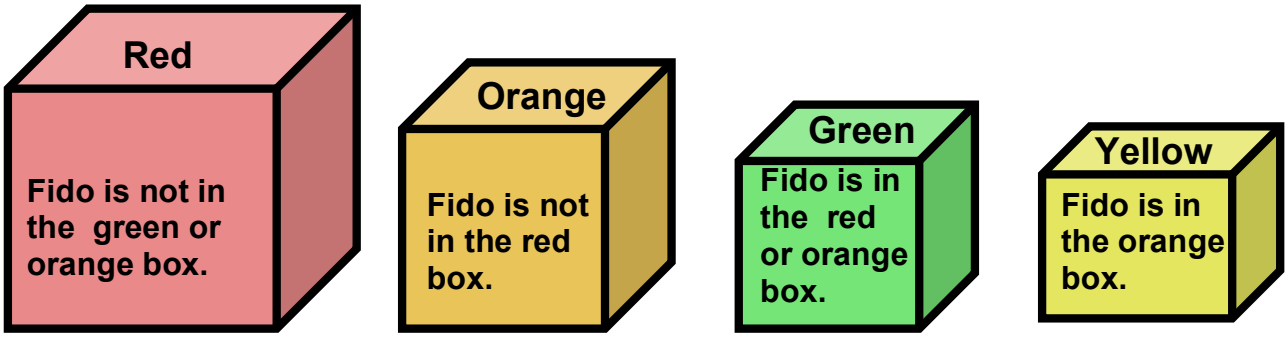
Math 46

Where's Fido?

Groups: Entire class
Materials: Set of four closed boxes or containers, each of which has different markings or is a different color, and signs written on index cards attached to each container. One of the boxes will contain a small dog doll!
Time: 20-40 minutes.



Write these statements on the board:
Fido is hidden in one of these boxes, and at most one of the statements on the boxes is true. Where is Fido?

This problem is inspired by, and based on, the logic problems of Raymond Smullyan. It is an enjoyable half hour activity with which to begin the section on logic and set theory. Students usually attempt to use a variety of techniques to solve this problem, and invariably there is considerable disagreement within the class as to where Fido actually is. I find it very useful actually to bring four colored boxes of different sizes to class, and to have hidden a small dog doll (Fido) in one of them. The actual boxes seem to focus the attention of the students and provide them a more concrete experience of the problem, since they know that Fido will actually be revealed at some point.. Typically I demand that students discuss the problem with the goal of convincing others in the class of their point of view. I announce that the discussion will continue until there is unanimous agreement. Unfortunately, it is usually not possible to reach consensus, but when a large majority are in agreement, I open the boxes and reveal Fido.

One of the techniques that is often successful in problems of this type, and reveals a method for easily constructing variations on this problem, is to create a chart showing where Fido might be if each particular statement were true.

	Red box	**Orange box**	**Green box**	**Yellow box**
Red statement	Yes			Yes
Orange statement		Yes	Yes	Yes
Green statement	Yes	Yes		
Yellow statement		Yes		
Total #:	**2**	**3**	**1**	**2**

Thus for example, if the red statement were true, Fido might be in the Red or the Yellow box. Conversely, if Fido were in the Yellow box, then the Red statement would be true.

Below we see that only in the case of the green box would 1 or fewer statements be true, and so Fido must be in the green box.

Often students assume that the statements are somehow tied to the boxes on which they are written, but this is a slight subterfuge, as there is no such logical connection. Many students try to perform a type of reasoning in which they proceed from one statement to the other, looking for something that forces Fido's presence in a particular box.; this is usually not successful, as they need to view the whole problem at once - and the chart method encourages that. Students also sometimes need to discuss the meaning of the word "or," and think about how that word is interpreted given the assumption of a two-valued logical system in this problem (in which a statement must either be true or false.) The use of the words "at most" adds difficulty, since it forces students to consider the two possibilities inherent in these words – either zero or one statement must be true.

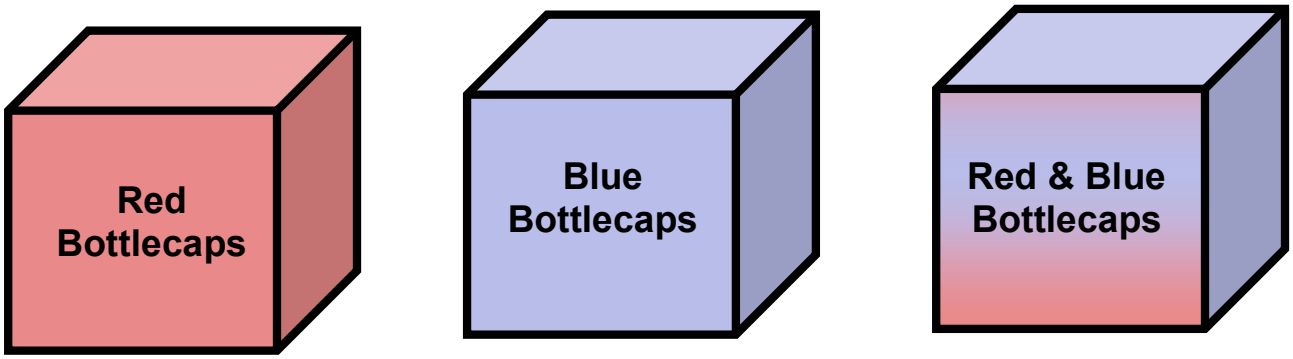
Extensions and variations are easily constructed using the chart. For example, using the same statements, we might hide Fido in the Orange box and announce that at least three statements are true. Or in order to construct an alternative problem, we might first place "Yes" in a number of the boxes at random, then count column totals to insure that one column can be distinguished from the other; finally, we use the pattern in each row to construct a corresponding statement. For example, the green statement might just as easily be "Fido is not in the green or yellow box," using the chart above.

Other extensions might involve using more or fewer boxes, depending on the age of the students or varying the number of statements or the number of objects. Actual objects – or prizes of some sort – might be hidden in the boxes as well. Have your students make up their own puzzles. Student teachers might be asked to construct modifications or extensions that are either more or less demanding, for different age students.

(2) Boxes of Bottlecaps

Three containers are shown, with one labled Red Bottlecaps, one labeled Blue Bottlecaps, and the third labeled Red and Blue Bottlecaps. One of the boxes does cntain red bottlecaps, and another blue, and the third a mixture; however, the signs have been rearranged so that none is true. Write the following on the board:

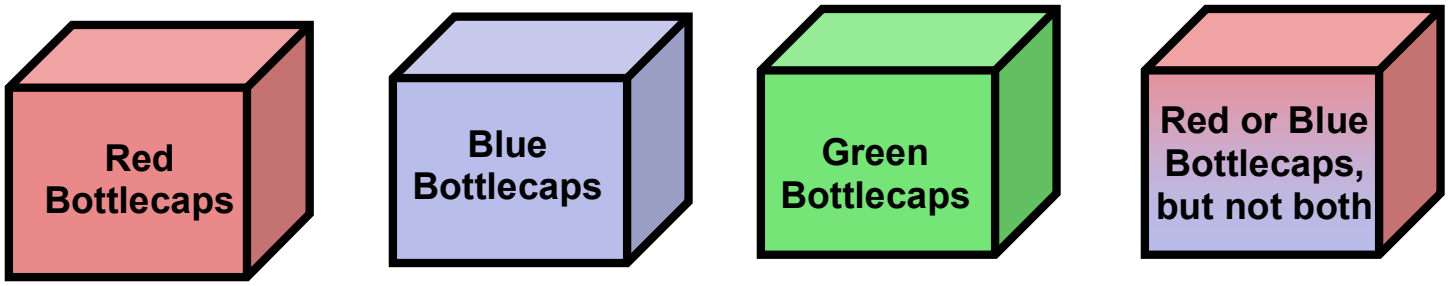
Each of these three boxes is incorrectly labeled.



Instructions: Draw one bottlecap out of one box, without looking in the box and then deduce wich box contains which color or colors. Which box should you check? (Source: Marilyn Burns)

(3) More Bottlecaps

Each of these four boxes is incorrectly labeled.
Draw one bottlecap out of one box, without looking in the box and then deduce which box contains which color or colors. Which box should you check?



This problem is a bit more difficult than the last.

Bibliography:

About Teaching Mathematics, by Marilyn Burns (especially the chapter on logic).
Any of the puzzle books by Raymond Smullyan, for example:
The Riddle of Scheherezade: and Other Amazing Puzzles, New York: Knopf, 1997.
The Lady or the Tiger? : and other logic puzzles, including a mathematical novel that features Gödel's great discovery. New York : Knopf, 1982.
To Mock a Mocking Bird and other logic puzzles : including an amazing adventure in combinatory logic. New York : Knopf, 1985.
What is the name of this book? : The riddle of Dracula and other logical puzzles. Englewood Cliffs, N.J. : Prentice-Hall, c1978.

- originally written for Raft workshop, Oct. 28, 2000, by Karl Schaffer