John Horton Conway —Talking a Good Game

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Quick now—what day of the week did December 4, 1602 fall on? Sorry, time's up. You have to give the answer (Saturday) in less than two seconds to compete with Professor John Conway of Princeton University. Conway enjoys mentally calculating days of the week so much that he has programmed his computer so he cannot log on until he does ten randomly-selected dates in a row. He usually does ten dates in about 20 seconds. His best time is 15.92 seconds. Conway says "the ability to do these lightning mental calculations is very important to me. You've no idea how fast you have to think to do them. The reason I do it is because it gives me a kick. The adrenaline spills all over you, and when you're thinking that quickly, it's really nice."

Conway, at age 56, is one of the world's most original mathematicians and is a member of the prestigious Royal Society of London. He is in the middle of a second career as professor of mathematics at Princeton University with his second family. It was a great coup for Princeton mathematicians when they lured Conway away from Cambridge University in 1986. We are visiting with him today (November 29, 1993) to gain a few insights into his work and what makes him tick.

Conway has made substantial contributions to several branches of mathematics: set theory, number theory, finite groups, quadratic forms, game theory, and combinatorics. He is best known, in a popular sense, for his work on the theory of games, especially the Game of "Life"

and his invention of a theory of numbers that has its origins in games. Conway's enchantment with games is reflected in the title of one of his papers, "All Games Bright and Beautiful." In Conway's theory of numbers, every two-person game is a number! Don Knuth, the noted computer scientist, was so taken with Conway's new theory of numbers that he wrote *Surreal Numbers*, a novel that explains the theory for students.

"Life"

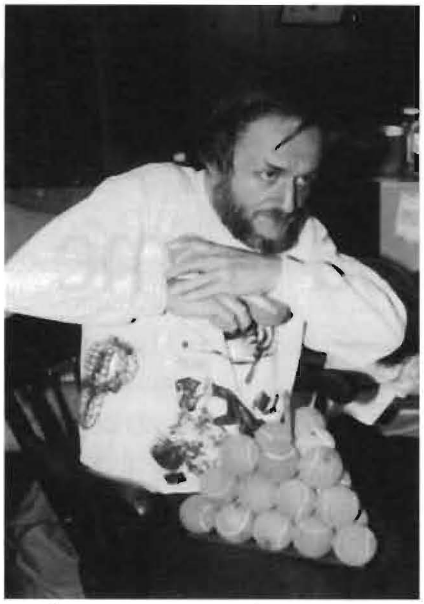
"Life," Conway's most famous game creation to date, burst on the scene in 1970 when Martin Gardner brought it to the attention of hundreds of thousands of readers of his "Mathematical Games" column of *Scientific American* magazine. "Life's" popularity was quick and far-reaching. Its great popularity spawned "Lifeline," a newsletter for "Life" enthusiasts, which was published for many years.

Gardner later wrote that his "column on Conway's 'Life' forms was estimated to have cost the nation millions of dollars in illicit computer time. One computer expert, whom I shall leave nameless, installed a secret switch under his desk. If one of his bosses entered the room he would press the button and switch his computer screen from its 'Life' program to one of the company's projects."

Conway says that "Life" arose out of "the aim to find a system in which you can see what happens in the future.... I always thought you ought to be able to design a system that was deterministic, but unpredictable."

Although he has co-authored with Berlekamp and Guy *Winning Ways for Your Mathematical Plays*, the two-volume classic on games, he asserts that he is not very interested in playing actual games. He claims that "I can't play chess, I know the rules, but you would be amazed at how badly I play. That's not the thing that turns me on. I'm interested in the theory of it, especially if it's simple and elegant. I really like to consider the simpler games, like checkers. I used to play checkers with my first wife, and she always used to beat me. Perhaps I would win one game in ten or twenty, and I was trying very hard.... I had a similar experience with my daughter, playing the game called Reversi."

Conway's mathematical abilities, especially his rapid calculating skills, were evident as a little boy. He says that "my mother found me reciting 2, 4, 8, 16, 32, 64,... —the powers of 2 when I was four." When he was eleven, he told the headmaster of his grammar school in Liverpool that "I want to go to Cambridge and study mathematics."



Conway illustrates sphere packing.

What was it about mathematics that attracted Conway so strongly? "I can't recall what started it," he says. "It's probably just the fact that I was good at it, and that was that. If you regard it as a competitive subject, then to stand out and beat the other kids was fun.

"When I was a teenager, I thought a lot about the different departments of knowledge, in some sense, and I know what turned me on to math was this feeling of objectivity. Consider other things you might do, like law. Then you're basing your life on essentially arbitrary decisions that have been taken by individuals, or by the way society has developed as a whole. I can't develop much interest in that.... I like the idea that with philosophical, mathematical, and scientific questions, there's a chance of communicating with beings on other planets, so to speak. There's a certain universality that definitely is central to mathematics.

"When I was young, things were quite difficult. It was quite a rough district we lived in, and some terrible things happened." Conway remembers being beaten up by older boys "because I hadn't chosen the right professional soccer team. Sometimes it didn't matter whether I chose the right one or not, they would beat me up anyway." He also remembers being taken into an ancient air raid shelter and having lighted cigarettes applied to his skin.—Ouch! He eventually got to Cambridge, but Conway says that "from ages 11–13, with the onset of adolescence, puberty, and all of that, I didn't do terribly well. I started to hang around with a bunch of lay-abouts. I had a hell of a time when I was a high school student.

"Teachers and my parents were getting concerned about me," he remembers. "I was given several good talkings to by various people, and by age 16, I started going to classes again and started being on top again."

The Real Me

Conway indeed got on top again to the point that he won a scholarship to Cambridge. He clearly remembers his train trip to Cambridge, and being rather introverted, quiet, and shy at the time. "I was on the train when I said to myself, 'You don't have to be like this anymore. Nobody at Cambridge knows you.' I had stepped out of the world I was previously in. So I decided to turn myself into an extrovert, and I did. I decided I was going to laugh with people, and make fun of myself. I got there, and that's what happened. For quite a long time, I felt like a fraud. I said to myself, 'This isn't the real me.' And then it ceased to be acting. Every now and then, I still feel shy on occasions, but not very often."

Anyone who watches Conway bounce around a classroom or organize a knot theory square dance would agree that the introvert is long gone.

Conway's Princeton office is an environment that clearly would appeal to children of all ages from two to one hundred. Pleasantly cluttered with books, bric-a-brac, and mathematical models hanging from the ceiling and walls, it bears a striking resemblance to a classroom in a progressive elementary school. He even has a home-built "quaternion machine" hanging on one wall, which he gleefully operates for visitors.

During our visit, Conway tells us about his new system for clarifying the mysteries of knot theory. He brings in two undergraduate students to join him and the interviewer in a special "square dance" using two colored ropes that do indeed serve to explicate what he calls the "theory of tangles." He recalls working out a good part of his theory of tangles while still a high school student.

At one point in our discussion, he brings out a few dozen tennis balls to illustrate a problem in sphere packing. Packing the maximum number of spheres in a given space, especially higher dimensional spaces, has been one of Conway's passions for several years. The sphere packing problem in eight-dimensional space is very important to transmitting data over telephone lines. He tells us that twenty-four dimensional space is wonderful for "there is really a lot of room up there among those packed spheres." His interest in sphere packing led to his writing, with Neil Sloane, the book entitled *Sphere Packings, Lattices, and Groups*. He is quite proud of a recent review of the book which describes it as "the best survey of the best work in the best fields of combinatorics written by the best people. It will make the best reading by the best students interested in the best mathematics that is now going on." He is so proud of the review that he has displayed the "best" parts of it in large letters on one of his walls. Of the many nice reviews he has had of his work, he says this one is the "best."

Tennis balls (sphere-packing), colored ropes (knot theory), and counters on a checker board (the game of "Life") all reflect Conway's intense need to make things simple. He claims that "lots of people are happy when they've understood something. And I'm usually not. I'm only happy when I've really made it simple. Moreover, I don't understand so many deep things as a lot of other people do. But I'm interested in getting a still deeper understanding of some simpler things."

The Splash

Conway became a mathematical star in 1969 when he discovered a new simple group, now called the Conway Group. At the time he was a junior faculty member at Cambridge University who was depressed because "I had been known as a quite-bright student at Cambridge, but I had never produced anything of any significance. I was feeling guilty and was almost suicidal. In addition I was married with four little girls, and we had very little money.

"About then John Leech discovered a beautifully symmetric structure in 24-dimensional space. It was believed that a special group corresponded to Leech's structure. I decided to take a crack at finding it since I knew a bit about groups. So I set up a schedule with my wife. We agreed that I would work on the problem on Wednesday nights from six until midnight and on Saturdays from noon until midnight. I started work on a Saturday, and made progress right away. At a half hour past midnight on that first Saturday, I came out with the problem solved! I had found the group!

"That did it. I immediately got offers to lecture on my new group all over the world. I became something of a mathematical jet-setter. My discovery really was a big splash."

So Much for Guilt

Conway went on to explain another plus that followed his discovery. "I suddenly realized that it was a good idea not to feel guilty; feeling guilty didn't do any good. Guilt just made it impossible to work.... So I decided for myself that from then on I wasn't going to work on something just because I felt guilty. If I was interested in some childish game, I would think about that childish game, whereas previously I would have sort of looked around and wondered what my colleagues were thinking."

John Conway in his Princeton office.

Last June, Andrew Wiles, one of Conway's Princeton colleagues, made an even bigger splash with his announcement of a proof of Fermat's Last Theorem. (See the premiere issue of *Math Horizons* for the story on Wiles.) Conway worked in number theory for several years and has been fascinated with the work of Wiles and the ensuing excitement. He says "I have slightly different opinions about the Fermat problem from most people. What I think is that it's quite likely that Fermat proved it, not just that he believed that he'd proved it.... There was no reason for him to deceive anybody. There's not much point in writing something, writing a note to yourself, and telling a lie in it. It wouldn't work.

"And now you're faced with a real problem. Even if Fermat deceived himself, what was his proof? What was that proof? Until you can solve that problem by exhibiting something that was available to Fermat and would fool Fermat, you're not really entitled to make any judgment.... The argument that says we haven't found a proof for over 300 years is not so fascinating. We're not all that clever. Many of the other things he wrote down lasted for 200 years.... When I die, I might knock on Fermat's door and see what happened. He'd be an interesting guy to talk to. I've often toyed with the idea of talking to people from the past."

Archimedes and Euler are two other mathematicians with whom Conway would like to chat. He adds that "I wouldn't like to talk with either Newton or Gauss, because neither of them seems to be my kind of person."

Conway also has a secret to pass on about producing mathematics. He advises us to "keep several things on the board, or at least on the back burner, at all times.... One of them is something where you can probably make progress.... If you work only on the really deep interesting problems, then you're not likely to make much progress. So it's a good idea to have some less deep, less significant things, that nevertheless are not so shallow as to be insulting." ■

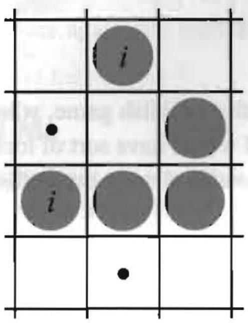
Photographs by Carol Baxter.

The Game of "Life"

The rules of Conway's Game of "Life" were chosen, after experimenting with many possibilities, to make the behavior of the population both interesting and unpredictable. The genetic laws are remarkably simple. The game is played on a chessboard, an infinite chessboard, where each cell has eight neighboring cells. The game begins with some arrangement of counters placed on the board (the live cells) as the initial generation. Each new generation is determined by the following rules:

1. Consider a live cell. If it has 0 or 1 live neighbors, then it dies from isolation. If it has 2 or 3 live neighbors, then it survives to the next generation. If it has 4 or more live neighbors, then it is crowded out and dies.
2. On the other hand, if a dead (unoccupied) cell has exactly 3 live neighbors, then it is a birth cell; a counter is placed on it in the next generation.

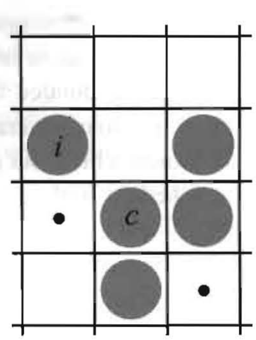
G_1

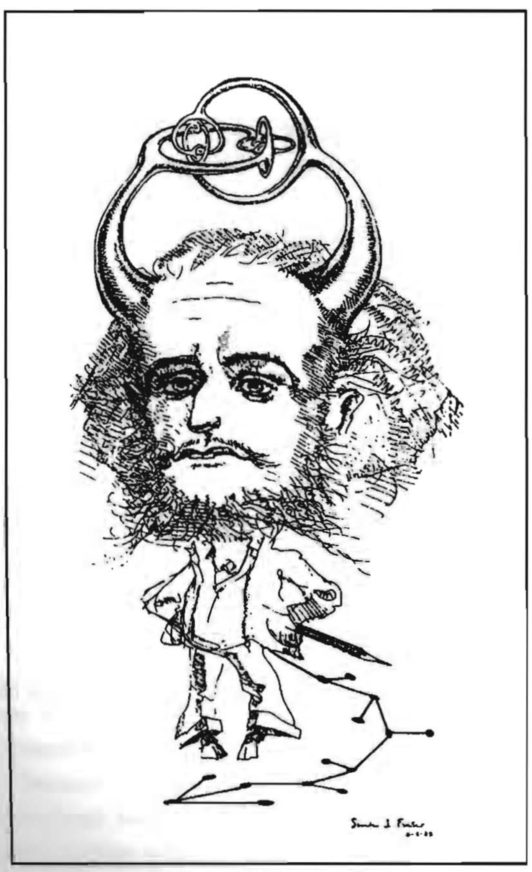


Here is a little example. The five circles in diagram G_1 indicate the live cells in the first generation. Those marked i will die in the next generation due to isolation (Rule 1). The cells with the black dots are empty cells that will become live in the next generation (Rule 2).

Diagram G_2 indicates what the second generation looks like. The cell marked c will die of crowding (Rule 1). To help you draw the pattern for the third generation we have used the marks i and • as before. You should continue to draw the pictures for several successive generations.

G_2





John 'Horned' (Horton) Conway

If you play the Game of "Life" with various initial generations you will find that the population undergoes unusual and unexpected changes. Patterns with no initial symmetry become symmetrical. Some initial configurations die out entirely (although this may take a long time), others become stable (still lifes), while others oscillate forever.

Conway originally conjectured that no finite initial pattern can grow without limit. But he was wrong. There is a "gun" that shoots out "gliders" and a "train" that moves along but leaves a trail of "smoke."

If you draw several more generations of the above example you will understand that it is called "glider" because it is a glide-reflection of an earlier generation (which one?).

To learn more about the Game of "Life," including the glider gun, see Martin Gardner's *Wheels, Life and Other Mathematical Amusements* (1983), which contains three chapters on "Life." You may want to write a program to play "Life" on your computer. ■