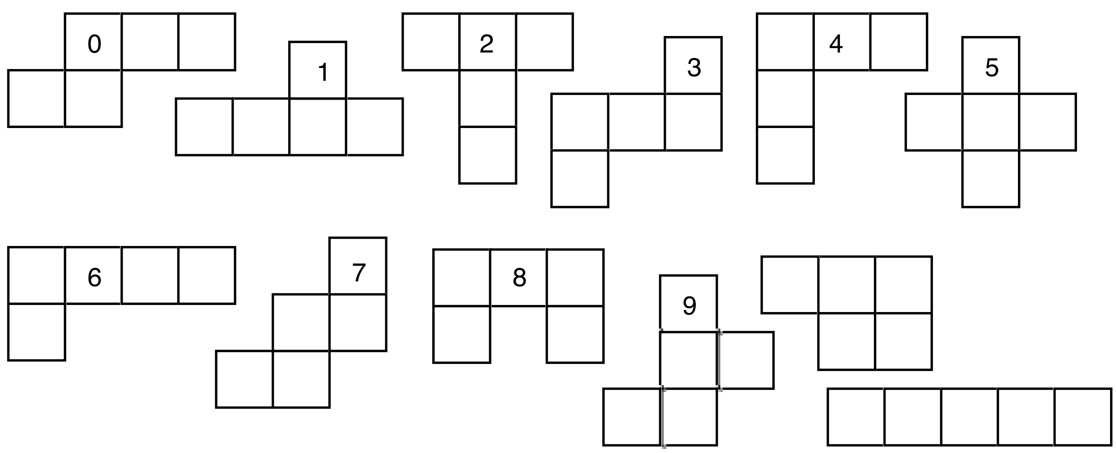
Math 44, Take Home problems, Spring 2009

(a) Due at the start of class, 1:30 PM, on Thu., June 4, 2009
(b) You are encouraged to work together; however if your work is identical to that of someone else in the class, neither will be counted.
(c) Only work appearing on these official pages will receive credit (except for problem 1 on tilings) - do not use extra sheets. Make your answers fit in the space provided, one problem per page (you may use the back if absolutely necessary!)
(d) If asked to explain why your result is correct, do not merely state an answer, give a full explanation in complete sentences.
(e) **For each problem**, list who you worked with and give details as to who did what. For example: "Jane found many examples and spotted the pattern, I solved the problem in general, then we both figured out how to write down an accurate explanation." No credit without this information.
(f) You may discuss these problems with others in this class, but not those not in the class. That means no tutors, other faculty, friends or family!! But please do discuss with your classmates.
(g) DO NOT hand in this cover sheet!

(1) Explain (in complete sentences) how and with whom you worked
on this problem, giving credit to anyone who contributed to your solution:

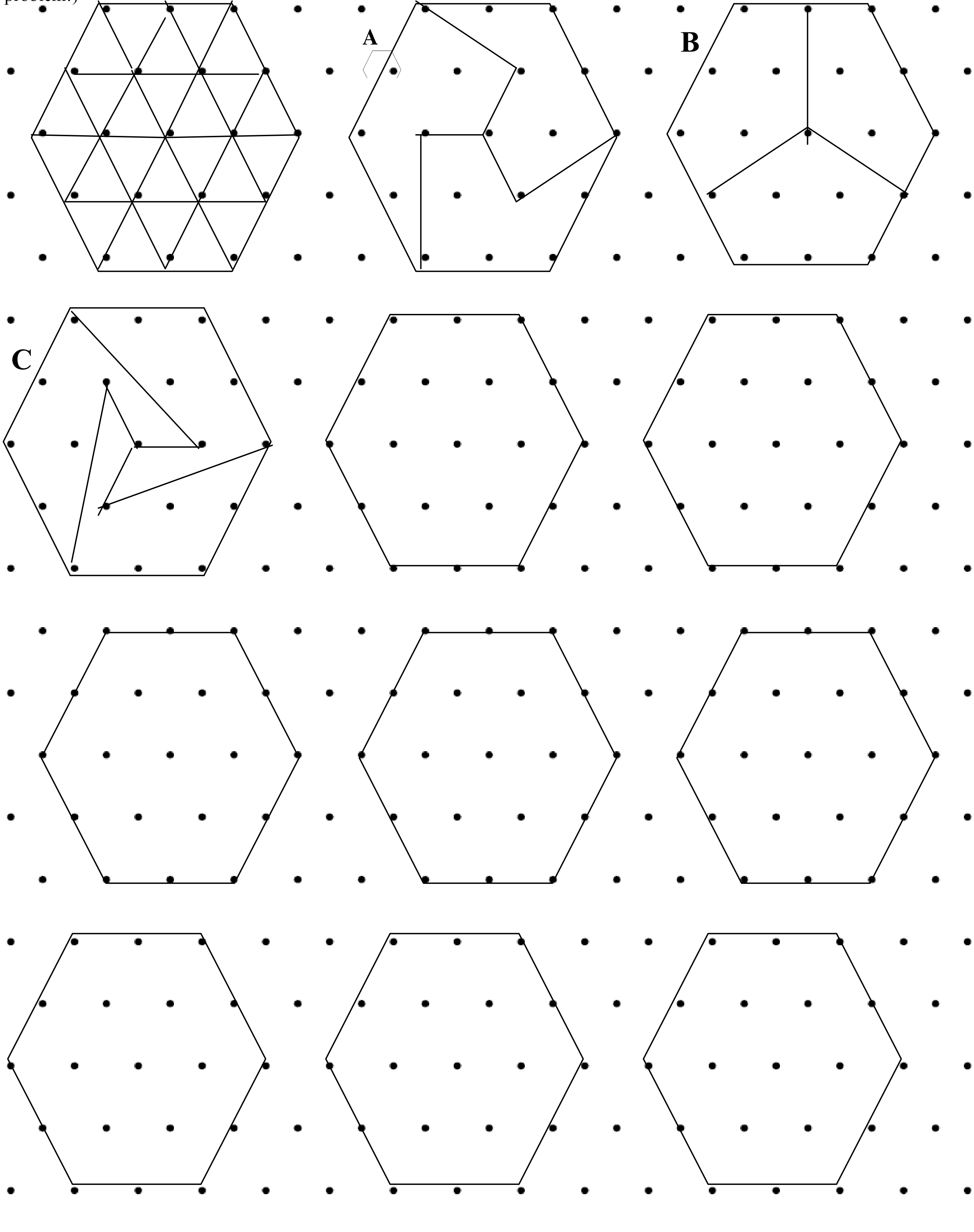
Let X,Y, and Z be the last three distinct (different) digits in your student ID number.
List those digits here: ______
(a) Use any two of those pentominoes labeled by X,Y, and Z to find a tiling of the plane. You must use BOTH of the pentominoes in your tiling. For this problem you may insert a sheet of graph paper or other paper that makes your tiling visible.

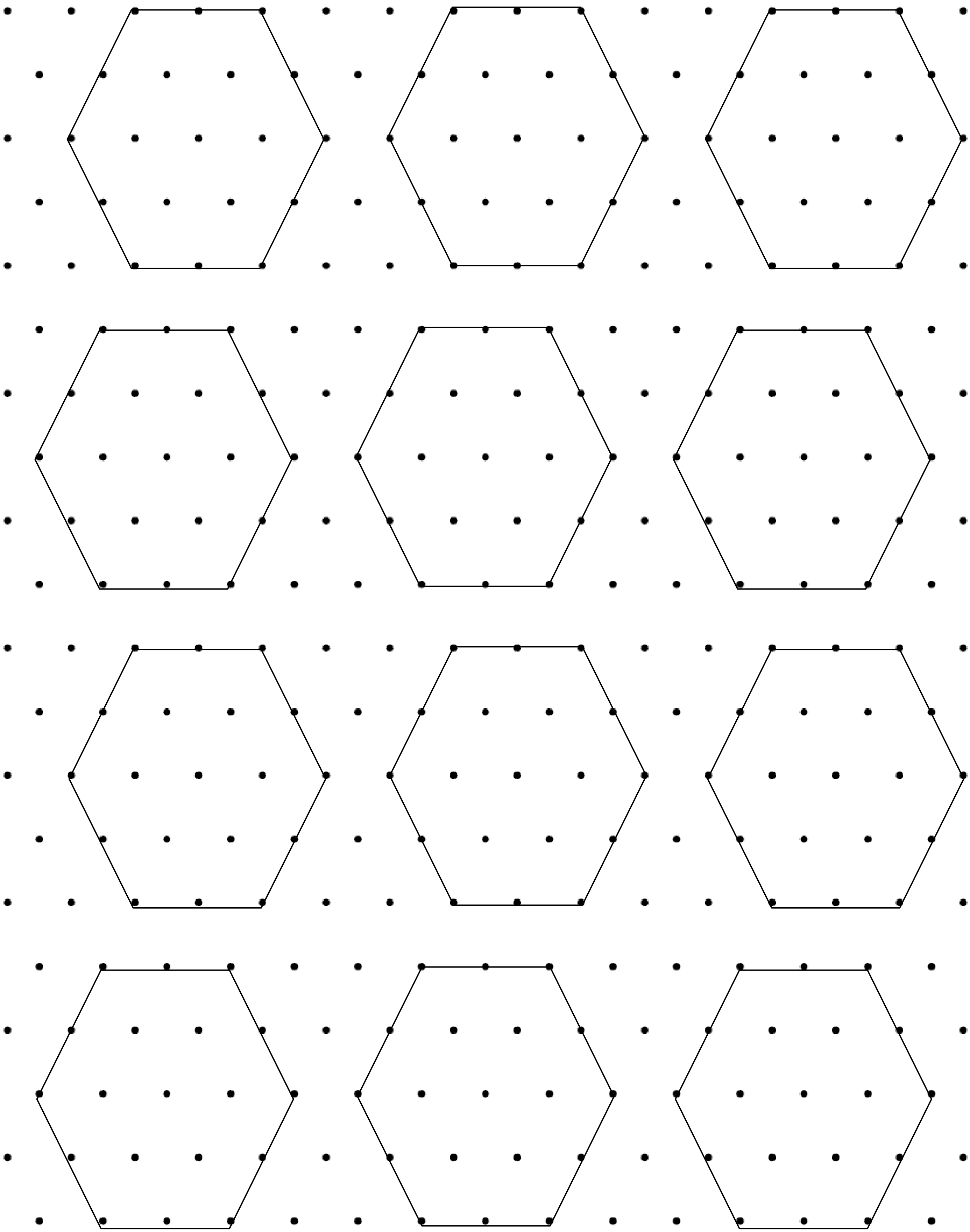


(b) Find three different symmetries in your tiling. If your tiling does not seem to have three, create another one that does! Show **exactly** where they are (as we demonstrated in class).

(2) The allowable lines are shown in the first hexagon. ..(Note: A, B, and C use lines that are not allowed in your problem!)

Count reflections and rotations as different. Show all patterns with 3-fold rotational symmetry that start at the center and go to the hexagon border, then stop, without lines crossing.





Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution: (**DO NOT consult those not in the class**!)

(3) Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution:

In this problem you will each be given a set of numbers, A, at the class site, with which to play a game like the take-away game we played in class. In the game, two players start with a pile of counters, and take turns choosing a number from set A (repeats are allowed) to remove from the pile. The first person to leave 0 counters wins. For example, if A =- {1,2,3,4}, we saw that when starting with a number that is not divisible by 5, assuming best play by both players, the second player can force a win by always bringing the running total to a multiple of 5.

Hint: suppose the numbers in your set were A={1,3}. Then work "backwards" from the lowest numbers to see what are the winning and losing positions. For example, in this case, 1 would be a winning number, since that player could remove 1 counter. 2 is a losing number, since the player with that number can only remove 1, leaving a winning number for her opponent. 3 is a winning number, since that player can remove 3. And 4 is a losing number, since that player can remove 1 or 3, leaving a winning number for her opponent. If you continue with this example, you will begin to see that odd numbers are losing numbers, and even numbers are winning numbers. Your problem will most likely have a more complicated analysis than this though!

List the four numbers in your set A:______________

(a) Find who has the winning strategy for each number 1 through 35. If a player has a winning strategy when left with a given number of counters, place W by that number; if the opposing player has the winning strategy, place L by the number. For example, the first two have been done for you:

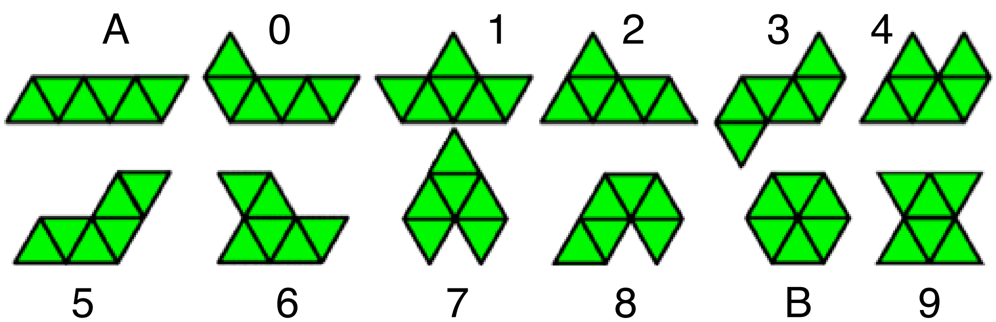
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
L	W																		

20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

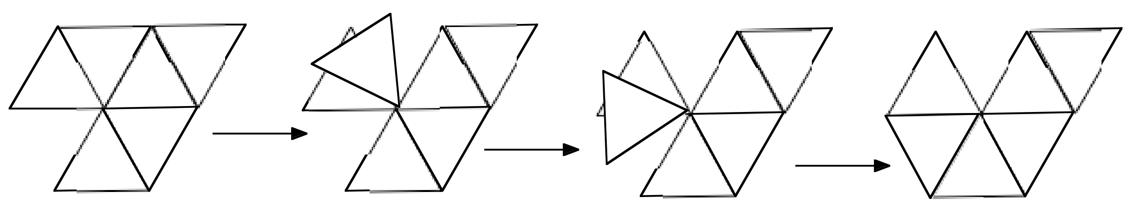
(b) Find and explain the general winning strategy, assuming best play by both opponents:

(4) Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution:

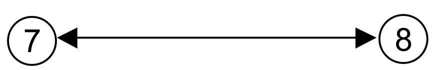
These are the twelve "hexiamonds" (made from six equilateral triangles).



Suppose you can transform one of the twelve hexaimonds to another by rotating one triangle around one of its vertices until that triangle "snaps" into place and forms another one of the hexiamonds. These hexiamonds are called "cousins." For example, the diagram below shows hexiamond 7 transformed into 8. Find the longest sequence of hexiamond cousins that you can, without repeats. You may abbreviate them using the numbers/letters introduced in problem 1.

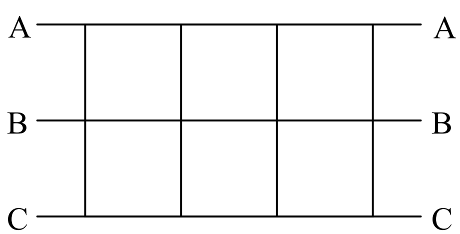


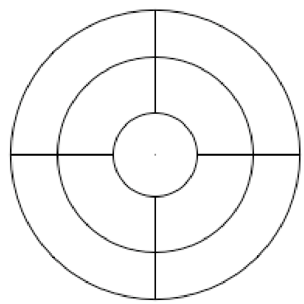
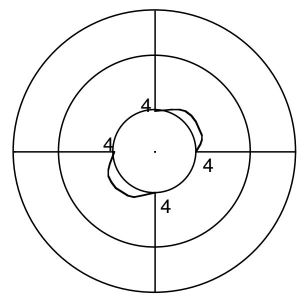
Hint: This problem will be much easier if you create a graph showing which hexaimonds are cousins of which others. For example, the diagram above shows that 7 is connected to 8. For full credit, create this graph showing which hexaimonds are cousins.



(5) Street Grids. Explain (in complete sentences) how and with whom you worked on this problem, giving credit to anyone who contributed to your solution: (**DO NOT consult those not in the class**!)

This is a 3 by 4 street "cylindrical" street grid graph, with 3 horizontal rows and 4 vertical columns. We imagine that the edges (streets) on the right connect around to the left, so that, for example, A is connected to A, B to B, and C to C. The result is a circular grid like the one in the center! The vertices of the graph are intersections of lines, and the edges are the short segments that join vertices. A Hamiltonian cycle is a path that starts and ends at the same vertex and goes to each vertex exactly once. An Euler cycle is a path that starts and ends at the same vertex and travels over each edge exactly once. If a graph does not have an Euler circuit, then by "doubling" edges we can "Eulerize" it so that all vertex degrees are even, and it will have an Euler circuit. For example, on the right, four degree 3 vertices have been changed to degree 4 by "doubling" two of the edges.



(a) For which values of m and n does an m by n street grid graph have a Hamiltonian circuit? (Hint: checkerboard color the vertices and use a "parity" argument.) Explain your answer.

(b) How do you use the values of m and n to calculate the minimum number of streets that have to be "double-backed" (covered twice) in order that an m by n grid have an Euler circuit. Explain why your formula or algorithm works.

It will be easier to do both of these problems if you work them for a series of grids first, and look for patterns. Investigate and fill in this table, placing an H in those columns for which the graph has a Hamiltonian cycle, and writing in the number of streets that have to be doubled. The first two cases have been done for you.

	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10
m=2	H,2								
m=3	H,2								
m=4									
m=5									
m=6									
m=7									
m=8									
m=9									
m=10									