

Modular or Clock Arithmetic

**Using multiples to understand
patterns**

What day of the week will it be 37 days from today?

$$37 = 7 \cdot 5 + 2,$$

2 days past “today.”

37 is 2 more than an exact multiple of 7.

What day of the week will it be 76 days from today?

$$76 = 7 \cdot 10 + 6,$$

6 days past “today.”

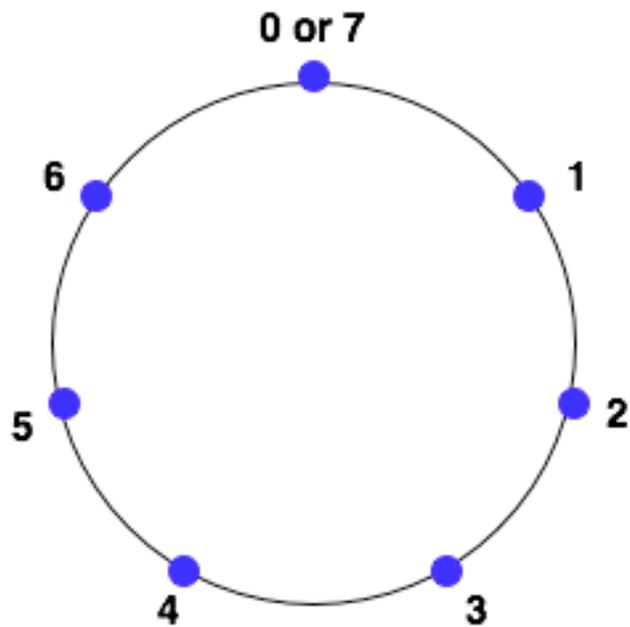
$$76 = 7 \cdot 11 - 1,$$

1 day before “today.”

We divide 37 by 7 and use remainder.

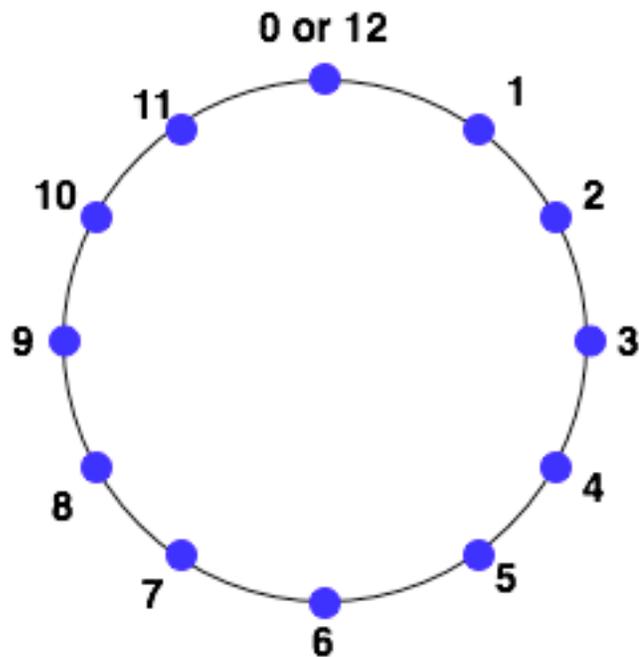
We divide 76 by 7 and use remainder.

Seven day week



Egypt, Sumeria, 5500 years ago.

12 hour clock



Egypt, Babylonia, and probably India 4,000 years ago.

Day and night each divided into 12 hour periods.

What time will it be 50 hours from now?

**$50 = 24 \cdot 2 + 2,$
2 hours past “now.”**

50 is 2 more than an exact multiple of 24.

What time will it be 37 hours from now?

**$37 = 12 \cdot 3 + 1,$
1 days past “now,” except at night instead of in day!**

We divide 50 by 24 and use remainder.

We divide 37 by 12 and use remainder, keeping track of the fact that since 36 is an odd multiple of 12, the time will be in the “opposite” part of the day (e.g. night versus day.)