Chapter Seven

ConcepTests for Section 7.1

1. Which of the following is an antiderivative of $x^2 + 7$?

   (a) $\dfrac{x^3}{3}$
   (b) $\dfrac{x^3}{3} + 7$
   (c) $\dfrac{x^3}{3} + 7x + 7$
   (d) $2x$
   (e) $2x + C$

   ANSWER:
   (c). The most general antiderivative is $\frac{x^3}{3} + 7x + C$, so (c) is one possible antiderivative.

2. For $t > 0$ which of the following is an antiderivative of $f(t) = \dfrac{1}{2t}$?

   (a) $\dfrac{1}{2}\ln(t)$
   (b) $\ln(2t)$
   (c) $2\ln(t)$
   (d) $-\dfrac{2}{t^2}$
   (e) None of these

   ANSWER:
   (a). We can rewrite the function as $f(t) = \frac{1}{2t} = \frac{1}{2} \cdot \frac{1}{t}$, so an antiderivative is (a) $\frac{1}{2}\ln(t)$.

3. Which of the following is an antiderivative of $g(x) = 6\sin(3x)$?

   (a) $-18\cos(3x)$
   (b) $18\sin(3x)$
   (c) $2\sin(3x)$
   (d) $-2\cos(3x)$
   (e) $6\cos(3x)$

   ANSWER:
   (d). If students get this wrong, have them find the derivative of each of the options.

ConcepTests for Section 7.2

1. Which of the following can be converted to the form $\int w^{10}\,dw$ using a substitution?

   (a) $\int (x^3+4)^{10}\,dx$
   (b) $\int x(x^3+4)^{10}\,dx$
   (c) $\int x^2(x^3+4)^{10}\,dx$
   (d) $\int x^3(x^3+4)^{10}\,dx$
   (e) $\int x^4(x^3+4)^{10}\,dx$

   ANSWER:
   (c). We use $w = x^3 + 4$, so $dw = 3x^2\,dx$ so we need x^2. The only one that can be converted to this form is (c).

2. Which of the following can be converted to the form $\int \frac{1}{w}\,dw$ using a substitution?

(a) $\int \frac{\cos x}{2+\sin x}\,dx$
(b) $\int \frac{1}{\sqrt{2x+1}}\,dx$
(c) $\int \frac{x^2}{x^2+1}\,dx$
(d) $\int \frac{x}{x^2+1}\,dx$
(e) $\int \frac{1}{e^x+2}\,dx$

ANSWER:
(a) and (d) only.

3. Which of the following can be converted to the form $\int e^w\,dw$ using a substitution?

(a) $\int e^{x^2}\,dx$
(b) $\int xe^{x^2}\,dx$
(c) $\int x^2e^{x^2}\,dx$
(d) $\int e^{\cos x}\,dx$

ANSWER:
(b) only.

For Problems 4–5, can the integral can be converted to the form $\int w^n\,dw$ by a substitution, where n is a constant?

4. (a) $\int \frac{4x^3+3}{\sqrt{x^4+3x}}\,dx$
(b) $\int \frac{e^x-e^{-x}}{(e^x+e^{-x})^3}\,dx$
(c) $\int \frac{2x}{x^2+1}\,dx$
(d) $\int \frac{\sin x}{x}\,dx$

ANSWER:
(a), (b), and (c). In (a) $w=x^4+3x$ yields $\int w^{-1/2}\,dw$, in (b) $w=e^x+e^{-x}$ yields $\int w^{-3}\,dw$, in (c) $w=x^2+1$ yields $\int w^{-1}\,dw$.

5. (a) $\int x^{16}(x^{17}+16)^{16}\,dx$
(b) $\int x^{16}(x^{17}+16x)^{16}\,dx$
(c) $\int \frac{18x}{1+6x^3}\,dx$
(d) $\int \frac{e^x}{e^x+6}\,dx$

ANSWER:
(a) and (d). In (a) $w=x^{17}+16$ yields $\frac{1}{17}\int w^{16}\,dw$, in (d) $w=e^x+6$ yields $\int w^{-1}\,dw$.
COMMENT:
You might note why $w=x^{17}+16x$ does not work for choice (b).

ConcepTests for Section 7.3

1. $\int_a^b 6x^2\,dx =$
   (a) $6b^2 - 6a^2$
   (b) $6a^2 - 6b^2$
   (c) $6b^2 + 6a^2$
   (d) $2b^3 - 2a^3$
   (e) $2a^3 - 2b^3$
   (f) $2b^3 + 2a^3$
   (g) $6(b-a)^2 + C$

   ANSWER:
   (d). We find the antiderivative $2x^3$ and use the Fundamental Theorem of Calculus.

2. Using a substitution, we have $\int_0^2 x(x^2+1)^3\,dx =$
   (a) $2\int_1^5 w^3\,dw$
   (b) $\int_1^5 w^3\,dw$
   (c) $\frac{1}{2}\int_0^2 w^3\,dw$
   (d) $\int_0^2 w^3\,dw$
   (e) $\frac{1}{2}\int_1^5 w^3\,dw$
   (f) $2\int_0^2 w^3\,dw$

   ANSWER:
   (e). We use the substitution $w = x^2 + 1$. For the new limits of integration, notice that $x = 0$ gives $w = 1$ and $x = 2$ gives $w = 5$.

3. If $\int_0^6 f(t)\,dt = 8$, then $\int_0^3 f(2x)\,dx =$
   (a) 4
   (b) 5
   (c) 6
   (d) 10
   (e) 16

   ANSWER:
   (a). We use the substitution $t = 2x$. Then $dt = 2\,dx$ and when $x = 0$ we have $t = 0$ and when $x = 3$ we have $t = 6$. Therefore
   $$\int_0^3 f(2x)\,dx = \frac{1}{2}\int_0^6 f(t)\,dt = \frac{1}{2}\cdot 8 = 4.$$

ConcepTests for Section 7.4

1. Which of the following graphs (a)–(d) could represent an antiderivative of the function shown in Figure 7.1?

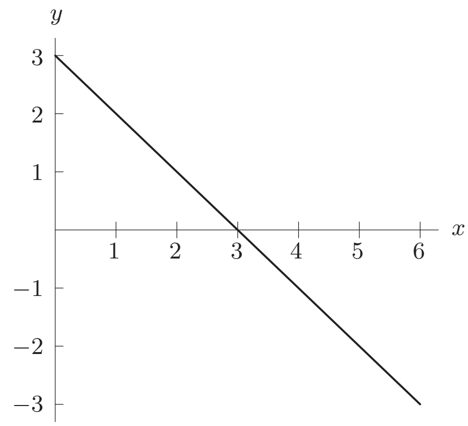
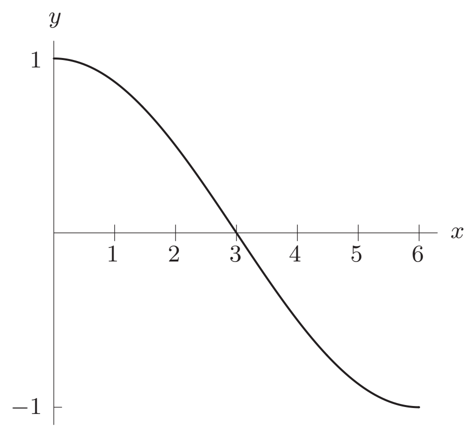
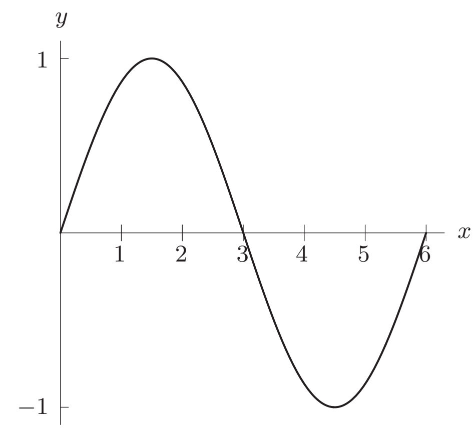


Figure 7.1

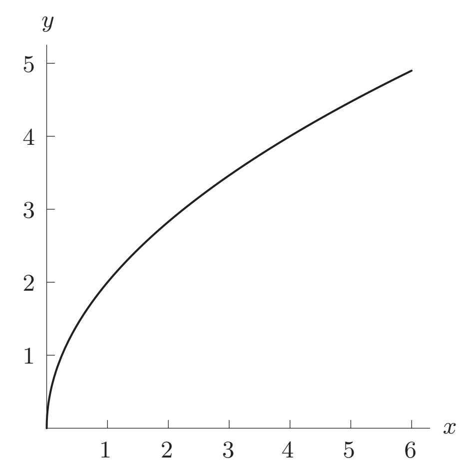
(a)



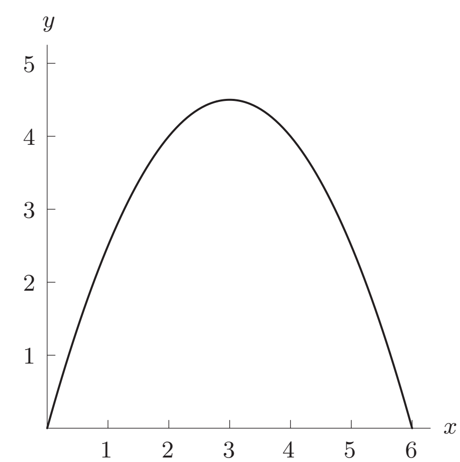
(b)



(c)



(d)



ANSWER:

(d). Because the graph in Figure 7.1 is decreasing, the graph of the antiderivative must be concave down. Because the graph in Figure 7.1 is positive for $x < 3$, zero for $x = 3$, and negative for $x > 3$, then $x = 3$ is a local maximum.

COMMENT:

You can mimic this problem with $y = x - 3$.

2. Which of the following graphs (a)–(d) could represent an antiderivative of the function shown in Figure 7.2?

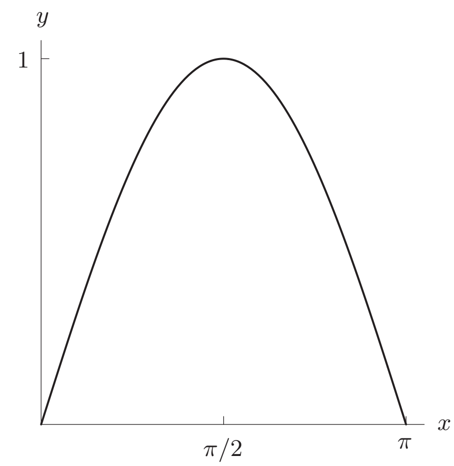
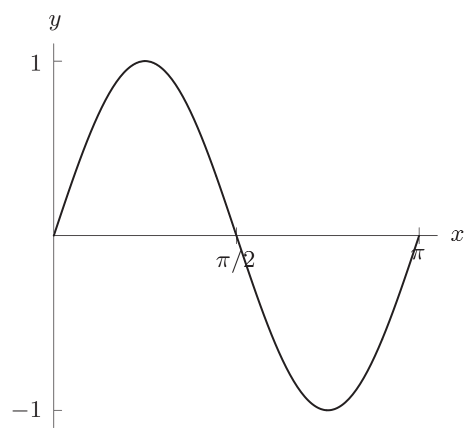
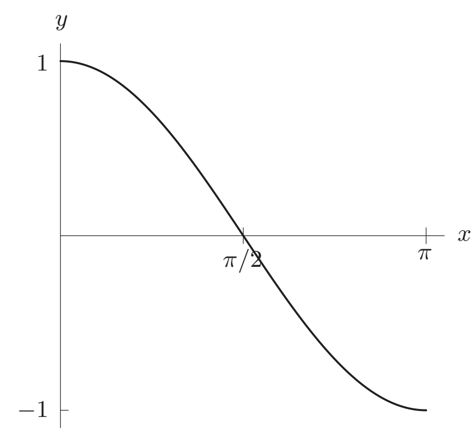


Figure 7.2

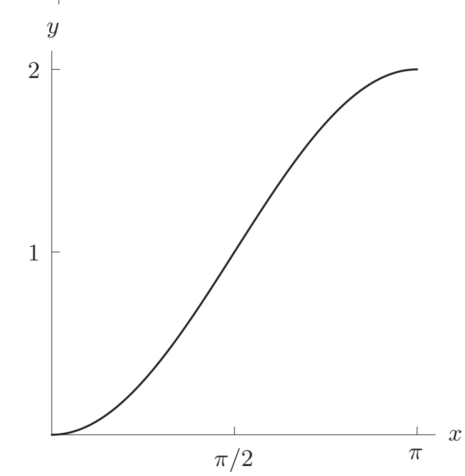
(a)



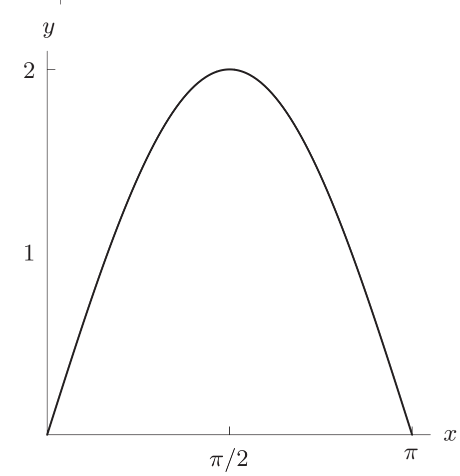
(b)



(c)



(d)



ANSWER:

(c). Because the graph in Figure 7.2 is always positive on this interval, the antiderivative must be increasing for this interval.

COMMENT:

You can mimic this problem with $y = -\sin x$. You could also ask your students to draw other possible antiderivatives of the function shown in Figure 7.2.

3. Which of the following graphs (a)–(d) could represent an antiderivative of the function shown in Figure 7.3?

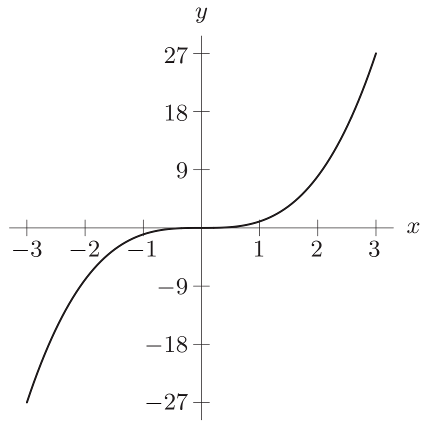
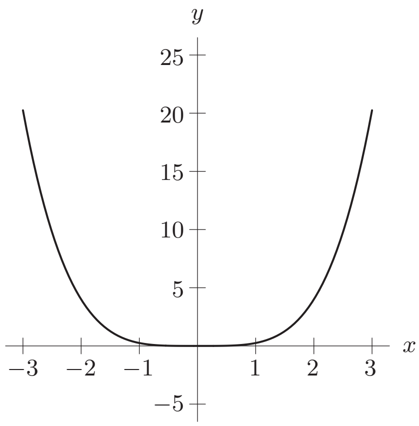
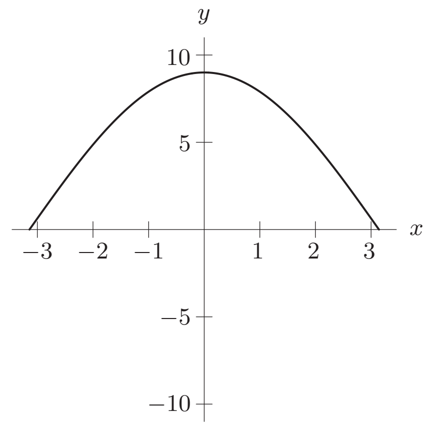


Figure 7.3

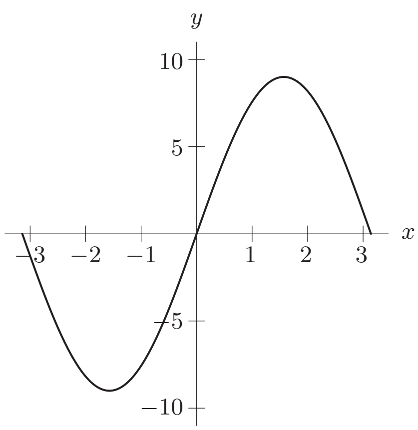
(a)



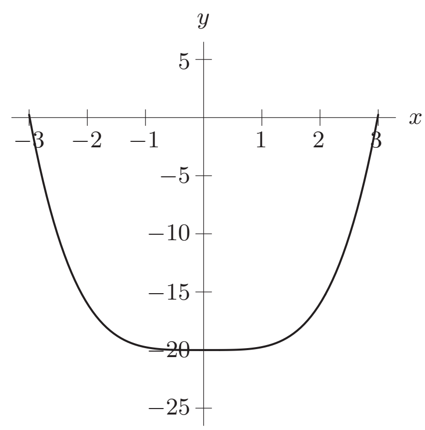
(b)



(c)



(d)



ANSWER:

(a) and (d). Because the graph in Figure 7.3 is continually increasing, the graph of its antiderivative is concave up. Notice that the graphs in (a) and (d) differ only by a vertical shift.

COMMENT:

You could also point out that since the graph in Figure 7.3 is negative from $-3 < x < 0$, then the graph of an antiderivative must be decreasing on this interval. Also, since the graph in Figure 7.3 is positive for $0 < x < 3$, then the graph of an antiderivative must be increasing on this interval.

4. Consider the graph of $f'(x)$ in Figure 7.4. Which of the functions with values from the Table 7.1 could represent $f(x)$?

Table 7.1

	x	0	2	4	6
(a)	$g(x)$	1	3	4	3
(b)	$h(x)$	5	7	8	7
(c)	$j(x)$	32	34	35	34
(d)	$k(x)$	-9	-7	-6	-7

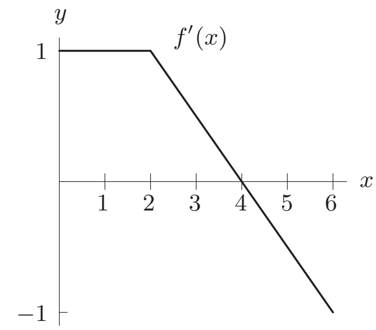
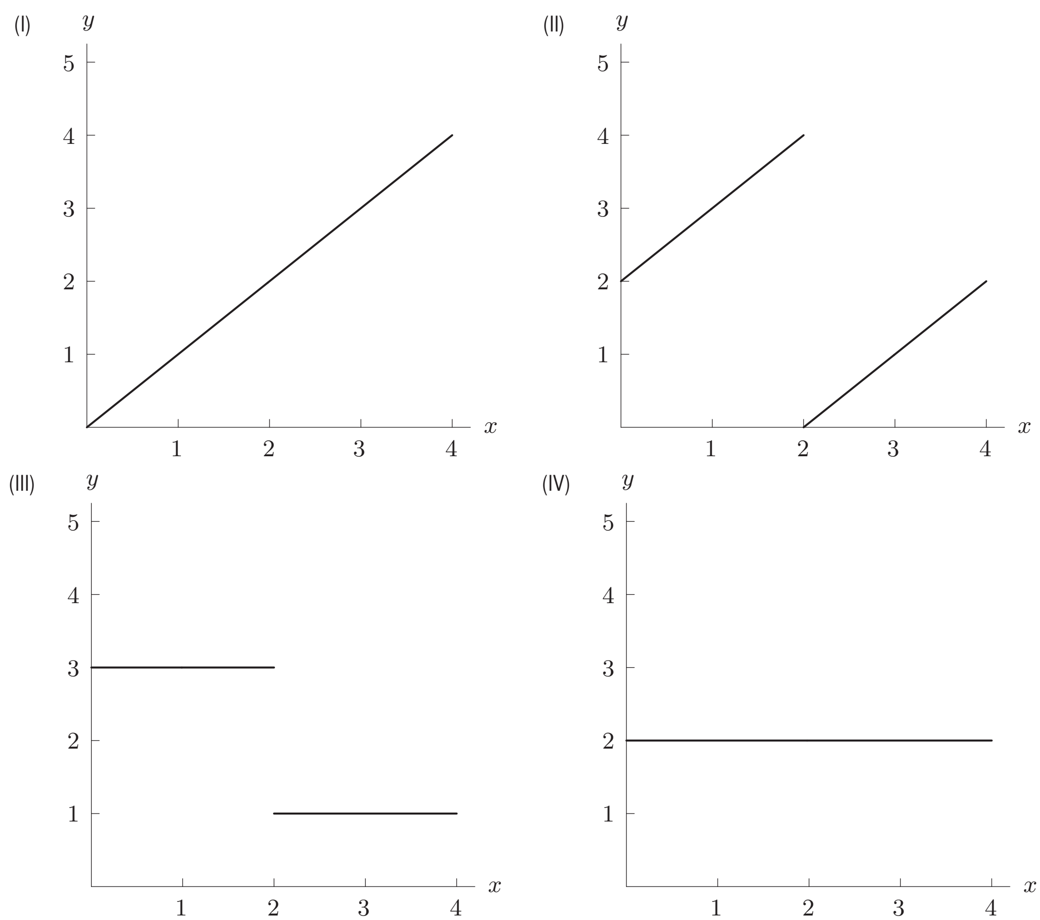


Figure 7.4

ANSWER:
(a), (b), (c), (d)
COMMENT:
You might point out only relative values of functions are important for this problem, not the actual values.

5. Graphs of the *derivatives* of four functions are shown in (I)–(IV). For the *functions* (not the derivative) list in increasing order which has the greatest change in value on the interval shown.

(a) (I), (IV), (III), (II)
(b) (I), (IV), (II), (III)
(c) (I) = (II), (IV), (III)
(d) (I) = (II), (III) = (IV)
(e) (I) = (II) = (III) = (IV)



ANSWER:

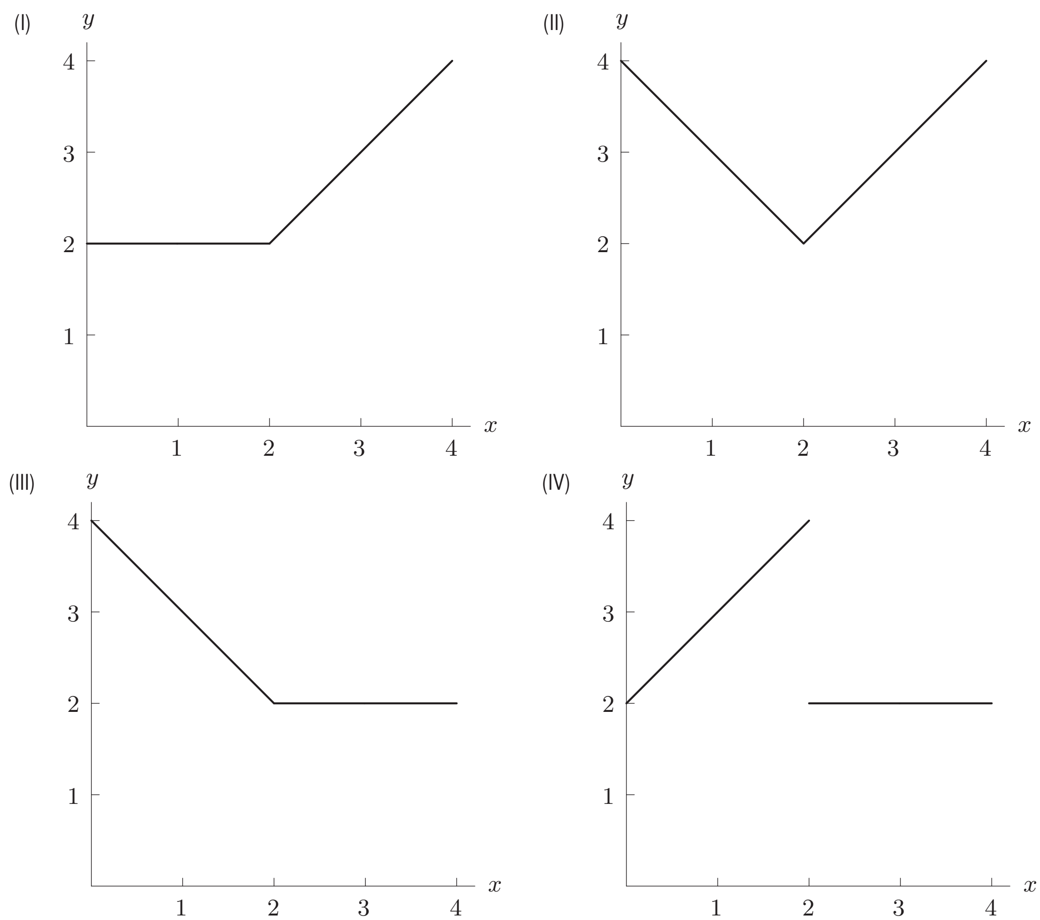
(e). The ordering will be given by the values of the area under the derivative curves. These areas are (I) $(1/2)(4)(4) = 8$, (II) $(1/2)(2)(4+2) + (1/2)(2)(2) = 8$, (III) $2(3) + 2(1) = 8$, and (IV) $4(2) = 8$.

COMMENT:

You can draw some other graphs of derivatives and ask which function has the greatest change over a specified interval.

6. Graphs of the *derivatives* of four functions are shown in (I) - (IV). For the *functions* (not the derivative) list in increasing order which has the greatest change in value on the interval shown.

(a) (I), (III), (IV), (II)
(b) (I) = (III), (IV), (II)
(c) (IV), (I) = (III), (II)
(d) (I) = (III) = (IV), (II)
(e) (I) = (II) = (III) = (IV)



ANSWER:

(d). The ordering will be given by the values of the area under these curves. These areas are (I) $4(2)+(1/2)(2)(2) = 10$, (II) $4(2) + 2(1/2)(2)(2) = 12$, (III) same as (I), 10, and (IV) $(1/2)(2)(2+4) + 2(2) = 10$.

COMMENT:

You could ask students how to change graph (II) to have area equal to 10 and not be identical to any of the other graphs.

7. Figure 7.5 shows a graph of $y = f(x)$ with some areas labeled. Assume $F'(x) = f(x)$ and $F(0) = 10$. Then $F(5) =$
   (a) 17
   (b) 4
   (c) 1
   (d) 13
   (e) 11
   (f) 16

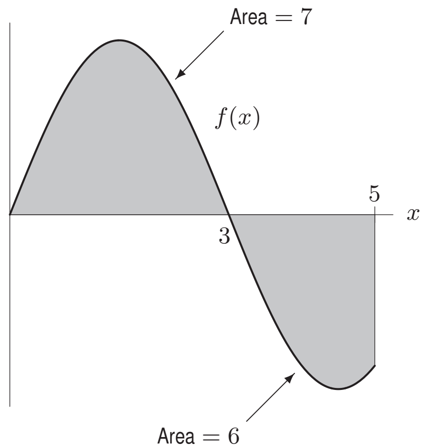


Figure 7.5

ANSWER:
(e). Since $\int_0^5 f(x)\,dx = 7 - 6 = 1$, we know that the total change in the antiderivative $F(x)$ between $x = 0$ and $x = 5$ is +1, so

$$F(5) = 10 + 1 = 11.$$

8. Figure 7.6 shows a graph of $y = F'(x)$. Where is there a local maximum on $F(x)$? A local minimum?
   (a) 7 ; 4
   (b) 1 and 7 ; 4.5
   (c) 2 ; 6
   (d) 6 ; 2
   (e) 4.5 ; 1 and 7
   (f) 7 ; 4

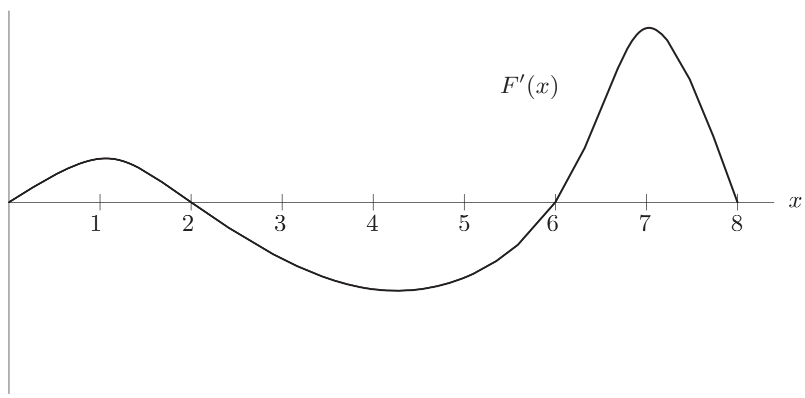


Figure 7.6

ANSWER:
(c). Since we see that $F'(x)$ is positive before 2, negative between 2 and 6, then positive after 6, we know that $F(x)$ is increasing before 2, decreasing between 2 and 6, and then increasing again after 6. Therefore, we know that $F(x)$ has a local maximum at $x = 2$ and a local minimum at $x = 6$.