Homework problem 4.3 #2:
$$\begin{aligned} x_1 \qquad\qquad\quad + x_4 &= 0 \\ x_1 + x_2 + x_3 + x_4 &= 0 \end{aligned}$$

Subtract equation 1 from equation 2:
$$\begin{aligned} x_1 \qquad\qquad\quad + x_4 &= 0 \\ x_2 + x_3 \qquad\quad &= 0 \end{aligned}$$

We can represent this using a matrix equation:
$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The leading 1s in this matrix are associated with variables x_1 and x_2. Let the other non-leading variables be represented by parameters r and s:

x_3 = r and x_4 = s.

The original equations can now be written:
$$\begin{aligned} x_1 \qquad\qquad &= -x_4 = -s \\ x_2 \qquad &= -x_3 = -r \end{aligned}$$

Then all four variables can be represented using these two parameters:
$$\begin{aligned} x_1 &= -s \\ x_2 &= -r \\ x_3 &= r \\ x_4 &= s \end{aligned} \quad \text{so} \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -s \\ -r \\ r \\ s \end{pmatrix} = r\begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Therefore the solution set of vectors has a basis with the two vectors shown, and the set of solutions to this homogeneous pair of equations is the null space of dimension two, spanned by the two vectors shown.

Note that these equations represent the homogeneous case for a linear transformation from R^4 to R^2, and the rank of the matrix can be no larger than 2, since that is the number of rows in the matrix. Therefore the nullity = dimension of the null space = number of parameters = 4 – rank = at least 2.