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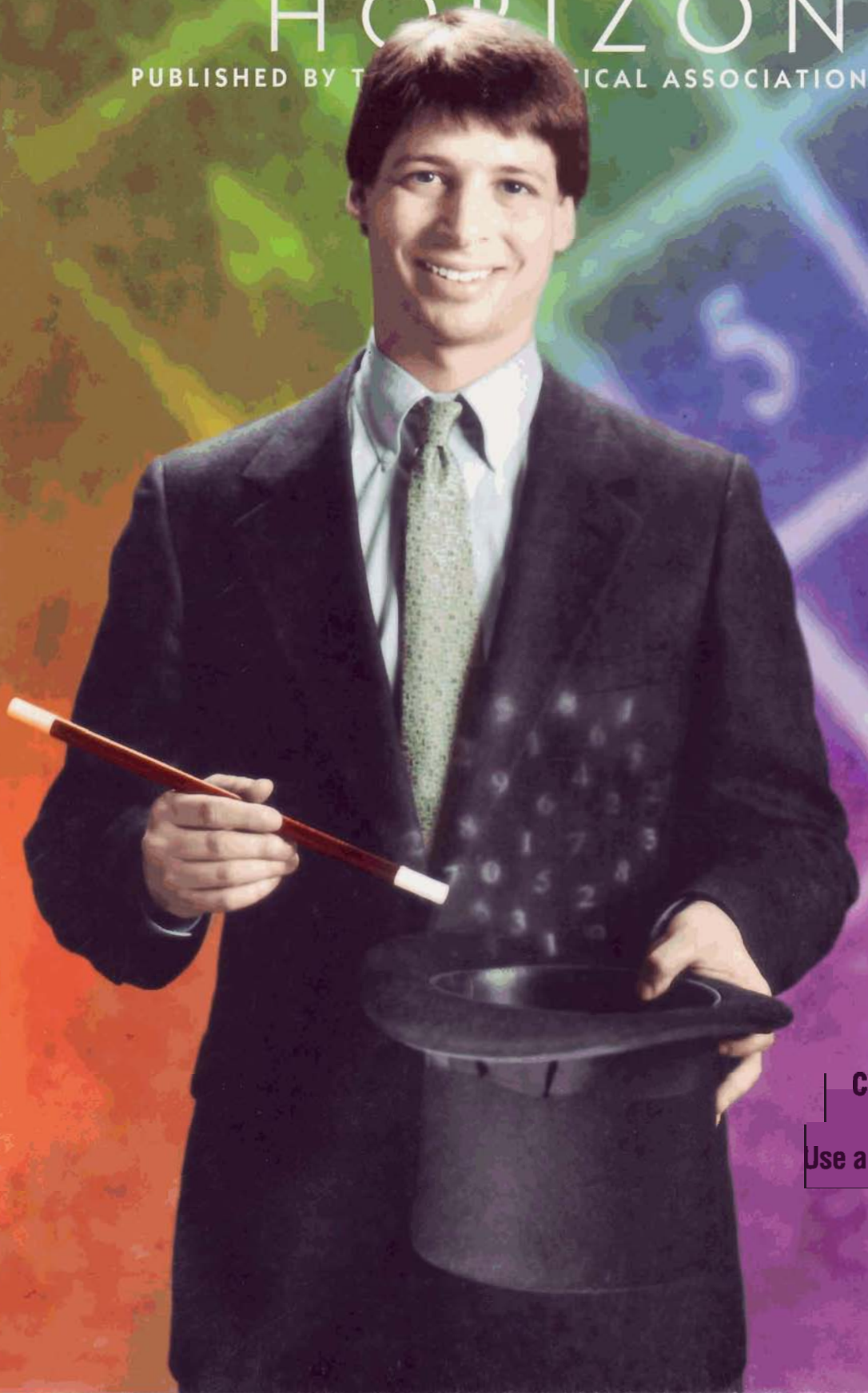

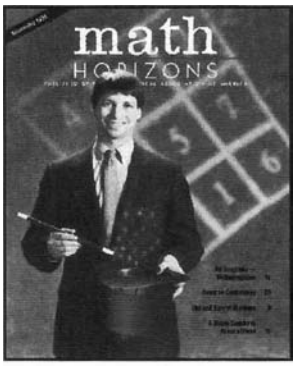

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DONALD J. ALBERS

Art Benjamin—Mathemagician

Mathematics professor Arthur (Art) Benjamin, 37, is a friendly person, a popular teacher, happily married, and soon to be a father. But as a child growing up in Cleveland, Ohio, all bets were off about his future. Little Art Benjamin was so rambunctious in his first nursery school that he was thrown out! Ditto for his second nursery school, and the third, ... "At nap time, all of the students except me would lie down on their blankets," he recalls. "I would be running around the room. I was hyperactive. A lot of the problem may have stemmed from the fact that I was bored with many things. I usually understood things the first time they were explained, and by the time something was being taught the fourth or fifth time, I'd rather get silly than pay attention." During the first five years of life, his hyperactivity resulted in several periods of hospitalization, often for weeks at a time. In today's language (and less understood during Art's childhood), his problem would be called ADHD, attention deficit hyperactivity disorder. He was prescribed the medication Valium, which he took for the next ten years. When Art was finally taken off Valium in the eighth grade, he promptly went out for the track team. He had always liked sports, but the Valium had for years slowed him down.

As a child, Benjamin remembers wanting a lot of attention and doing lots of things to get it. "I took up magic as a hobby, I sang, I danced, I got good at different games, and I learned to calculate quickly. When I was five, I memorized the states and their capitals in alphabetical order. I even learned the presidents in order."

Today, Professor Benjamin certainly is not lacking for attention. For starters, he gets lots of it from his students at Harvey Mudd College in Claremont, California. He is a popular teacher with a knack for involving students in his subject. He has written several research papers, including many co-authored with his students, and he is the Editor of the Spectrum Book series that is published by the Mathematical Association of America. During his spare time, he performs on numerous stages as Art Benjamin—Mathemagician. He is a regular at Hollywood's Magic Castle, the leading club for magicians in the world.

Could you kick this sweet little boy out of nursery school?

He has appeared on numerous television shows in the US, England, Canada, and Japan, including The Today Show, Evening Magazine, Square One, Live! Dick Clark Presents, and CNN Headline News. He also has given hundreds of performances for school children, high school and college students, adults, and even mathematicians.

As a premier "mathemagician", he regularly dazzles audiences with his calculating feats. He begins a typical performance by asking members of the audience with calculators to assist him. In order to test the accuracy of the calculators, members of the audience will call out two numbers for him to multiply, for example, 68 and 92. "Make sure you get 6,256," Benjamin says, before his assistants have even entered the numbers. The race between Benjamin and the machines is on. He then challenges them in squaring two-digit numbers, for example, 47, 59, 63, 89. He beats the machines every time with ease. Benjamin then moves on to squaring three-digit and four-digit numbers, and again the calculators lose. By this time, most members of the audience are thoroughly awed by his feats of lightning calculation.

He Likes All Numbers

Benjamin's interest in numbers, calculation, and number patterns goes back to his early childhood. He says that he's never met a number that he hasn't liked. One of his first toys resembled a slot machine. When he pressed the lever of the machine, four arithmetic problems appeared in each of four windows — one in addition, one in subtraction, one

DONALD J. ALBERS is the editor of *Math Horizons* as well as co-author of *Mathematical People.*

in multiplication, and one in division. He recalls learning his multiplication tables with that toy. By the time he was in third grade, he was discovering numerical properties resulting from multiplying two-digit numbers. "One day in school my classmates and I were in the hallway ready to go to lunch. I remember standing up against the wall daydreaming about numbers. I was marveling over the fact that 2,520 is the smallest number that all numbers between one and ten divide into perfectly. I tried a smaller number, like 1,260, and realized that eight did not divide into it. I then wondered what would be the smallest number that one through eleven would go into, and figured out that would probably be 2,520 times 11."

A few years later he hung a velcro dartboard on the wall at the end of his bed. The darts were actually velcro balls and the object of the game was to throw them at the numbered regions, 1 through 10, on the dartboard. "Sometimes I would lie awake at night staring at the dartboard. I would see the numbers, and I would multiply them together, getting various combinations as large as ten factorial (3,628,800)."

One day he took a bus into downtown Cleveland to visit his father at work. To pass the time, he started thinking about numbers that add up to 20. He started at the center with 10 and 10, and then formed the product 10×10 to get 100; 9×11 was 99; 8×12 was 96; 7×13 was 91. He then noticed that 9×11 was one less than 100; 8×12 was 4 less; 7×13 was 9 less. The pattern was 1, 4, 9; he guessed that 16 and 25 would come next. "Oh wow," he said to himself, "I wonder if this works for other numbers, and then I tried it for big numbers, small numbers, fractions. It worked!" With elementary algebra, it is easy to verify his discovery, but for a young boy with no knowledge of algebra, it was a big accomplishment.

In middle school, Benjamin's interest in games blossomed, and he started to play both chess and backgammon in a serious way. When he started college, he gave up chess in favor of backgammon because it was faster and easier to play. He soon got to be very good at backgammon. In 1997, he won the

The Raisin

Once upon a day quite cheery,
Far from lusterless Lake Erie,
On a beach somewhere southwest of the City of Singapore,
'Twas a grape that had a notion.
If he'd rest close to the ocean,
He would tan without his lotion,
Just by resting near the shore,
And he'd meet up with adventures that he never dreamt before,
All of this and much, much more.

All of this had happened one day
It was on a sunny Monday
As he soaked up every sun ray going into every pore.
Not a gust of wind was breezing,
And the warmth was, oh, so easing
And so beautifully pleasing.
He was filled with joy galore.
"Oh, how I wish," he said, "that I could stay here on the shore,
Remaining here forever more."

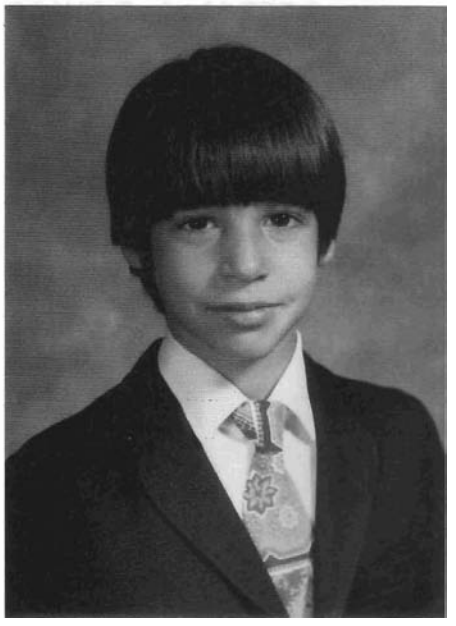
Well, that grape who had that notion,
Slowly motioned to the ocean.
He then discovered something which filled him with much gore.
For he saw by his reflection,
Nature made a small correction.
He was further from perfection,
Meaning worse off than before.
He had changed into a raisin while he rested on the shore,
And that he'd be forever more.

Well, one day a man in yellow
Gazed down at the little fellow
And looked at him and other raisins resting on the shore.
For you see his occupation
Was to go to this location,
Meaning that it was his station
To pick raisins off the shore.
So he knelt down towards that raisin on the beach near Singapore,
Picked him up with many more.

All of them were squished together
In a packet made of leather
In a factory southwest of that city, Singapore.
And that raisin was a snooper
Then he saw a spacious, super,
Massive, mammoth, monstrous scooper,
Scooping raisins by the score.
Packaged them in Raisin Bran and sold them to the store,
Scooped him up with many more.

And that raisin now is well aware
He's in a bowl in Delaware
Ready to be eaten by a child not yet four.
And the milk was slowly dropping
And the raisin heard it plopping,
Then the raisin heard it stopping
For the milk had ceased to pour.
Au revoir sweet life, he cried, life which I truly do adore,
Quoth the raisin never more.

Benjamin wrote "The Raisin" in his freshman year of high school. It is a parody of "The Raven" by Edgar Allen Poe.



Mr. Benjamin, dressed for success

American Backgammon Tour (the ABT is a series of tournaments played around the country throughout the year), and he is the all-time point leader in the history of the ABT. Benjamin says his early interest in games and calculation ties directly to his research areas of combinatorics ("clever ways of counting") and operations research ("the math of doing things efficiently").

Dreams of Broadway

After finishing high school, Benjamin attended Carnegie Mellon University (CMU). He majored in applied mathematics and statistics, and graduated in three and one-half years. During his freshman year, however, his direction was less than clear, for he spent much of his time writing song lyrics. He claims that, "if you had asked me in ninth grade and throughout much of high school, what I wanted to be when I grew up, I would have said a Broadway lyricist."

It's not surprising that young Benjamin entertained Broadway ambitions. His brother, also a teacher, was an actor for several years and remains active in Cleveland community theater as an actor and director. His sister is also active in local theater. "My father was an accountant by day and an actor and director by night. I think he wished that he had taken a shot at a career in the theater. He passed on to all of his children his love of the stage."

Each year the Scotch and Soda Society (a theater group) of CMU produced an original musical, and over the space of a few months, Benjamin and another freshman wrote the lyrics and music for the 19 songs of "Kije" (based on the story "Lieutenant Kije Suite"). According to Benjamin, it was the biggest moneymaker that Scotch and Soda ever had. "It was a big hit. I had delusions of Broadway." As he sat telling me of that experience, he suddenly broke into song, remembering perfectly a song he had written 19 years ago (the song is performed by the villain as he attempts to seduce the princess in the second act of "Kije").

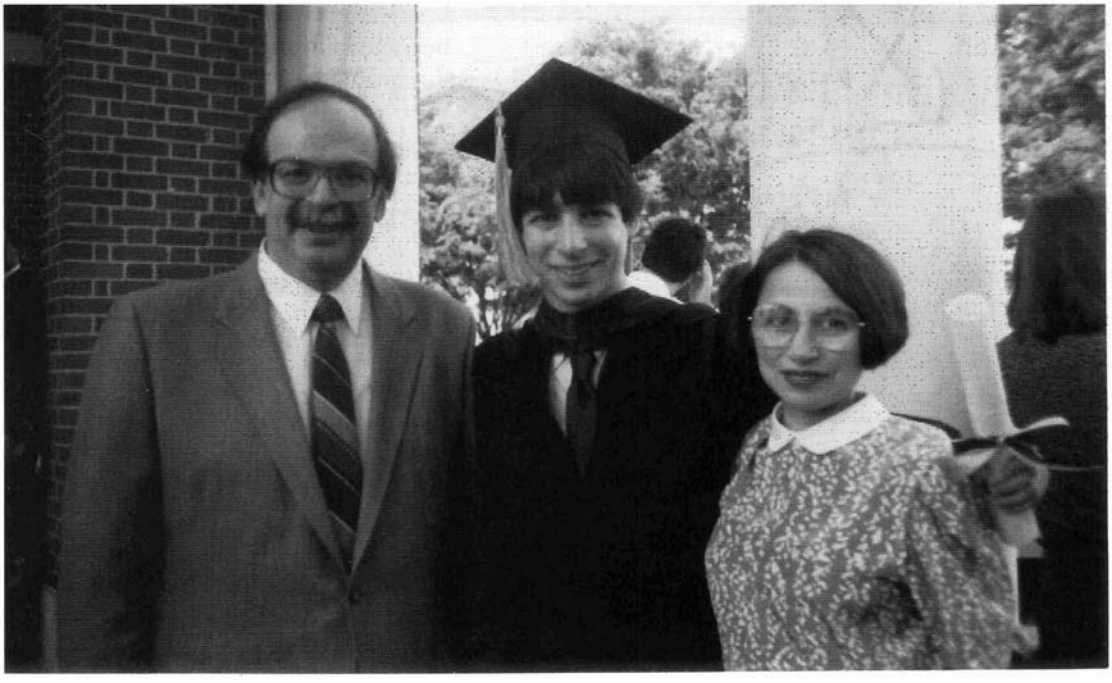
The Great Benjamini

During his middle school and high school days, Benjamin was known primarily for his magic and he performed under the name of The Great Benjamini. Initially, he performed at birthday parties for children, mostly between the ages of five and seven. Those shows involved only magic, no calculations. "I was almost a clown. I was the Great Benjamini. I would fall down and get hit over the head with wands, and make them laugh. That was my goal."

When he started doing shows for adults, he felt the need for more sophisticated material. He added card tricks and mentalism—sort of fake ESP. After he got into mentalism, his father suggested that he put some of his mental calculations into his act. Young Benjamin was skeptical of his father's suggestion and told him, "But that's not magic, that's real, Dad!" Nonetheless, he took his father's advice, and the rapid calculations part of his act got the best response. "I decided that this must be the future of my act."

"I was something of a celebrity in high school, and not always in a positive sense. Everybody knew me as the magician." He decided that when he started college, he wanted people to know him for himself. But fate intervened twice during that year to frustrate his goal. In his first month at CMU, a magic convention was held in Pittsburgh. He attended, impressed some well-known magicians with his mental calculations, and was hired to perform at the Dove and Rabbit, a new magic nightclub in Pittsburgh. He did not inform his classmates about his off-campus job, for he was determined to have them see the "other" Art Benjamin. So on campus, he threw himself into writ-

High school days: "The Great Benjamini" in action!



The newly minted Dr. Benjamin with his parents, Larry and Lenore Benjamin, on the occasion of his graduation from Johns Hopkins in 1989.

ing lyrics for "Kije" and keeping up with classes.

But in the Spring fate struck again. Benjamin was taking a course in cognitive psychology from Marcel Just and the topic of the day was "lightning calculators." Professor Just knew a few tricks himself that were designed to give the impression of mental talent. He then asked the class, "We have lots of math and science students in this room. Does anyone here know any tricks?"

"I really shouldn't have," Benjamin recalls, "I was trying to keep my magic under wraps but when somebody asked that question... I couldn't resist. I did my act from the Dove and Rabbit. The last thing I did was to square a four-digit number, something I had just introduced in my nightclub act. While I was concentrating on the calculation and had already given out the first few digits of the answer, it was so quiet, you could hear a pin drop. I then gave the rest of the answer. It was right, and there was an audible gasp. My impromptu performance dazzled them." After class, Professor Just introduced Benjamin to Professor William Chase, who was doing research on skilled memory. For the next two years, Benjamin worked with Chase, first as a subject and later as his research assistant.

My Turtle Pancho

In spite of all his amazing mental calculating ability, Benjamin insists that his memory is only a little above average. How then can he hold so many digits in his memory when multiplying two distinct four-digit numbers in his head?

But that's not magic, that's real, Dad!

Most of us can hold about seven unrelated digits in our short-term memory. When Professor Chase first tested Benjamin, he found that he could only recall 8–10 digits, just a bit above average. Chase had studied other subjects, including one man who could recall strings of 80 digits. Chase knew that such feats of recall involved the use of the grouping of digits and mnemonics (memory tricks). We can routinely recall strings of seven digits, such as when we remember telephone numbers. If we see the string 3876254, and try to commit it to memory, it's much easier to do so if we break it up into groups: 387-6254. Benjamin's calculating feats involved remembering far more than 10 digits. Chase knew that he must be using some mnemonic system.

Art can square three-digit numbers in less than a second; four-digit numbers take him about 15 seconds. Chase asked Art to explain every step of what he was doing as he squared a four-digit number. He couldn't do it, because as he explains, "The steps were so rapid that I couldn't follow them in detail." In order to slow him down, Chase asked him to square a five-digit number, something he had never done before. And that slowed him down enough that he could describe what he was doing.

For starters, Art does his calculations from left to right instead of right to left. Asked to square 46,872, Art pinches his eyes shut, clenches his fists, and paces rapidly back and forth on the stage. He utters what seem like disconnected words (his mnemonics) — *fuzz, nunnery, mover* — and then says, "two billion one hundred ninety six million." After saying these digits out loud, he never thinks about them again, depending on his audience to remember them, and freeing up space

in his memory to continue with the calculation. After more pacing, hand wringing, and strings of bizarre words, he gives the answer: "2,196,984,384."

When Art started college, he did not think that multiplying distinct three-digit numbers in his head was something that he could ever do. "I could square them, but multiplying two different ones seemed beyond me." By the time he graduated from CMU, he was multiplying distinct six-digit numbers in his head! The problem would have to be called out slowly to him so that he could create the mnemonic to remember the 12 digits of the problem. The phonetic code that he employs can be found in his book *Mathemagics: How to Look Like a Genius Without Really Trying*: 0 is the "s" or "z" sound, 1 is the "t" or "d" sound, 2 is the "n" sound, 3 is the "m" sound, etc. Next he converts numbers into words by placing vowels around or between the consonant sounds (spelling doesn't matter). For example, the number 32 can become any of the following words: *man, men, mine, moon, money, menu*, etc. Here's the code he used to translate the first 24 digits of pi:

3 1415 926 5 3 58 97
"My turtle Pancho will, my love, pick

9 3 2 384 6264
up my new mover, Ginger."

With just a little bit of practice, using the phonetic code, you can memorize over a hundred digits of pi!

Choosing a Career: Math or Magic?

Benjamin's reputation as a mathemagician, researcher, and teacher continued to grow during his undergraduate days at CMU and graduate days at Johns Hopkins University. While at Hopkins, Benjamin worked on his dissertation ("Turnpike Structures for Optimal Maneuvers") under Alan Goldman, research that later earned him the prestigious Nicholson Prize from the Operations Research Society of America. Hopkins is also where he met Deena, a fellow mathematician, whom he married 4 years later at the Magic Castle in Hollywood. Before graduate school, Benjamin had already performed on several national television shows. It's clear that he could have been a full-time magician if he had chosen to do so. But he chose mathematics and teaching to be the center of his life. He says, "I wanted to be someone who would bring math to others, popularize it, and use my entertaining talents to get more people excited about it. On top of that, I had had some teaching experience as an undergraduate. Teaching was fun; it was just like performing, but better because your repertoire changed with every lecture. I sensed that if I were to become a full-time performer, it would become a bit repetitive after a while. I love applause, no question about that, but eventually I would say to myself, 'Yes, I can do this and I can do that. But what else?' I wanted something more substantial and intellectually satisfying. I also felt that if I were going to teach mathematics professionally, I would need to go on and earn my Ph.D." It appears that he made a very wise decision, for today he is teaching, doing research, performing, and receiving applause for all of those activities.

I wanted to be someone who would bring math to others, popularize it, and use my entertaining talents to get more people excited about it.

Secrets of Benjamin

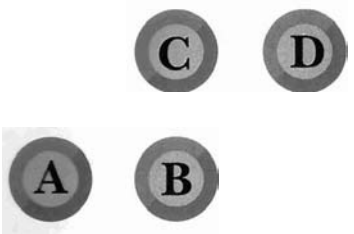
A particularly delightful aspect of Professor Benjamin is that he is eager to share his secrets of rapid mental calculations with others. Most magicians jealously guard their secrets, but not him. Several years ago he distributed within the magic community his booklet, "The Secrets of Benjamin." In 1993, he and Michael Shermer wrote *Mathemagics*, which lays out his secrets in detail. He clearly wants others to learn the art of rapid mental calculation and to come to love mathematics in the process. Can they get as good as Art Benjamin — Mathemagician? Perhaps, but that will require enormous practice and prodigious motivation. You can count on it!■

Art and Deena Benjamin relaxing at home.

Graphs, Maneuvers, and Turnpikes

If you wish to drive from my mother's house in Cleveland, Ohio to my office in Claremont, California, here's what you do. Get on the I-71 freeway South to I-70 West. Go about 1700 miles until you get to I-15 (around Beaver, Utah). Take I-15 South about 700 miles to I-10 West. Go another 20 miles and you're practically there. The point is that you will spend almost all of your time travelling on high speed roads.

A similar phenomenon takes place when maneuvering a collection of objects from one location to another. For instance, to move 4 checkers from one corner of a checkerboard to the diagonally opposite corner, you should spend almost all of your time moving the configuration



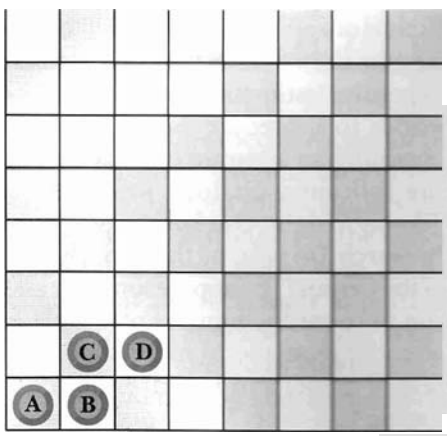
up and over in two (double-jump) moves. In general, it can be proved that the solution to any maneuvering problem on the plane obeying a few natural conditions will spend almost all of its time repeatedly translating one or two efficient configurations.

For more details, see

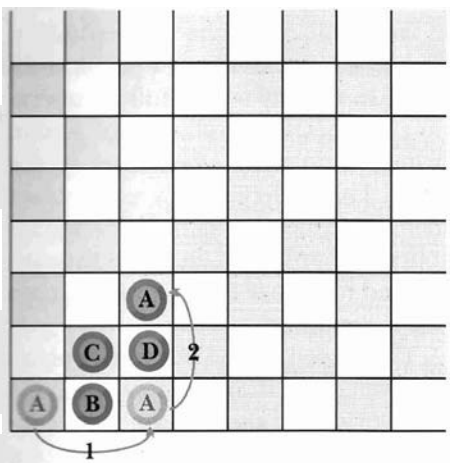
"Graphs, Maneuvers, and Turnpikes," Arthur Benjamin, *Operations Research*, vol. 38, #2 (1990), pp. 202–216.

"Optimal Leapfrogging," Arthur Benjamin with J. Auslander and D. S. Wilkerson, *Mathematics Magazine*, vol. 66, #1 (1993), pp. 14–19.

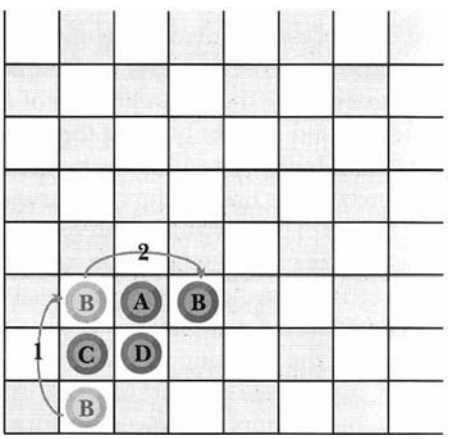
—Arthur Benjamin



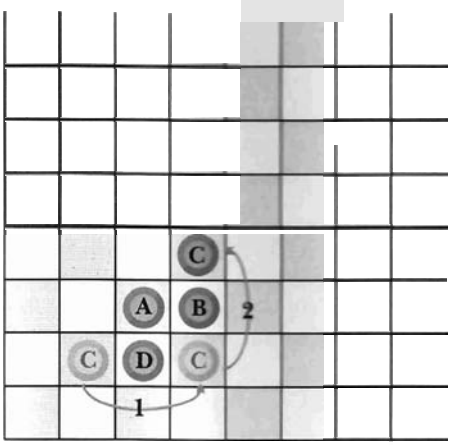
Starting position



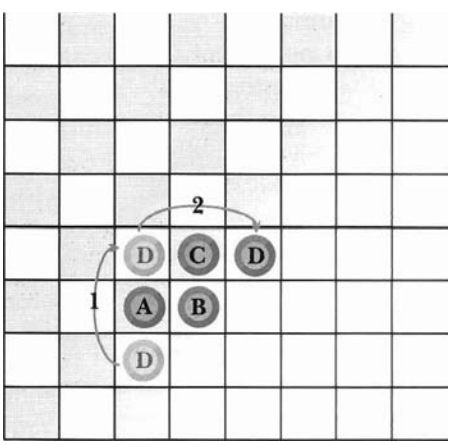
A does a double jump



B does a double jump



C does a double jump



D does a double jump