Modular Arithmetic
Math 44 and 46

We say that $16 \equiv 28 \pmod{3}$, which is read "*16 is congruent to 28, mod 3*," because 16 and 28 both leave remainder 1 when divided by the *modulus* 3:

$$16 = 3 \bullet 5 + 1$$
$$28 = 3 \bullet 9 + 1$$

In general, $a \equiv b \pmod{m}$ means that when you divide a by m you get the same remainder that you get when you divide b by m.

Note that, for example, $0 \equiv 8 \pmod{4}$, because both 0 and 8 leave a remainder of 0 when divided by 4.

We saw patterns using remainders in the Take Away Game, and in the pattern game during the first few classes of the quarter.

Thus, for example, 15 is a winning position for the first player in the "1,2,3 Take Away Game," who wins by first taking 3, leaving 12, which is a multiple of 4, and thereafter "completing" groups of 4 no matter what the second player does.

(1) Who has a winning strategy in the "1,2,3,4 Take Away Game," in which each player takes 1,2,3,or 4 buttons, and the player taking the last button wins, if the game starts with 20 buttons? _____ Describe the strategy:

(2) Who wins if the "1,2,3,4 Take Away Game" starts with 100 buttons, and what is the strategy?

(3) Fill in the blank with one of the numbers 0,1,2,3,4, or 5 to make this statement correct: $20 \equiv$ ____ (mod 6)

$100 \equiv$ ____ (mod 6)

(4) Find three solutions to _________ $\equiv 2 \pmod{12}$

(5) Find three solutions to $17 \equiv 5$ (mod _______). (Hint: try numbers starting with 2 and see which works!)