US Pop 2010: 308,745,538
        2000. 282,171,900
predict in 2020 using linear + exp fus
Let t = year

REVIEW PROBLEMS FOR CHAPTER ONE   71

REVIEW PROBLEMS FOR CHAPTER ONE

1. Which graph in Figure 1.105 best matches each of the following stories?[59] Write a story for the remaining graph.

   (a) I had just left home when I realized I had forgotten my books, and so I went back to pick them up.
   (b) Things went fine until I had a flat tire.
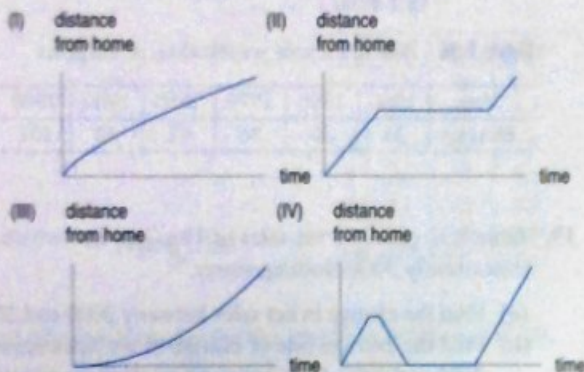   (c) I started out calmly but sped up when I realized I was going to be late.

Figure 1.105

2. The population of Washington DC grew from 1900 to 1950, stayed approximately constant during the 1950s, and decreased from about 1960 to 2000. Graph the population as a function of years since 1900.

3. It warmed up throughout the morning, and then suddenly got much cooler around noon, when a storm came through. After the storm, it warmed up before cooling off at sunset. Sketch temperature as a function of time.

4. A gas tank 6 meters underground springs a leak. Gas seeps out and contaminates the soil around it. Graph the amount of contamination as a function of the depth (in meters) below ground.

5. (a) The graph of $r = f(p)$ is in Figure 1.106. What is the value of r when p is 0? When p is 3?
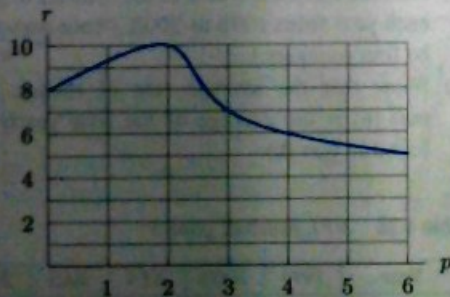   (b) What is $f(2)$?

Figure 1.106

6. The yield, Y, of an apple orchard (in bushels) as a function of the amount, a, of fertilizer (in pounds) used on the orchard is shown in Figure 1.107.

   (a) Describe the effect of the amount of fertilizer on the yield of the orchard.
   (b) What is the vertical intercept? Explain what it means in terms of apples and fertilizer.
   (c) What is the horizontal intercept? Explain what it means in terms of apples and fertilizer.
   (d) What is the range of this function for $0 \leq a \leq 80$?
   (e) Is the function increasing or decreasing at $a = 60$?
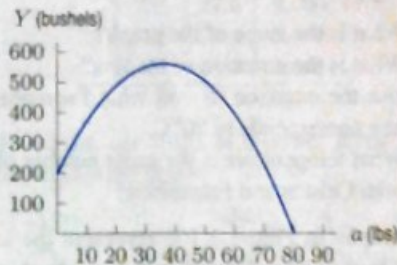   (f) Is the graph concave up or down near $a = 40$?

Figure 1.107

7. Let $y = f(x) = 3x - 5$.

   (a) What is $f(1)$?
   (b) Find the value of y when x is 5.
   (c) Find the value of x when y is 4.
   (d) Find the average rate of change of f between $x = 2$ and $x = 4$.

Find the equation of the line passing through the points in Problems 8–11.

8. $(0, -1)$ and $(2, 3)$ 9. $(-1, 3)$ and $(2, 2)$

10. $(0, 2)$ and $(2, 2)$ 11. $(-1, 3)$ and $(-1, 4)$

12. Find the linear equation used to generate the values in Table 1.34.

Table 1.34

x	5.2	5.3	5.4	5.5	5.6
y	27.8	29.2	30.6	32.0	33.4

13. The percentage of people, P, below the poverty level in the US[60] is given in Table 1.35. Find a formula for the percentage as a linear function of time since 2000.

Table 1.35

Year (since 2000)	0	1	2	3
P (percentage)	11.3	11.7	12.1	12.5

---

[59] Adapted from Jan Terwel, "Real Math in Cooperative Groups in Secondary Education." *Cooperative Learning in Mathematics*, ed. Neal Davidson, p. 234, (Reading: Addison Wesley, 1990).
[60] *The World Almanac and Book of Facts 2005*, p. 128 (New York).

14. Residents of the town of Maple Grove who are connected to the municipal water supply are billed a fixed amount yearly plus a charge for each cubic foot of water used. A household using 1000 cubic feet was billed $90, while one using 1600 cubic feet was billed $105.

(a) What is the charge per cubic foot?
(b) Write an equation for the total cost of a resident's water as a function of cubic feet of water used.
(c) How many cubic feet of water used would lead to a bill of $130?

15. The graph of Fahrenheit temperature, °F, as a function of Celsius temperature, °C, is a line. You know that 212°F and 100°C both represent the temperature at which water boils. Similarly, 32°F and 0°C both represent water's freezing point.

(a) What is the slope of the graph?
(b) What is the equation of the line?
(c) Use the equation to find what Fahrenheit temperature corresponds to 20°C.
(d) What temperature is the same number of degrees in both Celsius and Fahrenheit?

16. The graphs in Figure 1.108 represent the temperature, H, of four loaves of bread each put into an oven at time $t = 0$.

(a) Which curve corresponds to the bread that was put into the hottest oven?
(b) Which curve corresponds to the bread that had the lowest temperature at the time that it was put into the oven?
(c) Which two curves correspond to loaves of bread that were at the same temperature when they were put into the oven?
(d) Write a sentence describing any differences between the curves shown in (II) and (III). In terms of bread, what might cause this difference?
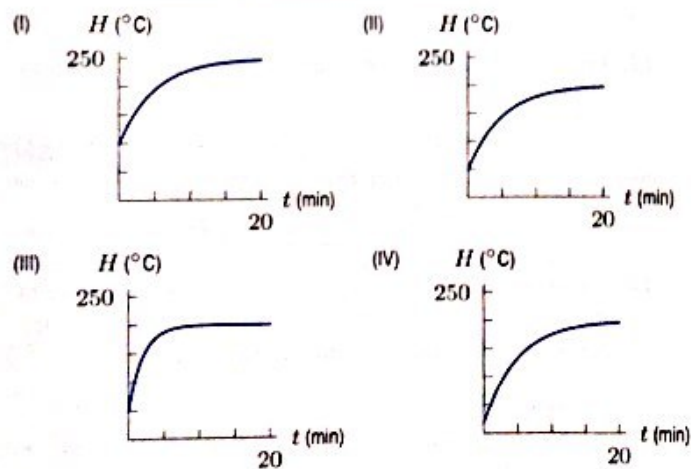

Figure 1.108

17. You drive at a constant speed from Chicago to Detroit, a distance of 275 miles. About 120 miles from Chicago you pass through Kalamazoo, Michigan. Sketch a graph of your distance from Kalamazoo as a function of time.

18. Table 1.36 shows world bicycle production.[61]

(a) Find the change in bicycle production between 1950 and 2000. Give units.
(b) Find the average rate of change in bicycle production between 1950 and 2000. Give units and interpret your answer in terms of bicycle production.

Table 1.36   World bicycle production, in millions

Year	1950	1960	1970	1980	1990	2000
Bicycles	11	20	36	62	92	101

19. Table 1.37 gives the net sales of The Gap, Inc, which operates nearly 3000 clothing stores.[62]

(a) Find the change in net sales between 2000 and 2003.
(b) Find the average rate of change in net sales between 2000 and 2003. Give units and interpret your answer.
(c) From 1998 to 2003, were there any one-year intervals during which the average rate of change was negative? If so, when?

Table 1.37   Gap net sales, in millions of dollars

Year	1998	1999	2000	2001	2002	2003
Sales	9054	11,635	13,673	13,848	14,455	15,854

20. Find the average rate of change of $f(x) = 3x^2 + 1$ between $x = -2$ and $x = 1$. Illustrate your answer graphically.

21. Table 1.38 shows attendance in millions at NFL football games.[63]

(a) Find the average rate of change in the number of games from 1999 to 2003. Give units.
(b) Find the annual increase in the number of games for each year from 1999 to 2003. (Your answer should be four numbers.)
(c) Show that the average rate of change found in part (a) is the average of the four yearly changes found in part (b).

Table 1.38   NFL football games

Year	1999	2000	2001	2002	2003
Attendance	20.76	20.95	20.59	21.51	21.64

[61] www.earth-policy.org/Indicators/indicator11_data1.htm, accessed April 19, 2005.
[62] www.gapinc.com/financmedia/AR_proxy.htm, accessed February 2, 2005.
[63] Statistical Abstracts of the United States 2004–2005, Table 1239.

14. Residents of the town of Maple Grove who are connected to the municipal water supply are billed a fixed amount yearly plus a charge for each cubic foot of water used. A household using 1000 cubic feet was billed $90, while one using 1600 cubic feet was billed $105.

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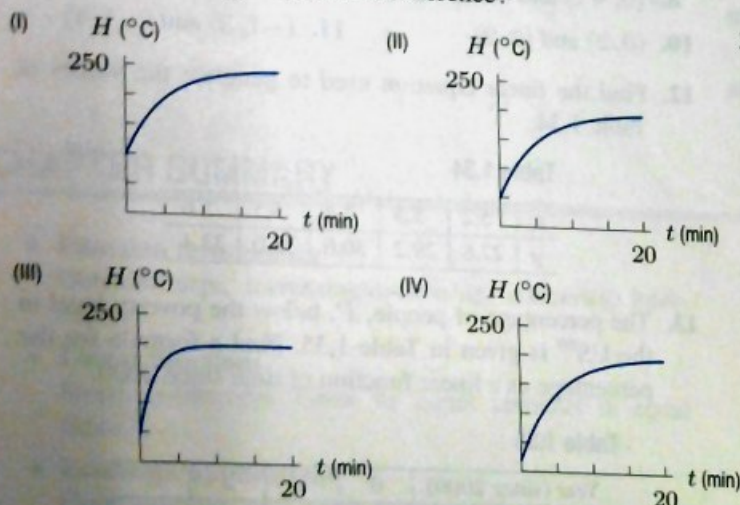

Figure 1.108

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Table 1.36 *World bicycle production, in millions*

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Table 1.37 *Gap net sales, in millions of dollars*

Year	1998	1999	2000	2001	2002	2003
Sales	9054	11,635	13,673	13,848	14,455	15,854

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(b) Find the annual increase in the number of games for each year from 1999 to 2003. (Your answer should be four numbers.)

(c) Show that the average rate of change found in part (a) is the average of the four yearly changes found in part (b).

Table 1.38 *NFL football games*

Year	1999	2000	2001	2002	2003
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[62] www.gapinc.com/financmedia/AR_proxy.htm, accessed February 2, 2005.
[63] *Statistical Abstracts of the United States 2004–2005, Table 1239.*

22. Sketch reasonable graphs for the following. Pay particular attention to the concavity of the graphs.

(a) The total revenue generated by a car rental business, plotted against the amount spent on advertising.

(b) The temperature of a cup of hot coffee standing in a room, plotted as a function of time.

23. Each of the functions g, h, k in Table 1.39 is increasing, but each increases in a different way. Which of the graphs in Figure 1.109 best fits each function?

(a) (b)

(c)

Figure 1.109

Table 1.39

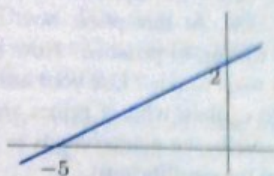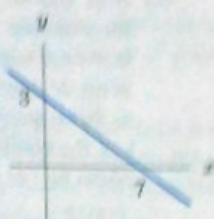| t	$g(t)$	$h(t)$	$k(t)$
1	23	10	2.2
2	24	20	2.5
3	26	29	2.8
4	29	37	3.1
5	33	44	3.4
6	38	50	3.7
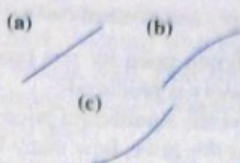
Find possible formulas for the graphs in Problems 24–29.

24.

25.
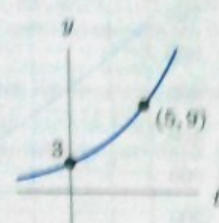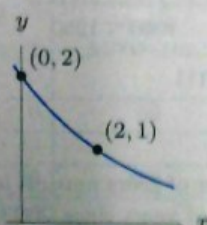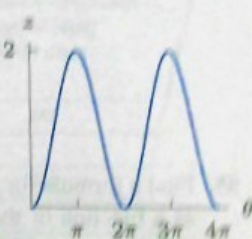

26.
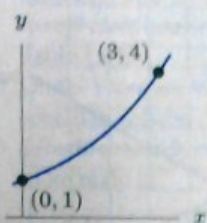
27.

28.

29.

30. Find the average rate of change between $x = 0$ and $x = 10$ of each of the following functions: $y = x$, $y = x^2$, $y = x^3$, and $y = x^4$. Which has the largest average rate of change? Graph the four functions, and draw lines whose slopes represent these average rates of change.

31. The rate, R, at which a population in a confined space increases is proportional to the product of the current population, P, and the difference between the carrying capacity, L, and the current population. (The carrying capacity is the maximum population the environment can sustain.)

(a) Write R as a function of P.

(b) Sketch R as a function of P.

32. Table 1.40 gives values for three functions. Which functions could be linear? Which could be exponential? Which are neither? For those which could be linear or exponential, give a possible formula for the function.
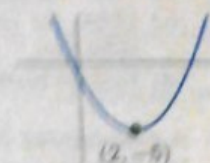
Table 1.40

x	$f(x)$	$g(x)$	$h(x)$
0	25	30.8	15,000
1	20	27.6	9,000
2	14	24.4	5,400
3	7	21.2	3,240

In Problems 33–34, use shifts of a power function to find a possible formula for the graph.

33.
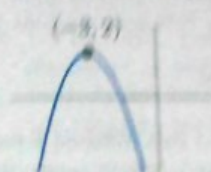
34.

For Problems 35–38, solve for x using logs.

35. $3^x = 11$

36. $20 = 50(1.04)^x$

37. $e^{5x} = 100$

38. $25e^{3x} = 10$

39. Write the exponential functions $P = e^{0.08t}$ and $Q = e^{-0.3t}$ in the form $P = a^t$ and $Q = b^t$.

40. If $h(x) = x^3 + 1$ and $g(x) = \sqrt{x}$, find

(a) $g(h(x))$ (b) $h(g(x))$

(c) $h(h(x))$ (d) $g(x) + 1$

(e) $g(x + 1)$

41. Let $f(x) = 2x + 3$ and $g(x) = \ln x$. Find formulas for each of the following functions.

(a) $g(f(x))$ (b) $f(g(x))$ (c) $f(f(x))$

42. For $f(n) = 3n^2 - 2$ and $g(n) = n + 1$, find and simplify:

(a) $f(n) + g(n)$

(b) $f(n)g(n)$

(c) The domain of $f(n)/g(n)$

(d) $f(g(n))$

(e) $g(f(n))$

Simplify the quantities in Problems 43–46 using $m(z) = z^2$.

43. $m(z + 1) - m(z)$

44. $m(z + h) - m(z)$

45. $m(z) - m(z - h)$

46. $m(z + h) - m(z - h)$

For Problems 47–48, determine functions f and g such that $h(x) = f(g(x))$. [Note: There is more than one correct answer. Do not choose $f(x) = x$ or $g(x) = x$.]

47. $h(x) = (x + 1)^3$ **48.** $h(x) = x^3 + 1$

49. An amusement park charges an admission fee of $7 per person as well as an additional $1.50 for each ride.

    (a) For one visitor, find the park's total revenue $R(n)$ as a function of the number of rides, n, taken.
    (b) Find $R(2)$ and $R(8)$ and interpret your answers in terms of amusement park fees.

50. A company that makes Adirondack chairs has fixed costs of $5000 and variable costs of $30 per chair. The company sells the chairs for $50 each.

    (a) Find formulas for the cost and revenue functions.
    (b) Find the marginal cost and marginal revenue.
    (c) Graph the cost and the revenue functions on the same axes.
    (d) Find the break-even point.

51. A photocopying company has two different price lists. The first price list is $100 plus 3 cents per copy; the second price list is $200 plus 2 cents per copy.

    (a) For each price list, find the total cost as a function of the number of copies needed.
    (b) Determine which price list is cheaper for 5000 copies.
    (c) For what number of copies do both price lists charge the same amount?

52. You have a budget of k to spend on soda and suntan oil, which cost p_1 per liter and p_2 per liter respectively.

    (a) Write an equation expressing the relationship between the number of liters of soda and the number of liters of suntan oil that you can buy if you exhaust your budget. This is your budget constraint.
    (b) Graph the budget constraint, assuming that you can buy fractions of a liter. Label the intercepts.
    (c) Suppose your budget is doubled. Graph the new budget constraint on the same axes.
    (d) With a budget of k, the price of suntan oil doubles. Graph the new budget constraint on the same axes.

53. A corporate office provides the demand curve in Figure 1.110 to its ice cream shop franchises. At a price of $1.00 per scoop, 240 scoops per day can be sold.

    (a) Estimate how many scoops could be sold per day at a price of 50¢ per scoop. Explain.
    (b) Estimate how many scoops per day could be sold at a price of $1.50 per scoop. Explain.
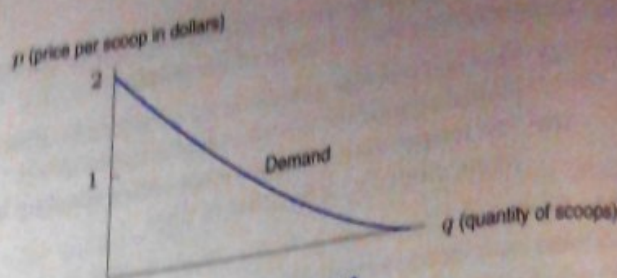

Figure 1.110

54. Figure 1.111 shows supply and demand curves.

    (a) What is the equilibrium price for this product? At this price, what quantity is produced?
    (b) Choose a price above the equilibrium price—for example, $p = 300$. At this price, how many items are suppliers willing to produce? How many items do consumers want to buy? Use your answers to these questions to explain why, if prices are above the equilibrium price, the market tends to push prices lower (toward the equilibrium).
    (c) Now choose a price below the equilibrium price—for example, $p = 200$. At this price, how many items are suppliers willing to produce? How many items do consumers want to buy? Use your answers to these questions to explain why, if prices are below the equilibrium price, the market tends to push prices higher (toward the equilibrium).
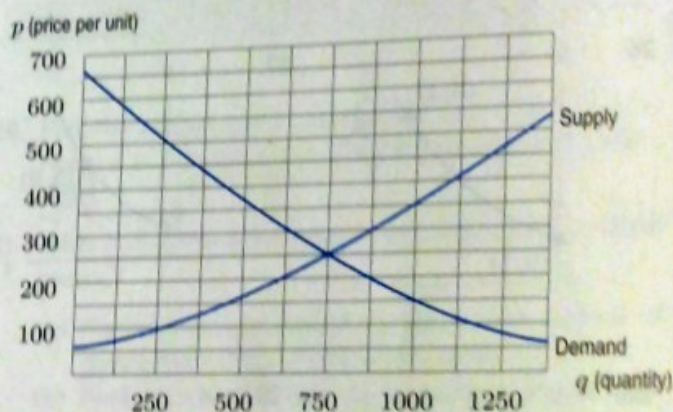

Figure 1.111

55. Find a formula for the number of zebra mussels in a bay as a function of the number of years since 2003, given that there were 2700 at the start of 2003 and 3186 at the start of 2004.

    (a) Assume that the number of zebra mussels is growing linearly. Give units for the slope of the line and interpret it in terms of zebra mussels.
    (b) Assume that the number of zebra mussels is growing exponentially. What is the percent rate of growth of the zebra mussel population?

56. Worldwide, wind energy[64] generating capacity, W, was 13,559 megawatts in 1998 and 39,151 megawatts in 2003.

   (a) Use the values given to write W, in megawatts, as a linear function of t, the number of years since 1998.

   (b) Use the values given to write W as an exponential function of t.

   (c) Graph the functions found in parts (a) and (b) on the same axes. Label the given values.

57. If the world's population increased exponentially from 4.478 billion in 1980 to 5.423 billion in 1991 and continued to increase at the same percentage rate between 1991 and 2004, calculate what the world's population would have been in 2004. How does this compare to the actual population of 6.378 billion, and what conclusions, if any, can you draw?

58. When the Olympic Games were held outside Mexico City in 1968, there was much discussion about the effect the high altitude (7340 feet) would have on the athletes. Assuming air pressure decays exponentially by 0.4% every 100 feet, by what percentage is air pressure reduced by moving from sea level to Mexico City?

59. The median price, P, of a home rose from $60,000 in 1980 to $180,000 in 2000. Let t be the number of years since 1980.

   (a) Assume the increase in housing prices has been linear. Give an equation for the line representing price, P, in terms of t. Use this equation to complete column (a) of Table 1.41. Use units of $1000.

   (b) If instead the housing prices have been rising exponentially, find an equation of the form $P = P_0 a^t$ to represent housing prices. Complete column (b) of Table 1.41.

   (c) On the same set of axes, sketch the functions represented in column (a) and column (b) of Table 1.41.

   (d) Which model for the price growth do you think is more realistic?

Table 1.41

t	(a) Linear growth price in $1000 units	(b) Exponential growth price in $1000 units
0	60	60
10		
20	180	180
30		
40		

60. (a) A population, P, grows at a continuous rate of 2% a year and starts at 1 million. Write P in the form $P = P_0 e^{kt}$, with P_0, k constants.

   (b) Plot the population in part (a) against time.

61. (a) What is the continuous percent growth rate for the function $P = 10e^{0.15t}$?

   (b) Write this function in the form $P = P_0 a^t$.

   (c) What is the annual (not continuous) percent growth rate for this function?

   (d) Graph $P = 10e^{0.15t}$ and your answer to part (b) on the same axes. Explain what you see.

62. The half-life of a radioactive substance is 12 days. There are 10.32 grams initially.

   (a) Write an equation for the amount, A, of the substance as a function of time.

   (b) When is the substance reduced to 1 gram?

63. Air pressure, P, decreases exponentially with the height, h, in meters above sea level:

$$P = P_0 e^{-0.00012h}$$

where P_0 is the air pressure at sea level.

   (a) At the top of Mount McKinley, height 6194 meters (about 20,320 feet), what is the air pressure, as a percent of the pressure at sea level?

   (b) The maximum cruising altitude of an ordinary commercial jet is around 12,000 meters (about 39,000 feet). At that height, what is the air pressure, as a percent of the sea level value?

64. A radioactive substance has a half-life of 8 years. If 200 grams are present initially, how much remains at the end of 12 years? How long until only 10% of the original amount remains?

65. One of the main contaminants of a nuclear accident, such as that at Chernobyl, is strontium-90, which decays exponentially at a rate of approximately 2.5% per year.

   (a) Write the percent of strontium-90 remaining, P, as a function of years, t, since the nuclear accident. [Hint: 100% of the contaminant remains at $t = 0$.]

   (b) Graph P against t.

   (c) Estimate the half-life of strontium-90.

   (d) After the Chernobyl disaster, it was predicted that the region would not be safe for human habitation for 100 years. Estimate the percent of original strontium-90 remaining at this time.

66. The number of people living with HIV infections increased worldwide approximately exponentially from 2.5 million in 1985 to 37.8 million in 2003.[65] (HIV is the virus that causes AIDS.)

   (a) Give a formula for the number of HIV infections, H, (in millions) as a function of years, t, since 1985. Use the form $H = H_0 e^{kt}$. Graph this function.

   (b) What was the yearly continuous percent change in the number of HIV infections between 1985 and 2003?

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[64] www.wwindea.org/wind_energy.htm, accessed April 24, 2005.
[65] *The World Almanac and Book of Facts 2005*, p. 89 (New York).

67. The size of an exponentially growing bacteria colony doubles in 5 hours. How long will it take for the number of bacteria to triple?

68. Interest is compounded annually. Consider the following choices of payments to you:

Choice 1: $1500 now and $3000 one year from now

Choice 2: $1900 now and $2500 one year from now

(a) If the interest rate on savings were 5% per year, which would you prefer?

(b) Is there an interest rate that would lead you to make a different choice? Explain.

69. A person is to be paid $2000 for work done over a year. Three payment options are being considered. Option 1 is to pay the $2000 in full now. Option 2 is to pay $1000 now and $1000 in a year. Option 3 is to pay the full $2000 in a year. Assume an annual interest rate of 5% a year, compounded continuously.

(a) Without doing any calculations, which option is the best option financially for the worker? Explain.

(b) Find the future value, in one year's time, of all three options.

(c) Find the present value of all three options.

70. Using Figure 1.112, find $f(g(x))$ and $g(f(x))$ for $x = -3, -2, -1, 0, 1, 2, 3$. Then graph $f(g(x))$ and $g(f(x))$.
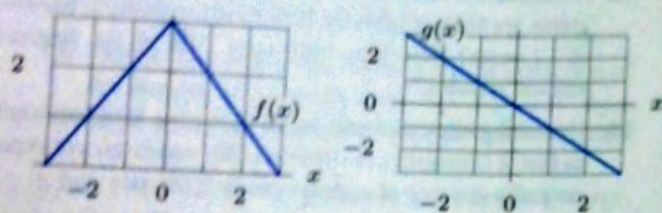

Figure 1.112

71. The Heaviside step function, H, is graphed in Figure 1.113. Graph the following functions.

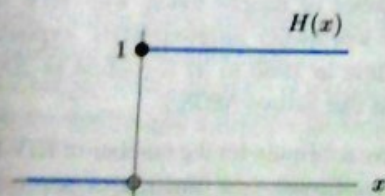(a) $2H(x)$ (b) $H(x) + 1$ (c) $H(x + 1)$

(d) $-H(x)$ (e) $H(-x)$

Figure 1.113

72. A population of animals varies periodically between a low of 700 on January 1 and a high of 900 on July 1. Graph the population against time.

73. When a car's engine makes less than about 200 revolutions per minute, it stalls. What is the period of the rotation of the engine when it is about to stall?

For Problems 74–75, find a possible formula for each graph.

74.

75.
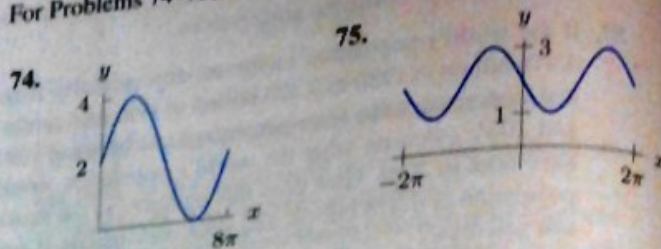
76. Figure 1.114 shows quarterly beer production during the period 1997 to 1999. Quarter 1 reflects production during the first three months of the year, etc. [66]

(a) Explain why a periodic function should be used to model these data.

(b) Approximately when does the maximum occur? The minimum? Why does this make sense?

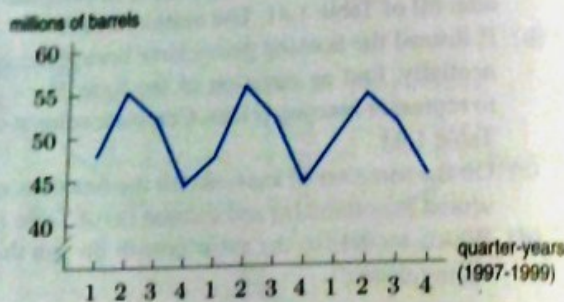(c) What are the period and amplitude for these data?

Figure 1.114

77. In a US household, the voltage in volts in an electric outlet is given by

$$V = 156 \sin(120\pi t),$$

where t is in seconds. However, in a European house, the voltage is given (in the same units) by

$$V = 339 \sin(100\pi t).$$

Compare the voltages in the two regions, considering the maximum voltage and number of cycles (oscillations) per second.

[66] www.beerinstitute.org/pdfs/PRODUCTION_AND_WITHDRAWALS_OF_MALT_BEVERAGES_1997_1999.pdf, accessed May 7, 2005.