

## CHAPTER SUMMARY

- **Function terminology**

Domain/range, increasing/decreasing, concavity, intercepts.

- **Linear functions**

Slope,  $y$ -intercept. Grow by equal amounts in equal times.

- **Economic applications**

Cost, revenue, and profit functions, break-even point. Supply and demand curves, equilibrium point. Depreciation function. Budget constraint. Present and future value.

- **Change, average rate of change, relative change**

- **Exponential functions**

Exponential growth and decay, growth rate, the number  $e$ , continuous growth rate, doubling time, half-life, compound interest. Grow by equal percentages in equal times.

- **The natural logarithm function**

- **New functions from old**

Composition, shifting, stretching.

- **Power functions and proportionality**

- **Polynomials**

- **Periodic functions**

Sine, cosine, amplitude, period.

## REVIEW PROBLEMS FOR CHAPTER ONE

1. The time  $T$ , in minutes, that it takes Dan to run  $x$  kilometers is a function  $T = f(x)$ . Explain the meaning of the statement  $f(5) = 23$  in terms of running.
2. Describe what Figure 1.103 tells you about an assembly line whose productivity is represented as a function of the number of workers on the line.

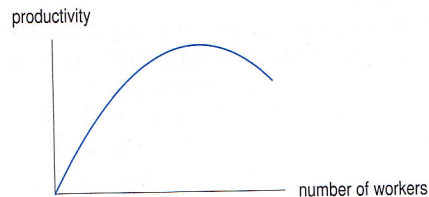


Figure 1.103

3. It warmed up throughout the morning, and then suddenly got much cooler around noon, when a storm came through. After the storm, it warmed up before cooling off at sunset. Sketch temperature as a function of time.
4. A gas tank 6 meters underground springs a leak. Gas seeps out and contaminates the soil around it. Graph the amount of contamination as a function of the depth (in meters) below ground.
5. The yield,  $Y$ , of an apple orchard (in bushels) as a function of the amount,  $a$ , of fertilizer (in pounds) used on the orchard is shown in Figure 1.104.
  - (a) Describe the effect of the amount of fertilizer on the yield of the orchard.
  - (b) What is the vertical intercept? Explain what it means in terms of apples and fertilizer.
  - (c) What is the horizontal intercept? Explain what it means in terms of apples and fertilizer.
  - (d) What is the range of this function for  $0 \leq a \leq 80$ ?
  - (e) Is the function increasing or decreasing at  $a = 60$ ?

- (f) Is the graph concave up or down near  $a = 40$ ?

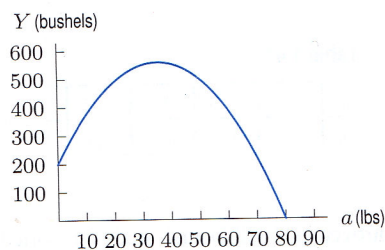


Figure 1.104

6. Let  $y = f(x) = 3x - 5$ .
  - (a) What is  $f(1)$ ?
  - (b) Find the value of  $y$  when  $x$  is 5.
  - (c) Find the value of  $x$  when  $y$  is 4.
  - (d) Find the average rate of change of  $f$  between  $x = 2$  and  $x = 4$ .
7. You drive at a constant speed from Chicago to Detroit, a distance of 275 miles. About 120 miles from Chicago you pass through Kalamazoo, Michigan. Sketch a graph of your distance from Kalamazoo as a function of time.
 

Find the equation of the line passing through the points in Problems 8–11.
8.  $(0, -1)$  and  $(2, 3)$
9.  $(-1, 3)$  and  $(2, 2)$
10.  $(0, 2)$  and  $(2, 2)$
11.  $(-1, 3)$  and  $(-1, 4)$
12. Match the graphs in Figure 1.105 with the following equations. (Note that the  $x$  and  $y$  scales may be unequal.)

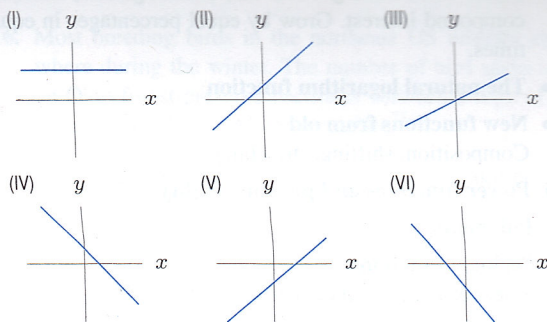


Figure 1.105

13. Find a linear function that generates the values in Table 1.41.

Table 1.41

| $x$ | 5.2  | 5.3  | 5.4  | 5.5  | 5.6  |
|-----|------|------|------|------|------|
| $y$ | 27.8 | 29.2 | 30.6 | 32.0 | 33.4 |

14. A controversial 1992 Danish study<sup>78</sup> reported that men's average sperm count has decreased from 113 million per milliliter in 1940 to 66 million per milliliter in 1990.

- Express the average sperm count,  $S$ , as a linear function of the number of years,  $t$ , since 1940.
- A man's fertility is affected if his sperm count drops below about 20 million per milliliter. If the linear model found in part (a) is accurate, in what year will the average male sperm count fall below this level?

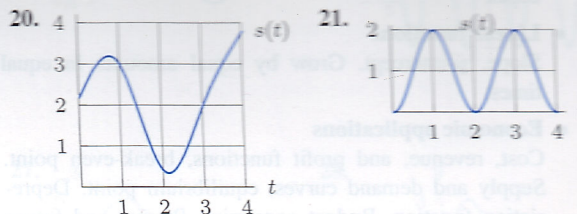
15. Residents of the town of Maple Grove who are connected to the municipal water supply are billed a fixed amount monthly plus a charge for each cubic foot of water used. A household using 1000 cubic feet was billed \$40, while one using 1600 cubic feet was billed \$55.

- What is the charge per cubic foot?
- Write an equation for the total cost of a resident's water as a function of cubic feet of water used.
- How many cubic feet of water used would lead to a bill of \$100?

In Problems 16–21, find the average velocity for the position function  $s(t)$ , in mm, over the interval  $1 \leq t \leq 3$ , where  $t$  is in seconds.

16.  $s(t) = 12t - t^2$       17.  $s(t) = \ln(t)$

<sup>78</sup>"Investigating the Next Silent Spring," *US News and World Report*, pp. 50–52 (March 11, 1996).



22. The graphs in Figure 1.106 represent the temperature,

$H$ , of four loaves of bread each put into an oven at time  $t = 0$ .

- Which curve corresponds to the bread that was put into the hottest oven?
- Which curve corresponds to the bread that had the lowest temperature at the time that it was put into the oven?
- Which two curves correspond to loaves of bread that were at the same temperature when they were put into the oven?
- Write a sentence describing any differences between the curves shown in (II) and (III). In terms of bread, what might cause this difference?

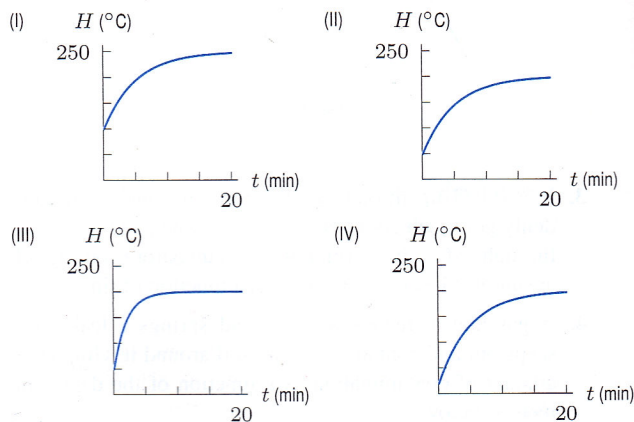


Figure 1.106

23. Sketch reasonable graphs for the following. Pay particular attention to the concavity of the graphs.

- The total revenue generated by a car rental business, plotted against the amount spent on advertising.
- The temperature of a cup of hot coffee standing in a room, plotted as a function of time.

24. Each of the functions  $g, h, k$  in Table 1.42 is increasing, but each increases in a different way. Which of the graphs in Figure 1.107 best fits each function?

Table 1.42

| $t$ | $g(t)$ | $h(t)$ | $k(t)$ |
|-----|--------|--------|--------|
| 1   | 23     | 10     | 2.2    |
| 2   | 24     | 20     | 2.5    |
| 3   | 26     | 29     | 2.8    |
| 4   | 29     | 37     | 3.1    |
| 5   | 33     | 44     | 3.4    |
| 6   | 38     | 50     | 3.7    |

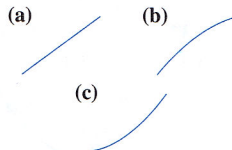


Figure 1.107

25. When a new product is advertised, more and more people try it. However, the rate at which new people try it slows as time goes on.

- (a) Graph the total number of people who have tried such a product against time.  
 (b) What do you know about the concavity of the graph?

26. Figure 1.108 shows the age-adjusted death rates from different types of cancer among US males.<sup>79</sup>

- (a) Discuss how the death rate has changed for the different types of cancers.  
 (b) For which type of cancer has the average rate of change between 1930 and 1967 been the largest? Estimate the average rate of change for this cancer type. Interpret your answer.  
 (c) For which type of cancer has the average rate of change between 1930 and 1967 been the most negative? Estimate the average rate of change for this cancer type. Interpret your answer.

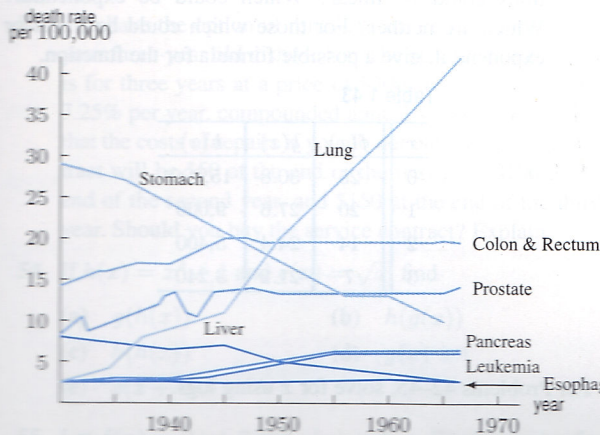


Figure 1.108

27. The volume of water in a pond over a period of 20 weeks is shown in Figure 1.109.

- (a) Is the average rate of change of volume positive or negative over the following intervals?

- (i)  $t = 0$  and  $t = 5$     (ii)  $t = 0$  and  $t = 10$   
 (iii)  $t = 0$  and  $t = 15$     (iv)  $t = 0$  and  $t = 20$

- (b) During which of the following time intervals was the average rate of change larger?

- (i)  $0 \leq t \leq 5$  or  $0 \leq t \leq 10$   
 (ii)  $0 \leq t \leq 10$  or  $0 \leq t \leq 20$

- (c) Estimate the average rate of change between  $t = 0$  and  $t = 10$ . Interpret your answer in terms of water.

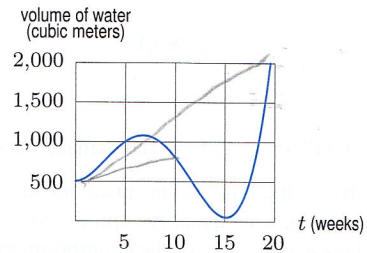


Figure 1.109

28. Find the average rate of change between  $x = 0$  and  $x = 10$  of each of the following functions:  $y = x$ ,  $y = x^2$ ,  $y = x^3$ , and  $y = x^4$ . Which has the largest average rate of change? Graph the four functions, and draw lines whose slopes represent these average rates of change.

29. (a) What are the fixed costs and the marginal cost for the cost function in Figure 1.110?

- (b) Explain what  $C'(100) = 2500$  tells you about costs.

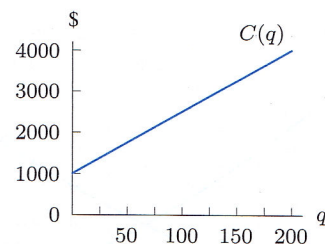


Figure 1.110

30. The Quick-Food company provides a college meal-service plan. Quick-Food has fixed costs of \$350,000 per term and variable costs of \$400 per student. Quick-Food charges \$800 per student per term. How many students



46. Write the exponential functions  $P = e^{0.08t}$  and  $Q = e^{-0.3t}$  in the form  $P = a^t$  and  $Q = b^t$ .

47. (a) What is the continuous percent growth rate for the function  $P = 10e^{0.15t}$ ?

(b) Write this function in the form  $P = P_0a^t$ .

(c) What is the annual (not continuous) percent growth rate for this function?

(d) Graph  $P = 10e^{0.15t}$  and your answer to part (b) on the same axes. Explain what you see.

48. You need \$10,000 in your account 3 years from now and the interest rate is 8% per year, compounded continuously. How much should you deposit now?

49. If  $Q_0$  is the quantity of radioactive carbon-14 in an organism at the time of death, the quantity,  $Q$ , remaining  $t$  years later is given by

$$Q = Q_0e^{-0.000121t}$$

(a) A skull uncovered at an archeological dig has 15% of the original amount of carbon-14 present. Estimate its age.

(b) Calculate the half-life of carbon-14.

50. A radioactive substance has a half-life of 8 years. If 200 grams are present initially, how much remains at the end of 12 years? How long until only 10% of the original amount remains?

51. The size of an exponentially growing bacteria colony doubles in 5 hours. How long will it take for the number of bacteria to triple?

52. When the Olympic Games were held outside Mexico City in 1968, there was much discussion about the effect the high altitude (7340 feet) would have on the athletes. Assuming air pressure decays exponentially by 0.4% every 100 feet, by what percentage is air pressure reduced by moving from sea level to Mexico City?

53. You have the option of renewing the service contract on your three-year old dishwasher. The new service contract is for three years at a price of \$200. The interest rate is 7.25% per year, compounded annually, and you estimate that the costs of repairs if you do not buy the service contract will be \$50 at the end of the first year, \$100 at the end of the second year, and \$150 at the end of the third year. Should you buy the service contract? Explain.

54. If  $h(x) = x^3 + 1$  and  $g(x) = \sqrt{x}$ , find

(a)  $g(h(x))$  (b)  $h(g(x))$

(c)  $h(h(x))$  (d)  $g(x) + 1$

(e)  $g(x + 1)$

55. Let  $f(x) = 2x + 3$  and  $g(x) = \ln x$ . Find formulas for each of the following functions.

(a)  $g(f(x))$  (b)  $f(g(x))$  (c)  $f(f(x))$

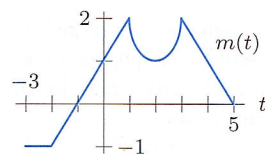


Figure 1.113

56.  $n(t) = m(t) + 2$

57.  $p(t) = m(t - 1)$

58.  $k(t) = m(t + 1.5)$

59.  $w(t) = m(t - 0.5) - 2.5$

In Problems 60–62, use Figure 1.114 to graph the function.

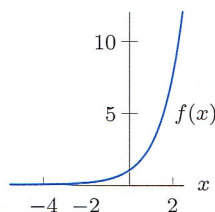


Figure 1.114

60.  $5f(x)$

61.  $f(x + 5)$

62.  $f(x) + 5$

63. A plan is adopted to reduce the pollution in a lake to the legal limit. The quantity  $Q$  of pollutants in the lake after  $t$  weeks of clean-up is modeled by the function  $Q = f(t)$  where  $f(t) = A + Be^{Ct}$ .

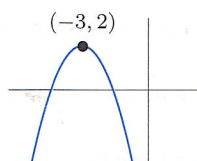
(a) What are the signs of  $A$ ,  $B$  and  $C$ ?

(b) What is the initial quantity of pollution in the lake?

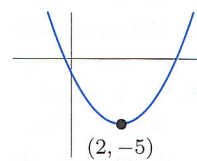
(c) What is the legal limit of pollution in the lake?

In Problems 64–65, use shifts of a power function to find a possible formula for the graph.

64.



65.



66. The following table gives values for a function  $p = f(t)$ . Could  $p$  be proportional to  $t$ ?

|     |   |    |    |     |     |     |
|-----|---|----|----|-----|-----|-----|
| $t$ | 0 | 10 | 20 | 30  | 40  | 50  |
| $p$ | 0 | 25 | 60 | 100 | 140 | 200 |

In Problems 56–59, use Figure 1.113 to graph the functions.

67. Zipf's Law, developed by George Zipf in 1949, states that in a given country, the population of a city is inversely proportional to the city's rank by size in the country.<sup>81</sup> Assuming Zipf's Law:

- (a) Write a formula for the population,  $P$ , of a city as a function of its rank,  $R$ .
- (b) If the constant of proportionality  $k$  is 300,000, what is the approximate population of the largest city (rank 1)? The second largest city (rank 2)? The third largest city?
- (c) Answer the questions of part (b) if  $k = 6$  million.
- (d) Interpret the meaning of the constant of proportionality  $k$  in this context.

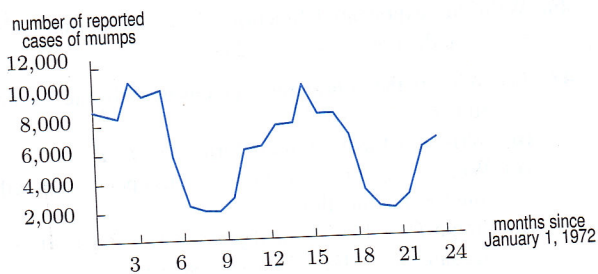


Figure 1.115

Find the period and amplitude in Problems 68–70.

68.  $y = 7 \sin(3t)$

69.  $z = 3 \cos(u/4) + 5$

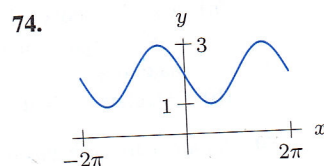
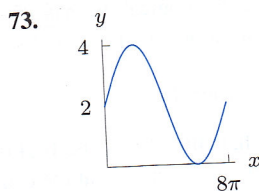
70.  $r = 0.1 \sin(\pi t) + 2$

71. Figure 1.115 shows the number of reported<sup>82</sup> cases of mumps by month, in the US, for 1972–73.

- (a) Find the period and amplitude of this function, and interpret each in terms of mumps.
- (b) Predict the number of cases of mumps 30 months and 45 months after January 1, 1972.

72. A population of animals varies periodically between a low of 700 on January 1 and a high of 900 on July 1. Graph the population against time.

For Problems 73–74, find a possible formula for each graph.



### CHECK YOUR UNDERSTANDING

In Problems 1–105, indicate whether the statement is true or false.

- 1. The domain is the set of outputs of a function.
- 2. If  $V = f(a)$ , where  $V$  is the value of a car (in thousands of dollars) and  $a$  is the car's age (in years), then  $f(10)$  is the age of a car valued at \$10,000.
- 3. If  $f(x) = x^2$ , then the point  $(2, 4)$  is on the graph of  $f(x)$ .
- 4. The set of numbers between 3 and 4, including 3 and 4, is written  $(3, 4)$ .
- 5. The function  $D = f(r)$  given by  $D = -3r + 10$  has vertical intercept 10.
- 6. A function is always given by a formula.
- 7. If  $f(x) = x^2 + 2x + 1$  then  $f(3) = 16$ .
- 8. The graph of a function can have more than one horizontal intercept.
- 9. The graph of a function can have more than one vertical intercept.

- 10. The vertical intercept on the graph of  $C = f(q)$ , where  $C$  is the cost to produce  $q$  items, represents the cost to produce no items.
- 11. The slope of the graph of a linear function  $f(p)$  is  $(f(p_2) - f(p_1))/(p_2 - p_1)$ .
- 12. The graph of the linear function with formula  $m(x) = 3x + 2$  has slope 2.
- 13. The slope of the graph of a linear function can be zero.
- 14. An equation of a line with slope  $-1$  that passes through the point  $(2, 5)$  is  $y = -x + 7$ .
- 15. The function whose values are shown in the following table could be linear:

|        |    |    |    |     |
|--------|----|----|----|-----|
| $s$    | 2  | 4  | 6  | 8   |
| $h(s)$ | -1 | -5 | -9 | -13 |

<sup>81</sup>Strogatz, S., "Math and the City", The New York Times, May 20, 2009.  
<sup>82</sup>Center for Disease Control, 1974, *Reported Morbidity and Mortality in the United States 1973*, Vol. 22, No. 53. Prior to the licensing of the vaccine in 1967, 100,000–200,000 cases of mumps were reported annually. Since 1995, fewer than 1000 cases are reported annually. Source: CDC.