

You could ask why there could not be another horizontal intercept outside this viewing window.

8. The graph of a function f is given in Figure 3.5. If f is a polynomial of degree 3, then the values of $f'(0)$, $f''(0)$, and $f'''(0)$ are (respectively)

(a) $-, +, +$ (b) $-, -, -$ (c) $-, +, -$
(d) $-, +, +$ (e) $+, -, +$ (f) $+, +, +$

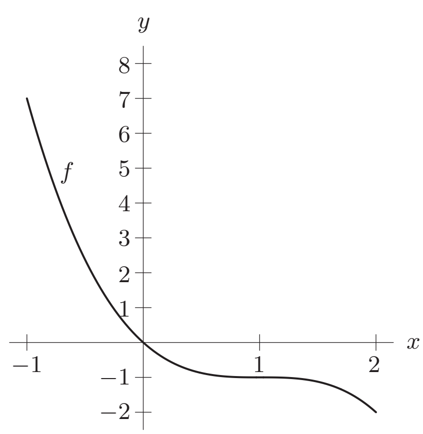

Figure 3.5

ANSWER:

(c). At $x = 0$, the graph is decreasing and concave up, so $f'(0) < 0$ and $f''(0) \geq 0$. Because the graph becomes more negative as x increases beyond 1, the sign of the coefficient of x^3 (and thus the sign of $f'''(0)$) is negative. (Recall the third derivative of a polynomial of degree 3 is a constant.)

COMMENT:

You could ask why the graph of this function could not become positive for larger values of x.

9. The graph of a function f is given in Figure 3.6. If f is a polynomial of degree 3, then the values of $f'(0)$, $f''(0)$, and $f'''(0)$ are (respectively)

(a) $-,-,+$ (b) $-,0,-$ (c) $-,+,-$
(d) $-,+,+$ (e) $+,-,+$ (f) $+,+,+$

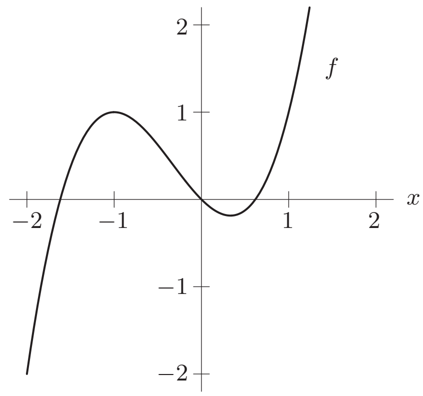

Figure 3.6

ANSWER:

(d). At $x=0$, the graph is decreasing and concave up, so $f'(0)<0$ and $f''(0)\geq 0$. Because the graph is positive as x increases beyond 0.5, the sign of the coefficient of x^3 (and thus the sign of $f'''(0)$) is positive. (Recall the third derivative of a polynomial of degree 3 is a constant.)

COMMENT:

You could ask why the graph of this function could not become negative for larger values of x.

ConcepTests for Section 3.2

1. Figure 3.7 shows the graph of $f(x)=a^x$ and its derivative $f'(x)$. Which of the following are possible values for the parameter a?

(a) $a=1$
(b) $a=2$
(c) $a=3$
(d) $a=4$
(e) None of the above

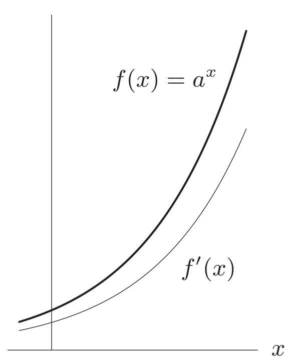

Figure 3.7

ANSWER:

(b) The only possible value of a of the options listed is $a=2$. The value $a=1$ is not possible, since the graph of $f(x)=1^x=1$ is a horizontal line. Since the derivative of $f(x)=a^x$ lies below the function, we know that $a<e=2.71828...$ since the graph of the derivative of $f(x)=e^x$ lies exactly on the graph of $f(x)=e^x$. The only values of a that give a graph similar to the one shown (with the derivative below the function) are values of a between 1 and e.

Alternately, we notice that $\ln a$ must be less than 1 for the graph of the derivative $f'(x)=\ln a\cdot a^x$ to lie below the graph of $f(x)=a^x$. Since $\ln 3>1$ and $\ln 4>1$, the only possible option is $a=2$.