Modular or Clock Arithmetic

Using multiples to understand patterns

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What day of the week will it be 37 days from today?

37 = 7•5 + 2,
2 days past "today."

37 is 2 more than an exact multiple of 7.

What day of the week will it be 76 days from today?

76 = 7•10 + 6,
6 days past "today."

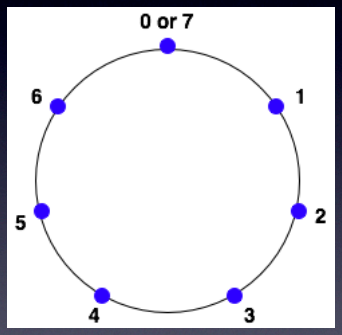
76 = 7•11 – 1,
1 day before "today."

We divide 37 by 7 and use remainder.

$$37 \equiv 2 \pmod{7}$$

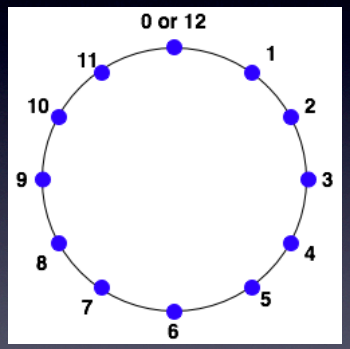
$$76 \equiv 6 \pmod{7}$$

Seven day week



Egypt, Sumeria, 5500 years ago.

12 hour clock



Egypt, Babylonia, and probably India 4,000 years ago.

Day and night each divided into 12 hour periods.

Chinese Remainder Theorem

Sun Zi suanjing (孫子算經 The Mathematical Classic by Sun Zi), 3rd center AD, by Sun Tzu

Later republished in a 1247 book by Qin Jiushao, the *Shushu Jiuzhang* (數書九章 Mathematical Treatise in Nine Sections)

What time will it be 50 hours from now?

$50 = 24 \bullet 2 + 2$,
2 hours past "now."

50 is 2 more than an exact multiple of 24.

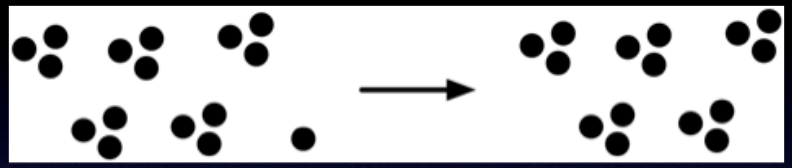
What time will it be 37 hours from now?

$37 = 12 \bullet 3 + 1$,
1 days past "now," except at night instead of in day!

We divide 50 by 24 and use remainder.
We divide 37 by 12 and use remainder, keeping track of the fact that since 36 is an odd multiple of 12, the time will be in the "opposite" part of the day (e.g. night versus day.)

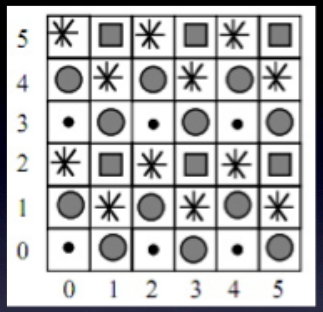
$$50 \equiv 2 \pmod{24}$$

$$37 \equiv 1 \pmod{12}$$



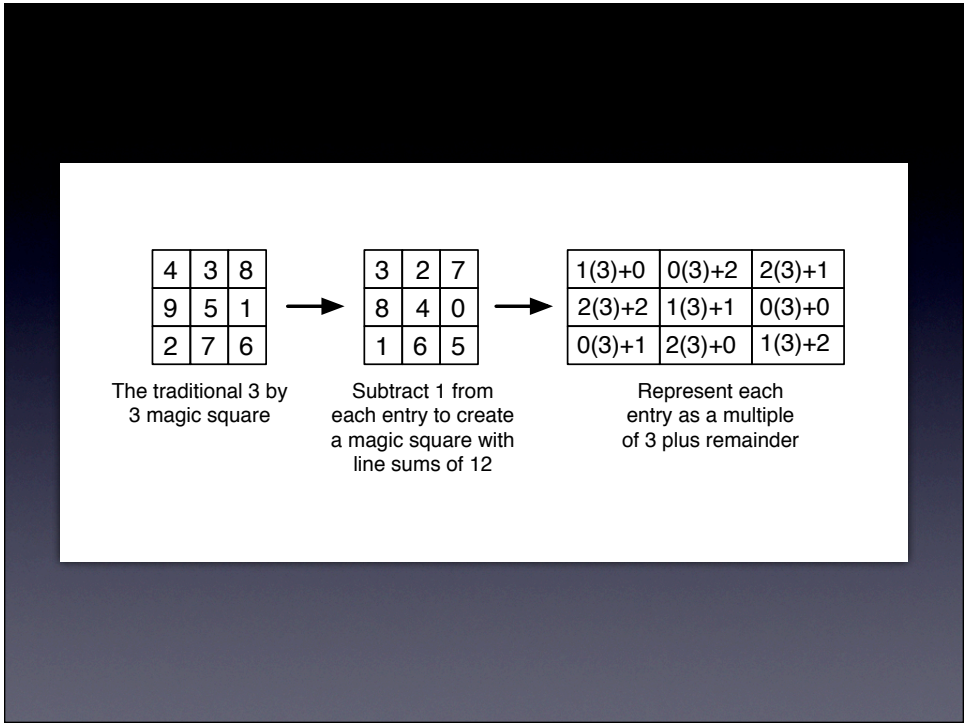
1,2-take-away game
Leave opponent with multiple of 3

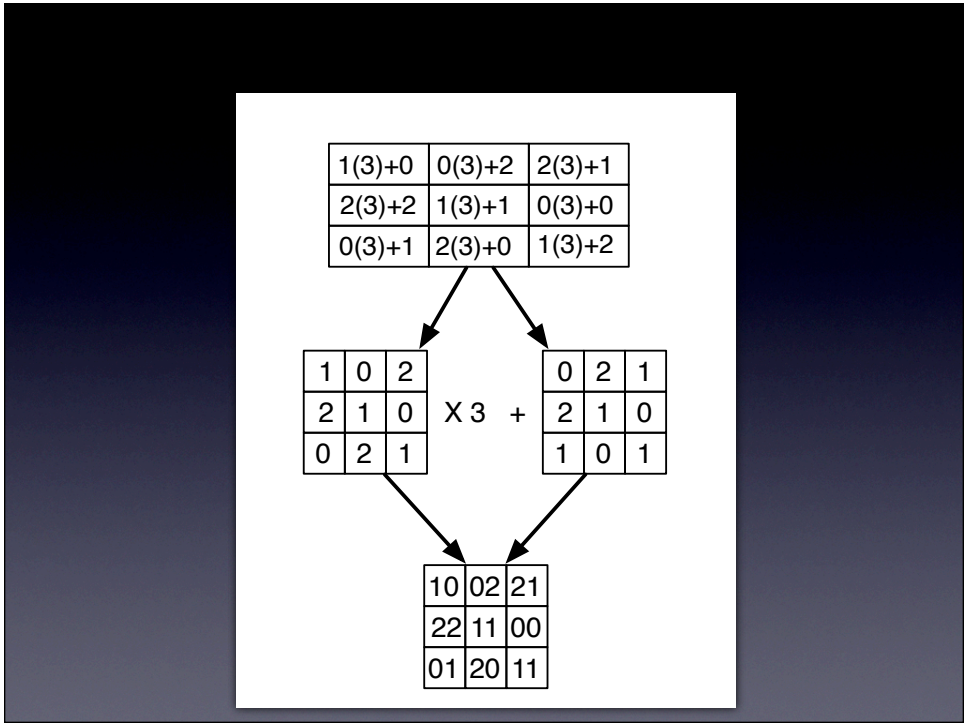
$$16 \equiv 1 \pmod{3}$$



Patterns repeat vertically every 3 rows

Row 16 is like which row: 0, 1, or 2?


4 3 8
9 5 1
2 7 6
The traditional 3 by 3 magic square
3 2 7
8 4 0
1 6 5
Subtract 1 from each entry to create a magic square with line sums of 12
1(3)+0 0(3)+2 2(3)+1
2(3)+2 1(3)+1 0(3)+0
0(3)+1 2(3)+0 1(3)+2
Represent each entry as a multiple of 3 plus remainder


1(3)+0 0(3)+2 2(3)+1
2(3)+2 1(3)+1 0(3)+0
0(3)+1 2(3)+0 1(3)+2
1 0 2
2 1 0
0 2 1
X 3 +
0 2 1
2 1 0
1 0 1
10 02 21
22 11 00
01 20 11

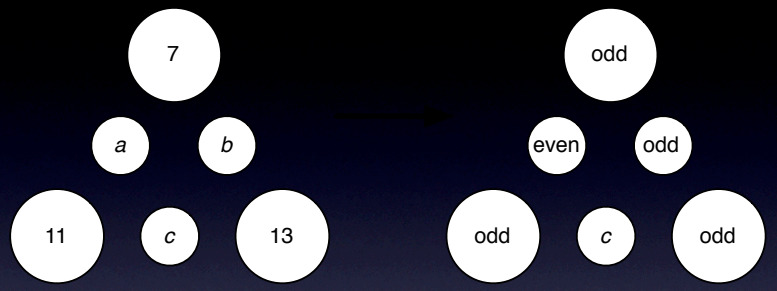
Experimental Design

10	02	21
22	11	00
01	20	11

	Low water	Medium water	High water
Low light	B0	A2	C1
Medium light	C2	B1	A0
High light	A1	C0	B1

Seed type: 0, 1, 2

Fertilizer: A, B, C



Chapter one number puzzles

One of *a* and *b* must be odd, the other even, since their sum is the odd number 7. But then is *c* even or odd?