A Conversation with Leon Bankoff

G. L. Alexanderson

Leon Bankoff was born in New York City, December 13, 1908. Thus, in the spring of 1986, he became one of the few who were able to witness for the second time the once-in-a-lifetime appearance of Halley's Comet. Though he had an early interest in mathematics, he did not choose the subject for a career, so after college at CCNY and dental school at NYU and USC, he became a prominent Los Angeles dentist whose list of patients includes rock stars and other luminaries of the Beverly Hills-Hollywood scene. With a resurgence of his interest in mathematics in the late 1940s he went on to publish regularly in mathematics and to edit the Problems Section of the *Pi Mu Epsilon Journal*. He numbers among his friends illustrious mathematicians from around the world.

The initial conversation recorded here took place in Santa Clara, California, August 22, 1986; it was continued in San Francisco in January 1991 at the annual Joint Mathematics Meetings where he was speaking at a special session on geometry.

MP: Let's start at the beginning. Were you born interested in mathematics? Or did that interest develop later?

Bankoff: I must have been about five when I became interested in mathematics.

MP: Numbers? Or figures?

Bankoff: Numbers. My mother gave me a problem, the old two jug problem about measuring out four gallons of a liquid using containers that could hold only five and three. I recall that I solved the problem and I got a pat on the back for it.

MP: Did your parents have a background in mathematics that would have caused them to encourage you in the field?

Bankoff: My parents were not professional people and were not scholars in the sense that they possessed great erudition. Yet, I could say that they were scholarly, always eager to learn and to learn quickly. My mother, an immigrant from Riga, and my father, who came from Pinsk, came to Manhattan at the turn of the century, where they met and married in January 1908. I did not know my grandparents on either side since they all remained in Europe. By the time I arrived in December of 1908 my parents had already learned to speak, read and write English. My father plied his trade as a manufacturer of custom hand-made cigars and continued his craft throughout his life, extending his activities as a merchant of related tobacco products and accessories. My mother, a talented seamstress, quickly found herself swamped with more work than she could handle thus forcing herself to expand by hiring a dozen seamstresses. She became a well-known Fifth Avenue couturier.

Bankoff, dressed as a Russian diplomat, Le Mans, 1970

Music at an Early Age

Both of my parents were music buffs, mostly in opera, and my earliest recollections involve open-air concerts in Central Park and the Lewisohn Stadium, plus almost incessant uproars emanating from our wind-up Victrola. My mother fostered my early interest in music and kept me busy with all sorts of math problems and puzzles. My father escaped early evening boredom by teaching me chess and checkers, pushing me way ahead in arithmetic, entertaining me with magic tricks and calligraphic flourishes. They provided me with all the books I wanted, and I didn't have to be coaxed to read them.

MP: Later on was there anyone in particular who was influential in developing your interest in mathematics, any high school teacher, any friend or relative?

Bankoff: My father taught me subtraction, multiplication and division before I entered school. By the time I entered the first grade, I was a little overqualified for it so I skipped two grades. As a result I graduated from elementary school—which ordinarily would have taken eight years—after six years. I had some teachers I remember. There was a man named Mr. Lawyer, who presented the Pythagorean theorem to these young kids who were not really ripe for it. I was terribly impressed with the fact that three squared plus four squared equals five squared.

MP: It's a beautiful theorem!

Bankoff: It really is. That theorem brightened my young brain; the appeal was geometric and number theoretic. I used to amaze my young friends by being able to generate Pythagorean triples by a simple, almost naive method. I would start with any odd number, multiply it by itself and divide the result by 2. The result, of course, was not an integer, but by adding $1/2$ and subtracting $1/2$ I could obtain two integers, which, together with the original odd number, produced the desired triple. I must confess that I didn't know how to generate triangles such as the 8-15-17, 23-45-58, etc. That came later.

I must add that another one of my great talents was the construction of magic squares. I learned the trick at a rather early age and used it to mystify anyone who needed to be mystified, mainly my pre-teenage friends.

Before entering high school I used to browse around in bookshops and I picked up Wells's *Algebra* where I found you could find out things about apples and oranges by using x's and y's. I went through the entire book before I had any formal classes in algebra. I spent a summer vacation solving all the problems in the book so by the time I got to school I didn't have to do any homework. All I had to do was copy my notes.

MP: So with this head start in mathematics, did you major in mathematics?

Bankoff: No, I did not. I majored in general science. I took whatever mathematics I was required to take: algebra, Euclidean geometry, trigonometry and some solid geometry, believe or it not. And in college, it was analytic geometry and calculus and that's where my formal mathematical education terminated.

Leon Bankoff, the student, drawn by Frank Netter, the famed medical illustrator, 1927

Dentists—"I Liked Their Lifestyle."

MP: Did you go directly from college to dental school?

Bankoff: Yes.

MP: Why dentistry?

Bankoff: That's a question that is frequently asked. It's not hard to answer really, because I entered college at the age of 15 and graduated at 18. I didn't know what career I wanted, really, but I very cleverly managed my courses so that I would qualify for anything I chose at the end of my college education. I could go into medicine, dentistry, engineering, pharmacy, or teaching. The reason I chose dentistry was that I knew quite a few dentists. I liked what they did. I liked their lifestyle. At that time physicians used to make housecalls at 2 o'clock in the

morning. I didn't care for that sort of life and for that reason medicine never attracted me. As far as all the other professions that were open, accounting, and so forth, they never appealed to me. But dentistry did and for another very good reason. I was always interested in sculpture. I noticed that dentistry was an outlet for my aptitudes for painting, sculpture, and so forth.

MP: Where did you go to college?

Bankoff: CCNY, followed by dental school at NYU for two years, then I transferred and took my last two years at USC.

MP: So that was the move to Los Angeles.

Bankoff: My family moved to California and I moved along with them.

MP: Between the time that you finished calculus and a certain number of years later, you were not doing any mathematics.

Bankoff: I used to dabble with various puzzles, the Sam Loyd and Dudeney stuff.

MP: What got you started on real mathematics? Was there a single event or a single person?

A College of Magic

Bankoff: Yes. Among my patients, way back in the forties, was a magician and he told me that he was studying in a college of magic run by a magician named Ben Chavez. Chavez had a college of magic that was the only one authorized by the United States government to give instruction in magic to GIs coming out of the service, people interested in that as a profession. I wasn't a GI but I was interested in magic and I enrolled in this school, took the course and got my diploma in prestidigitation. Naturally, as I do in anything I'm interested in, I accumulated a library. In that library of magic books were a number of periodicals. One of them was called *The Bat*. It had among other things a column called the Puzzle Corner to which various people contributed problems. Among them was Martin Gardner. This was in the middle forties, before he became famous for his column in *Scientific American*. In one issue there was a problem and they offered for the best solution a prize of a bound volume of the previous issues of *The Bat*. I won the contest. I got my prize and in reading subsequent issues I kept seeing references to a journal called *Mathematics Magazine*. I went to the newsstand but I couldn't find anything called *Mathematics Magazine*. But then I discovered that there was such a magazine, published locally in Los Angeles. I subscribed, tackled some of the problems and found that practically all of my contributions were accepted and published by the problems editor—at that time, Charles W. Trigg.

MP: Ah, Trigg enters the scene.

Bankoff: We became acquainted. I would solve a problem and tear down to Los Angeles City College, put it on his desk and say, "Look at this!" He put me onto means of acquiring more knowledge of mathematics. He told me to buy Nathan

Altshiller-Court's book on college geometry, various books on number theory—Oystein Øre, Uspensky and Heaslet—and books on trigonometry. I gradually started accumulating a library by searching bookstores for second-hand books. I wandered into a shop on Main Street in Los Angeles one day and asked whether they had any mathematics books. They said they had some down in the basement and I was surprised to find there about a dozen books called the *Ladies' and Gentlemen's Diary*. I bought them, of course. Later I had occasion to consult an issue of the *Diary* that I didn't have and I discovered that the William Andrews Clark Library (part of UCLA) had it. So I went down there and to my surprise I found that issues from my run were not in the Clark Library and the other way around. I told the librarian I would either have to buy his issues or will him my copies. But this served as a great impetus to my inquisitiveness in mathematics. I raided the bookshop of Zeitlin & Ver Brugge (the red barn on La Cienega), now gone, and I was able to get some rare books like Pappus in Latin and some Archimedes. I gradually accumulated an immense library. I bought everything that came along that was of interest, not only geometry but practically every area of mathematics.

MP: I remember Jake Zeitlin's bookshop very well. I recall that in London I was once looking for some obscure work in the history of mathematics and I was told there that my best bet for something like that would be to try Zeitlin in Los Angeles. It went against the conventional wisdom about locating scholarly books.

Bankoff: I knew Jake Zeitlin very well. I had lunch with him just two weeks before he died. We tried out a new restaurant and that night both of us took ill.

MP: That sounds ominous—more than a coincidence.

Bankoff: Well, not really. He went to the hospital for a heart condition and I had a gallbladder attack.

Soon after I met Jake he showed me a rare copy of Lewis Carroll's *Pillow Problems* which he wouldn't sell. It was so precious he wanted to keep it himself. But he lent it to me and I copied it by hand, word for word, even copying the style of the printing. Later, when Dover came out with an edition of it, Jake called me up and asked me whether I wanted to buy it!

Anyway, I might mention that when I took my mathematics in school, I had no trouble with it. For instance, before calculus I was on summer vacation. Since I was old enough then to start going with girls, I didn't want to waste my evenings doing homework. So that summer I solved practically every problem in the book. During the year when I got assignments all I had to do was copy out the solutions. Then I had my time free to pursue my social life.

MP: You learned calculus in college in 1926. And you encountered your magician, who indirectly got you interested in mathematics, in roughly the mid-1940's. So there was a 20-year period when you were not doing mathematics. You were, of course, doing dentistry. What else were you doing to occupy your time? Music?

Bankoff: I had been playing the piano since the age of 12. I used to play the violin but I gave that up when I went to college because I had started smoking and I couldn't tolerate the idea of ashes dropping into the f-holes. I became a chain smoker, and I always played the piano smoking a cigarette.

MP: It provided a spot for the ashtray.

Bankoff: No, at one time I had an ashtray constructed that I wore around my neck. A friend of mine made it so I could play the piano without any regard for the ashes. But I am glad to say that in 1969 I gave up smoking.

MP: So you played the piano, violin, and, I understand, somewhat later the classical guitar. And we've heard about sculpture.

Bankoff: I used to spend my Sundays taking the ferry across to New Jersey and painting with a cousin of mine who was quite a good painter and later owned a gallery in New York. Sculpture always interested me. But most of my time was spent as an equestrian. I belonged to a riding troop and we used to do all kinds of military maneuvers on horseback. I enjoyed that very much. And I spent a lot of time ice-dancing. There used to be numerous ice rinks in Los Angeles. Sonya Henie owned one in Westwood before Westwood was built up. There was a Pan-Pacific rink and one in Pasadena. I used to go Sundays with my daughters after they grew up. They all received an ice-skating education, figure skating. I also have been very interested in calligraphy. And I played chess a great deal. In the early days of my practice, I cannot say I was terribly busy. I started practice in 1932, in the depths of the depression—there was plenty of time to play chess. Salesmen would come in to sell me dental equipment and we sat and played chess.

MP: You sound a little bit wistful about the ease of life then with a lot of time to spend on your interests outside dentistry. I assume that life has gotten busier.

Bankoff: It has. I'm very intense.

MP: So all of these things were going on and then, mathematics. Since you got interested in mathematics, you have published quite a bit. Do you have any idea how many papers?

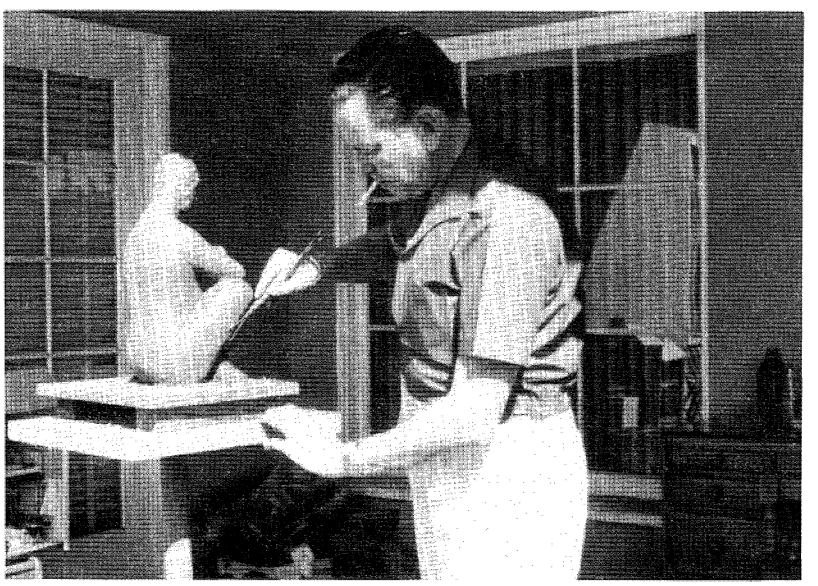
Bankoff: Not terribly many papers. Maybe about a dozen. My chief activity has been in posing and solving problems.

An Erdős Number of One

MP: You seem to know a good many of the important mathematicians of the mid-twentieth century. You have traveled in good mathematical circles. Not all that many mathematicians one meets, for example, have an Erdős number of one. But you have coauthored a paper with Erdős. Have you had other coauthors?

Bankoff: Well, there's Charles Trigg, of course, Jack Garfunkel and Murray Klamkin. I can trace my acquaintanceship with all those mathematicians to Charles Trigg. He encouraged me to write to Victor Thébault of France. There was a problem published in the early '50s in *Mathematics Magazine* about determining the radius of a circle inscribed in the figure known as the shoemaker's knife of Archimedes. I became fascinated with this problem and I sent in three separate solutions, which were published. Among the other solutions published were one by Howard Eves, one by Klamkin, as well as others. I asked Charles Trigg where I could learn more about this particular figure, and he told me that there was a French mathematician who was at that time about the same age I am now, who had written extensively about the shoemaker's knife of Archimedes. Thébault was

Bankoff with Paul Erdős, Beverly Hills, 1980



Bankoff, the sculptor, 1955

A Mere Coincidence

Dr. Leon Bankoff

The Arbelos, or the Shoemaker's Knife of Archimedes, is a curious geometrical figure that is chock-full of amazing coincidences. Many of the properties presented here seem contrary to intuition; yet all can be demonstrated mathematically.

The Shoemaker's Knife is the shaded area bounded by the three semicircles in Figure 1. Each arc touches two others, and the points of tangency, A, C and B, lie on a straight line. Needless to say, the centers 0_1, 0_2 and 0 lie on the same line. In constructing the figure, the point C may be chosen anywhere on line AB.

FIG. 1

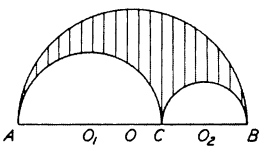
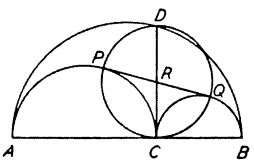


FIG. 2



In Figure 2, CD is perpendicular to AB; PQ, the common external tangent of the two smaller semicircles, cuts CD in R.

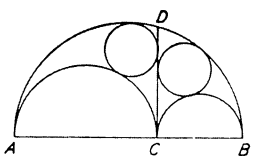
SURPRISE NO. 1: PQ = DC.

SURPRISE NO. 2: PR = CR = QR = DR. Hence, with R as center, a circle may be drawn passing through P, C, Q and D.

SURPRISE NO. 3: The area of circle PCQD is equal to the area of the arbelos. (Shaded area in Figure 1.)

Now let us consider an arbelos divided by the line CD into two unequal segments. Inscribe circles touching the boundaries of each space. (Figure 3.)

FIG. 3



SURPRISE NO. 4: The two inscribed circles are equal!

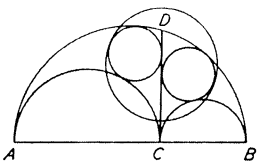


FIG. 4

SURPRISE NO. 5: The smallest circle that can enclose the two inscribed circles has a diameter equal to CD. Hence the area of this circumscribed circle is also equal to the area of the arbelos! (Figure 4.)

We now consider an arbelos having only one inscribed circle, touching arcs AC, CB and AB in P, Q and R, respectively. Let M be the mid-point of arc AC and N the mid-point of arc CB. With M as center and MA as radius, describe a circle. (Figure 5.)

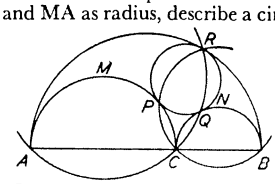


FIG. 5

SURPRISE NO. 6: The circumference of this circle will pass through C, Q and R. Similarly, the circle with center N and radius NB will pass through, C, P and R!

SURPRISE NO. 7: The perpendicular from the center of circle PQR upon the line AB is bisected by the circumference of PQR.

SURPRISE NO. 8: Draw AR, CR and BR. Angle ARB is bisected by RC.

If an arbelos is reflected in AB, as in a mirror, we obtain two congruent figures. (Figure 6.) In the lower arbelos draw CD and two inscribed circles, as in Figure 3. In the upper arbelos inscribe a circle as in Figure 5, touching the smaller semicircles in P and Q. Now draw a circle through P, C and Q.

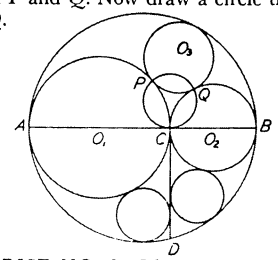


FIG. 6

SURPRISE NO. 9: The three small circles are equal!

How many of these "coincidences" can you prove? *WARNING:* Most of the proofs are not easy.

Reprinted from *Mathematics Newsletter*, Los Angeles City College, November 1954.

For more on this subject see Martin Gardner's column in the January 1979 *Scientific American* and Bankoff's "Are the twin circles of Archimedes really twins?," *Mathematics Magazine* 47 (1974) 214–218.

very well known in those days and, in fact, I contributed an article to the *American Mathematical Monthly* about him, along with Starke, Eves, and others. Anyway, when I found out about his work I wrote to Thébault and discussed this matter and he sent me numerous reprints of his. I made some discoveries of my own which I communicated to him. And after a short time it was decided that we would compile a ten-chapter book on this one subject, where he would contribute five chapters and I would contribute five.

MP: Had you met him at this point?

Bankoff: No. The correspondence took place very rapidly because in those days when you sent an airmail letter to France it would arrive in three days. If I wrote on Sunday, I would have a response the following Sunday.

MP: That's because the planes were a lot slower in those days!

Bankoff: And the mails were a lot faster!

A Visit to Tennie

Every summer he invited me out to visit him in his little chateau which he called "Le Paradis," which was located in Tennie, a small village—really not a village at all. It's not even large enough to be called a hamlet. It had a population of about 200 possibly, if that many. Thébault's telephone number was Tennie 2; the mayor had Tennie 1. At any rate, after a few years I decided something had to be done. I had developed a lot of material on the golden arbelos (shoemaker's knife) and various ramifications involving Pythagorean triangles, rational triangles connected with the arbelos, and in 1958 I wrote that I would be there. I went to Paris, then took a train to Le Mans, where Thébault was waiting for me with a young lady named Francine Laloue who served as an interpreter. He hired a taxi to take us 20 km away to his little hamlet, Tennie. We became acquainted. I showed him what I had done. He thought it was great stuff. After a visit of about three or four days, I left. I was supposed to continue work on this manuscript, but I got bogged down with so many other things, mainly dentistry, that the thing was delayed and delayed. Even to this day, 28 years later, the manuscript is still residing in my closet, unfinished.

MP: But one of these days...

Bankoff: One of these days I expect to retire and get this manuscript to the printers.

About five years ago Martin Gardner wrote about this unpublished manuscript in the *Scientific American*. I have had numerous inquiries about when it will be published. Someday I hope to finish the job. If I don't live long enough to do it, I have asked Clayton Dodge to get the material I have and to get it into print.

I admired Thébault's work greatly. His work on the shoemaker's knife was really incidental. He did a great deal of work on arithmetical recreation, intense work on number theory. In fact one of the first problems I solved was a problem that Thébault had proposed, to find a cube plus a square that is equal to 2,000,000. He was also very deeply into the geometry of the tetrahedron.

Victor Thébault and Bankoff, Tennie, France, 1958

I wouldn't say Thébault was penurious but he used to write on the back of calendars. He was retired when I met him. He did mathematics. He showed me around his estate and also suggested that I buy his property. I had no use for it, but I am sorry now I didn't. Ten years after he died his widow died. I used to make annual visits to her. Shortly after he died I was ushered into his library which she kept as a shrine. His desk was left exactly as he had left it, pen here, eyeglasses there, letter scale over there. And it was rather eerie to see there a letter of mine on the desk that he hadn't replied to. After his wife died I was invited by one of his sons to help myself to some of the items in Thébault's library. So I have a complete run of the *Educational Times* that belonged to him. And I have his moderately large run of the *Mathematical Gazette*. He was fond of making jokes and he smoked a great deal, as I did. I remember he once went to get champagne which he kept in a cupboard that looked like a grandfather's clock. But it wasn't really a clock. Before he retired he was a director of insurance, sort of an actuary. And for awhile he taught school. But when I knew him he did mathematics.

MP: Your interpreter on that visit has a special role now for you. How long after meeting her were you married?

Bankoff: Eight years. I'll tell you how that happened. In 1963, five years after my visit to Thébault, this young lady was invited by the University of California, Berkeley, to come and set up audio-visual language instruction in which she was expert. And when she arrived here she communicated with me and we reestablished our acquaintance. In 1965, because of visa regulations, she had to go back to France for a year. She came back again in 1966 and we were married that year.

MP: You are largely associated with geometry in people's minds, though you mentioned to me that you are really interested in inequalities. Have you published as much in inequalities? Why do we associate your name more with geometry?

Leon and Francine Bankoff in a restaurant in Paris, 1958

"I Disagreed with the Published Solution—I Found a Counterexample."

Bankoff: Well, geometry is more visible, for one thing. Inequalities probably do not attract as much attention. But my interest in inequalities started a long time ago. My first really good achievement was with a problem that was proposed by Thébault, by coincidence. He proposed a problem concerning some trigonometric inequality and I sent in a solution. My solution was not used but I had to disagree with the published solution for the simple reason that I found a counterexample. I communicated with Howard Eves, who was at the time the Problems Editor of the *Monthly* and, sure enough, he said I was right. Would I offer a correction? So that too was published in the *Monthly*.

MP: You mentioned that this work appears in a standard work on inequalities.

Bankoff: Yes, Bottema's book on geometric inequalities. When I say geometric inequalities, I always associate geometry with trigonometry. Practically all of my geometrical inequalities have trigonometrical consequences. And the trigonometrical inequalities, I generally visualize them geometrically.

MP: You mentioned the *Pi Mu Epsilon Journal*. You used to edit the Problems Section.

Bankoff: Sometime in the late '60s Klamkin was problems editor. But he was bogged down with many other duties. He was editor of the Problems Section of the *SIAM Review* so he asked me to take on the *Pi Mu Epsilon Journal*. I told him I didn't have the ability or the time. A few days later I received a big package from him saying: "You are the editor. Goodbye." I felt duty-bound to take it over. When I had been editor for a short time, I became aware of the difficulties that

have to be faced by problems editors (and you're very familiar with those). I wrote an essay on the problems of a problems editor; it appeared in the Fall 1975 issue of the *Journal*.

One of the earlier editions of Steinhaus's *Mathematical Snapshots* came out in 1950 and contained among many other things something that could be called the counterfeit coin problem, to ascertain which one of a number of coins is counterfeit and whether it is heavier or lighter. My daughter was a high school student. She looked over this particular solution and discovered an error. I checked it and sure enough there was something wrong with Steinhaus's solution. So I wrote to Oxford University Press and indicated where the error was. In 1960 an edition came out with the error corrected and an acknowledgment. Eight or nine years later another edition came out and they reinstated the error. It demonstrates what a dentristy professor of mine once said, "There is one entity that has claim to immortality and that is error."

The Man Who Corrected Einstein

There was another curious incident involving an error. It was the spring of 1952. My wife and I were at a movie one evening and when we came out I picked up a morning paper. There was a big hullabaloo about Einstein and a student who had requested an answer to a geometry problem she had sent him. It made the front page of the *Los Angeles Times*: "Stuck with Geometry, Girl Turns to Einstein: Sophomore Decides Famed Physicist Is Most Apt to Solve Problem—and He Does". Her school was not at all happy with her; the people there thought it looked bad for their teachers if she had to write to Einstein to get help with her geometry problems. There was a picture of Einstein's postcard (and a comment that he didn't put enough stamps on it!). She was pictured with the caption: "Still Puzzled. Johanna Mankiewicz, 15, daughter of Screen Writer Herman Mankiewicz, ponders over sketch supplied to her by Dr. Albert Einstein to help her solve a geometry problem. She said Einstein's sketch didn't help." Well, I looked at the postcard sketch pictured in the newspaper and Einstein wasn't right either. He gave an answer to a different problem. He described how to construct a tangent to a circle, but the problem was to compute the length of the tangent. All of this was in *The Mirror* as well, so I called the editors and told them the solution given did not solve the problem. They wanted to know my credentials to question Einstein. So they sent a reporter out in the early hours of the morning. The next morning I was on the front page of the *The Mirror*: "Einstein 'Fails' in Math, but Still Has Nobel Prize." I became known around Los Angeles as the man who corrected Einstein!

MP: Is your interest in mathematics directly related to your interest in dentistry at all? Are you interested in the geometry of the jaw or stress calculations or such?

Bankoff: No. Only indirectly. I am sort of a Martin Gardner-style debunker. I resent any introduction of the occult into dentistry where so many orthodontists try to attribute occult significance to the appearance of the golden ratio in the human skeleton and dental equipment.

MP: Are you a subscriber to the *Skeptical Inquirer*?

Bankoff: Of course.

MP: Does that come from your interest in mathematics or your interest in magic or just generally from your outlook on the world?

Bankoff: Before I knew of the *Skeptical Inquirer*, Bertrand Russell was one of my heroes.

MP: I know that you are quite close to Sol Golomb and see him regularly. When did you first meet him?

Bankoff: I met him in 1968 through Leo Moser. I met Leo through Charles Trigg.

MP: From Golomb in the other direction?

Bankoff: Oh, it began to fan out in all directions. For example, in the early days when I developed quite a lot of material on the shoemaker's knife, I was called upon to deliver lectures at a number of local colleges—Occidental, Pepperdine, Pomona, UCLA. I got to meet quite a few mathematicians.

One day I got a phone call from Nathan Altshiller-Court who was visiting Los Angeles. He wanted to meet me and, of course, I wanted to meet him. He spent a few days with me and he told me about a forthcoming book of his called *Mathematics in Fun and Earnest* which was just about to be published.

One day Leo Moser was in town. He called me and asked me to go over to Caltech with Charles Trigg and him because Erdős was to be there. I buttonholed Erdős and we took a walk. I had just published my proof of the Erdős-Mordell theorem which Eves had, I understood later, refereed. One thing led to another.

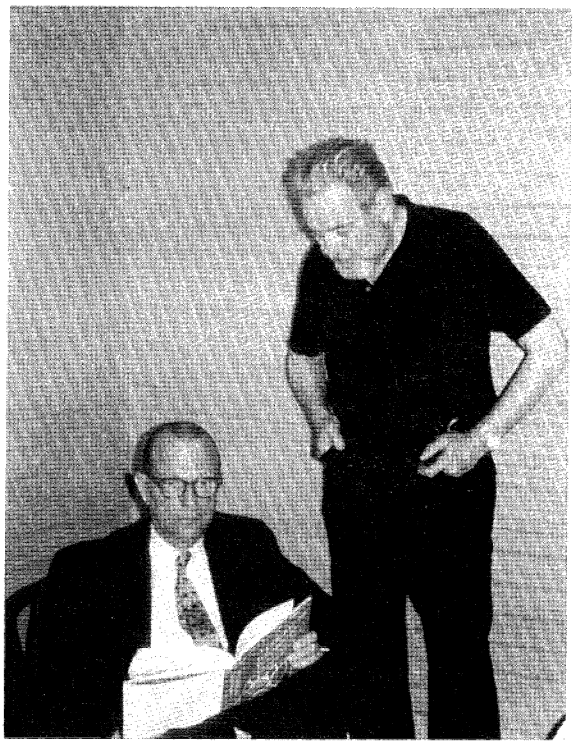
MP: What led to your joint paper with Erdős?

Bankoff: The Putnam Examination in 1967 (see G. L. Alexanderson et al., *The William Lowell Putnam Mathematical Competition/Problems and Solutions: 1965–1984*, Mathematical Association of America, 1985, p. 7) had a problem that was solved by the use of complex numbers. I received a letter from Donald Coxeter asking me to see if I couldn't solve it using ordinary Euclidean geometry. I was able to do that so I sent it to him and sent a copy to Klamkin. He showed it to Erdős and we put together this problem with various kinds of solutions, my geometric solution and Klamkin's and Erdős's complex numbers solution. The article appeared in *Mathematics Magazine* in November 1973. I have always been interested in various solutions to a single problem.

MP: This was the asymmetric propeller problem.

Bankoff: Yes. Later we discovered there are quite a few ramifications of this problem. I'm really ashamed of myself for not having completed this. But I did make a special trip to Edmonton to consult with Klamkin about it. He added a few notes about six-fold symmetry which I didn't understand. And because I didn't understand it I couldn't finish the paper.

MP: Is that one of those papers sitting in your desk drawer?



Bankoff with Charles W. Trigg, San Diego, 1971
(reading Pedoe's book on geometry)

Bankoff: No. It's sitting in my closet!

MP: Beginning in the '40s and '50s you started doing more mathematics. What got shoved aside? Chess? Piano? Photography? (I know you are also interested in photography, enough to own 13 cameras!)

Bankoff: Piano never got shoved aside. Chess did. In fact, many of these things went on concurrently anyway, but gradually I started spending more and more time on mathematics. I have at home about 25 huge notebooks, just my own notes on problems and solutions and such.

MP: You're working full-time as a dentist, playing the piano, sculpting, playing chess, taking pictures and doing mathematics. Do you sleep a negative number of hours per night?

Bankoff: No more than four. I understand Erdős sleeps only four hours per night too.

MP: But he pretends to sleep at lectures.

Bankoff: He pretends to, but he doesn't really. He closes his eyes and nods his head, but he's really wide awake.

MP: Do you have any new Erdős stories?

Erdős and Bach

Bankoff: The latest Erdős story that I can recount is this. About two years ago, Erdős visited Los Angeles, in the winter, to attend Caltech's Alaoglu Lecture and, as was his custom, he had dinner one evening with us. Invariably Erdős requests a background of Bach. So we put on some records of Bach and I happened to mention that the following evening we would be attending a recital at Ambassador Auditorium by Alexis Weissenberg. He would be playing the Goldberg Variations among other things. Would he like to attend as our guest? He agreed and the night of the recital, we picked him up at the Atheneum at Caltech. He was having dinner with Ron Graham. They had papers spread out all over their table, but when we arrived he gathered up his papers and we proceeded to the Ambassador Auditorium. He made his usual exploratory jaunts upstairs and down to see what was going on there. When the concert started—he was sitting next to me—his head dropped to his chest and he closed his eyes, as is his custom. To all intents and purposes he went to sleep. After an evening of Bach, we were in a group that went backstage to meet Weissenberg. Erdős wanted to meet him. And, as it turns out, Weissenberg wanted to meet Erdős. They got into a conversation. All of a sudden, Erdős said he had to make a phone call. When he finally got back we asked what took him so long. He said he had had to call Ron Graham. During the concert he had solved the problem they had been working on!

MP: What was your first publication?

Bankoff: My first were dental publications when I was still a student. They appeared in a journal, now defunct, called the *Dental Student's Magazine*. One of my pet topics related to occlusion and to changes in occlusal coordination resulting from the loss of teeth. Several articles described unusual clinical findings in my work in the school clinics. After graduation, I limited my writing to letters to the editor. Since I neither sought nor desired any academic connections, there was no point in my rehashing the findings of others just to get my name into print. In recent years my extramural activities in dentistry have been political, limited to service on a number of dental society committees.

My first mathematical publication came out around 1950, the solution to a problem.

MP: You had a nice article about the golden arbelos in *Scripta* in 1955. That's the first full article of yours I am aware of. And you have a nice recent publication in *Mathematics Magazine*, one on the history of the butterfly theorem. What do you have in the works, other than a drawer full of manuscripts?

Bankoff: Having relinquished my editorship of the Problems Section of the *Pi Mu Epsilon Journal*—I turned it over to Clayton Dodge—I have a little more time to spend on unfinished manuscripts that have been lying around.

MP: So we'll see a number of articles soon...

Bankoff: I have not only folders of material, but valises of material, large valises. In fact, we're thinking of adding a new structure to our house, to the garage behind the house, to hold all the books, and the manuscripts I hope to finish one day.

Bankoff, the classical guitarist, 1978

MP: I know that you spoke at the Strens Conference in Calgary. What did you talk about there?

Bankoff: The arbelos. I called it "The Marvelous Arbelos." It's not that it's the most important thing in geometry, but to me it represents something sentimental. It's the first geometric figure that I devoted great attention to. It is responsible for my trip to Europe and for my meeting my wife.

MP: Were you reporting on some recent investigations? Or was this a survey?

Bankoff: Just a survey.

MP: Shortly after you attended the Strens Conference you attended the International Congress of Mathematicians in Berkeley. And now you are in San Francisco for the joint meetings of the American Mathematical Society and the Mathematical Association of America where you are speaking in a special session on geometry.

What are you doing these days?

Bankoff: Well, not much mathematics. I spend a lot of time with my computers. I have seven, some hopelessly obsolete by now, but I still keep them around. One I use only for playing chess. My daughter, who died in 1985, bought me a Macintosh

for Father's Day, just two weeks before she died. So all these years I never even opened the package and you can understand why. But a few months ago I unpacked it and I'll have to learn how to use it. I'm still using my IBM 386 clone. I have one of the computer science teachers at Beverly Hills High come in and give me private instruction on the computer, to answer questions that I can't ferret out of books.

MP: How much of the day do you spend in the office these days? You still have patients who are counting on you.

Bankoff: I'm trying to cut back. I'm trying not to take on new patients. Of course, people do not really seek out 82-year-old dentists.

MP: Why not?

Bankoff: They like to be confident of a sense of continuity!

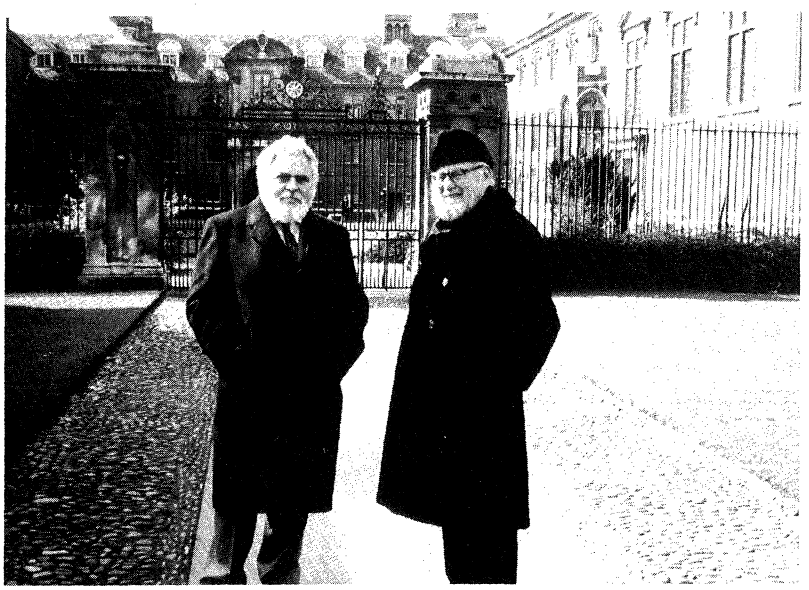
During the past four years I have taken up jazz piano. I had played classical piano since the age of 12 but I'm fascinated now with jazz piano. It's such a complex subject. I had studied harmony and counterpoint and felt I understood it pretty well, but I could never understand jazz. It's a different kind of music. It's entirely improvisation based on modern harmonies, voicings and rhythms—a sort of creative, spontaneous composition. So I take a lesson every Saturday afternoon in jazz piano. I find it quite hard, teaching my hands almost automatically to devise runs, licks, riffs and left-hand accompaniments.

Chopin's Tomb

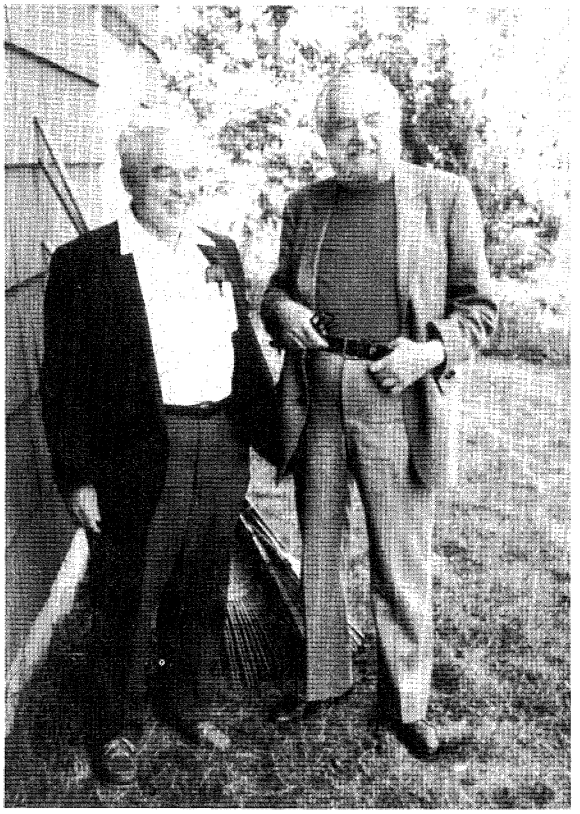
MP: You haven't lost interest in classical music, I suspect. I recall there's a story about your interest in Chopin.

Bankoff: When I first visited Paris in 1958, I decided that the first thing I wanted to do was visit Père Lachaise Cemetery to see Chopin's tomb. I was not in the best shape. I was recovering from a broken leg and had been on crutches till very recently. So I called a cab and asked the driver to take me into the cemetery, but that was not permitted, at least until I reached into my pocket for a few bills and suddenly things become possible. I got some flowers to take in. It was the only grave in that area to be covered with fresh flowers. I found that the muse on the tomb, with head bowed, was beautiful except for one thing. Somehow—maybe it was the work of a vandal—one finger had been broken off her hand. So when I got back to Los Angeles, I asked a patient of mine, the novelist Romain Gary, who was French consul-general there at that time, what I could do to repair the statue. He got me in touch with the estate of Chopin and they replied that they could find a sculptor to do the job. Actually, maybe they misunderstood my question. They also gave me an estimate on what it would take to restore the whole tomb. That was not my intent. So I sent them a check to have the finger repaired. I never heard from them, but a year later I was in France again and went out to Père Lachaise. I was dismayed to find that I could have done a much better job myself. Instead of having a nice graceful curve to the finger, it was sticking straight up in the air. I almost felt like breaking it off myself. But I didn't. A couple of years later I

Bankoff placing flowers at Chopin's tomb, Père Lachaise, Paris, 1968



Bankoff with Sol Golomb at Cambridge University, 1987



Bankoff with Howard Eves at Orono, Maine

returned to find that someone had broken off the offending finger and the thumb as well. That's it. I'm not repairing it again!

MP: Well, you did what you could.

So at this point in your career, do you have any regrets? Did you choose the right career?

Bankoff: My only regret is that I have so little time to spend on mathematics, the most beautiful thing in the world.

MP: But choosing dentistry as number one and mathematics as number two—is there any reason to believe that you should have chosen them the other way around?

Bankoff: Well, I am a mathematician without being a professional mathematician. I don't look upon myself as an amateur. Someone defined an amateur as one who does not earn money with what he is doing. But it is an activity that I indulge in as thoroughly as other mathematicians.

MP: But I like to associate "amateur" with its root, to mean that one loves the subject.

Bankoff: I certainly do. Of all of my interests, I put mathematics number one. How I make my living is another story.