De Anza College

Mechanics
Projectiles

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De Anza College

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Last time

- motion in 2 dimensions
- relative motion

Overview

- projectile motion
- time of flight and range of a projectile

Projectiles

projectile

Any object that is thrown. We will use this word specifically to refer to thrown objects that experience a vertical acceleration g.

Assumption

Air resistance is negligible.

Why do we care?

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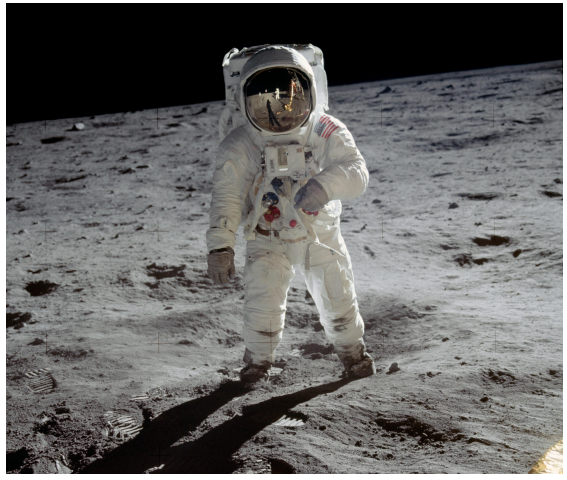

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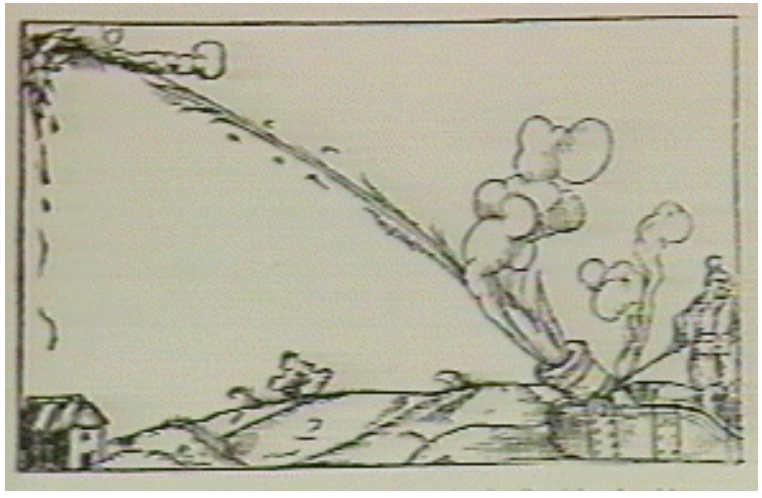
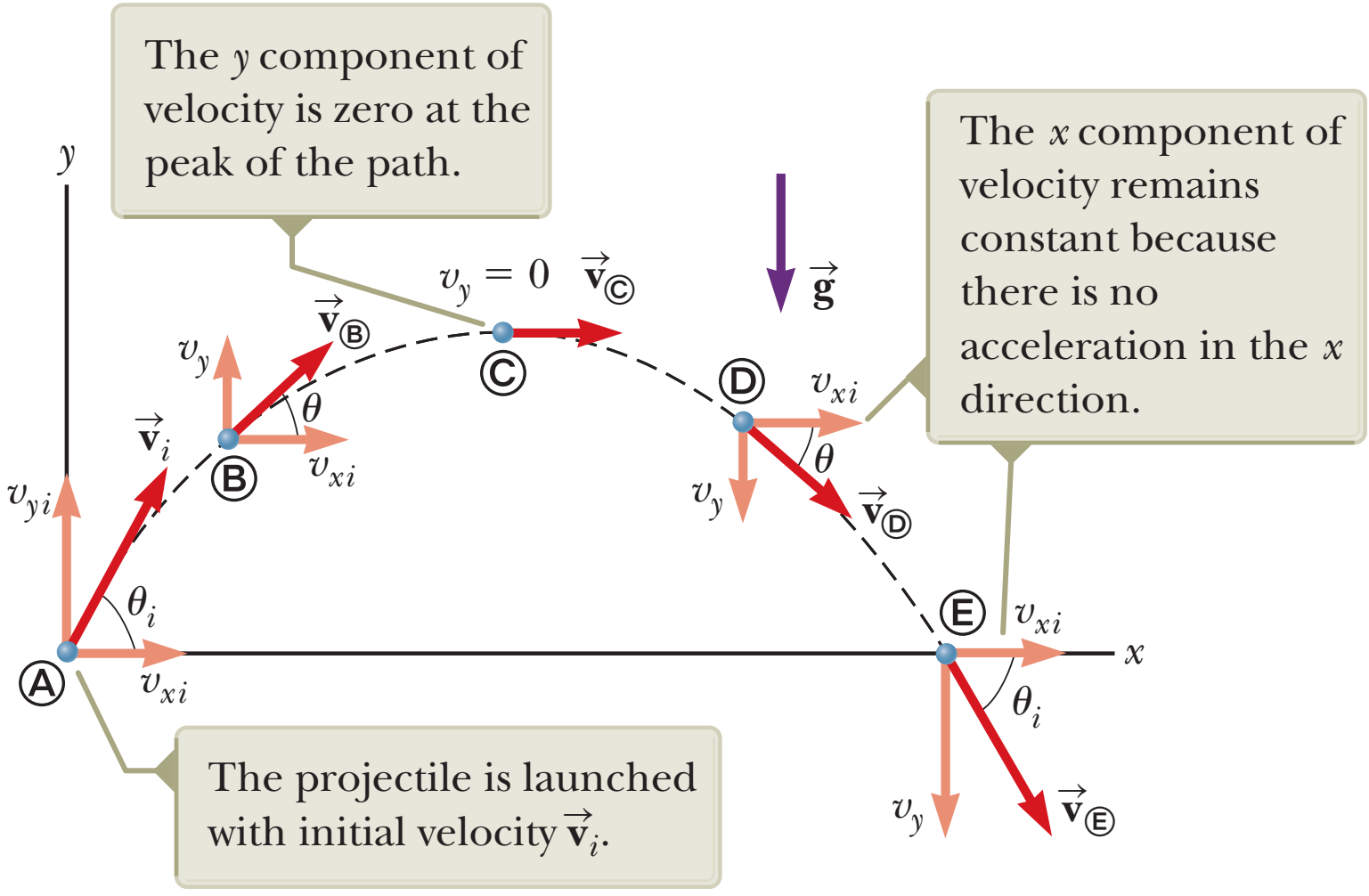


Projectile Velocity



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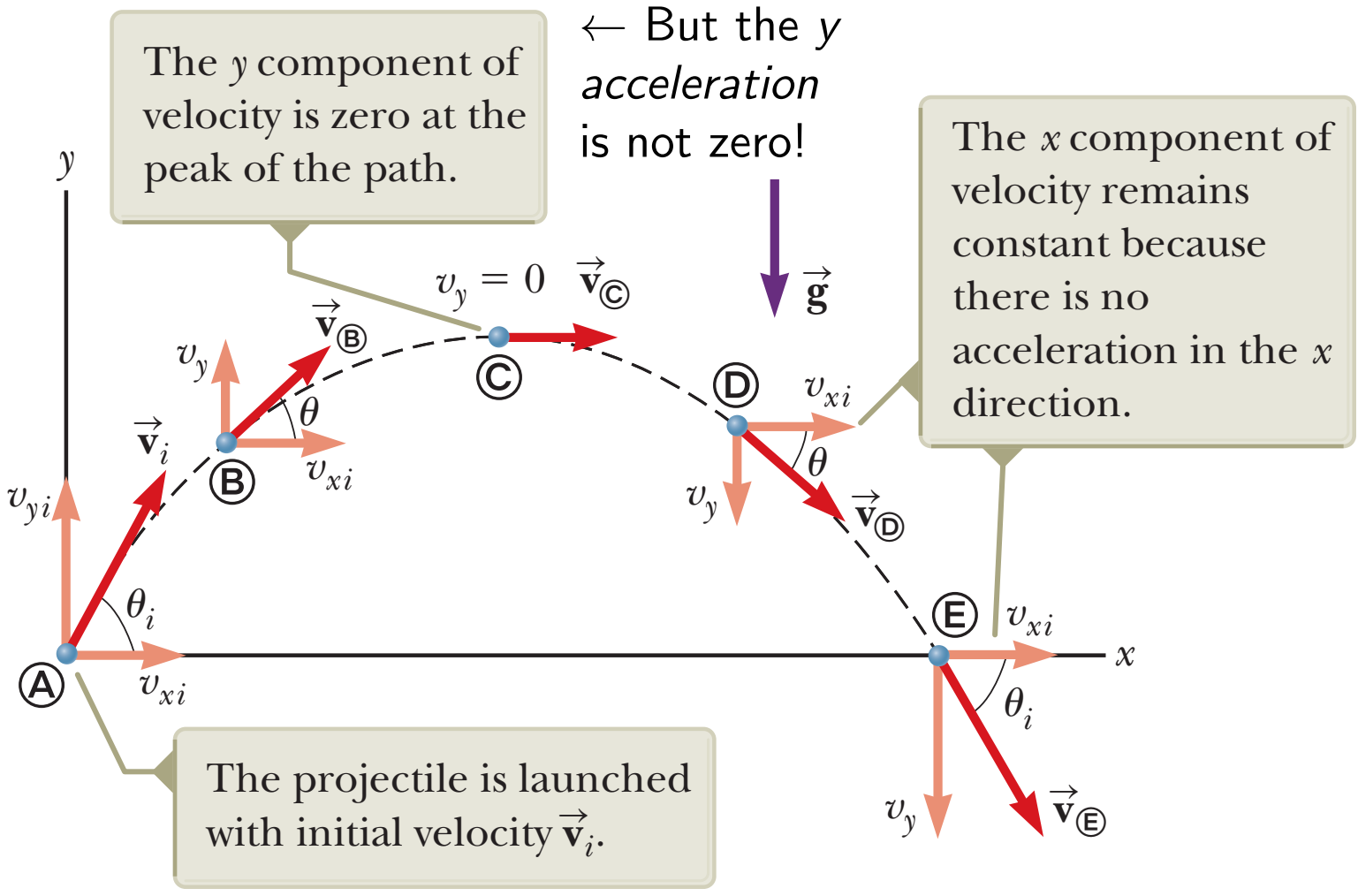
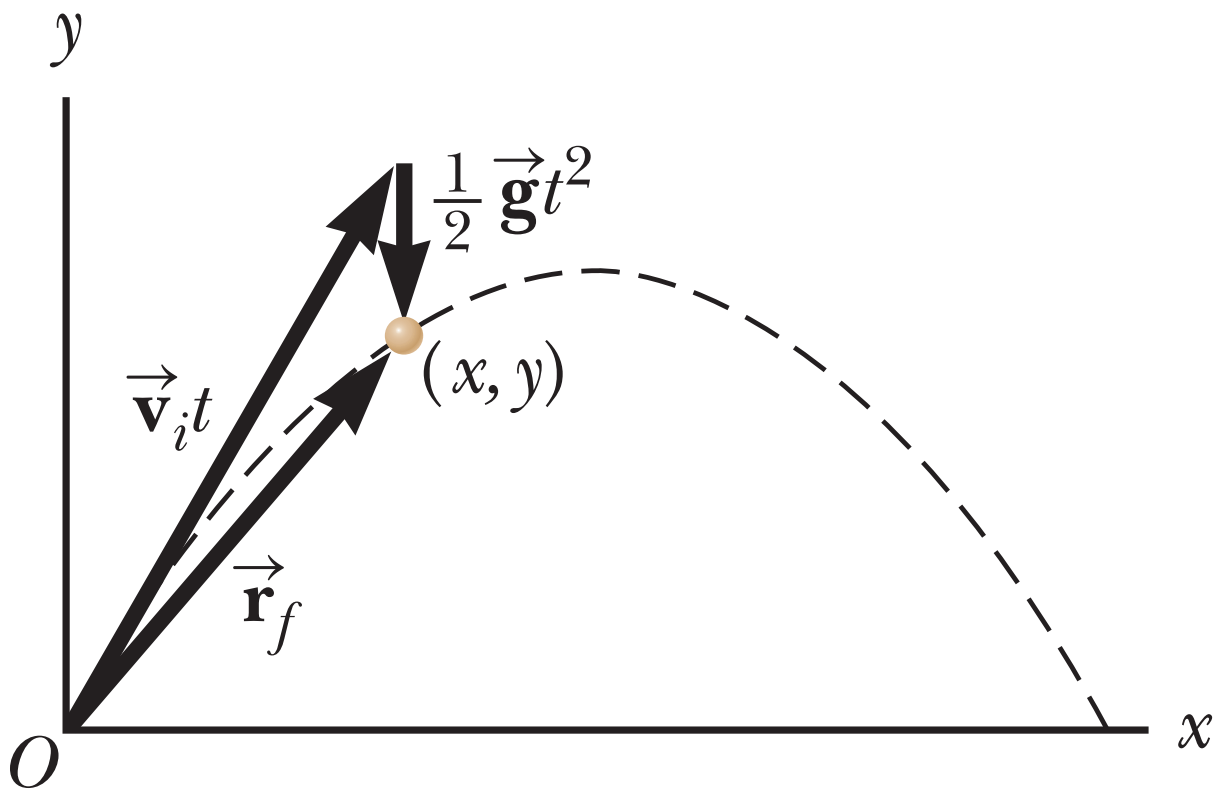


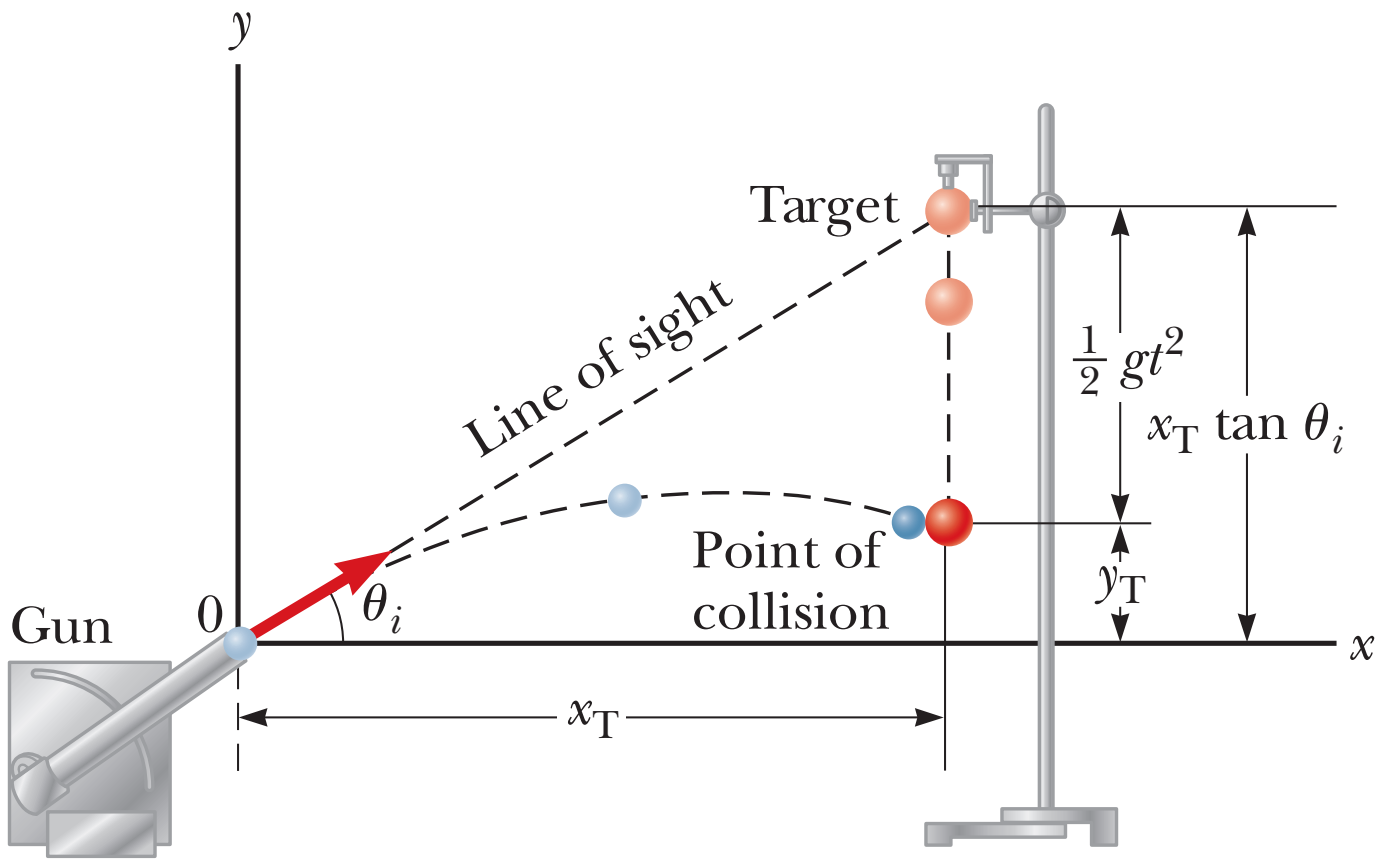
Vector Addition can give a Projectile's Trajectory



$$\Delta \boldsymbol{r} = \mathbf{r}_f - 0 = \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$$

Motion in 2 Dimensions

A method of testing that the vectors add as asserted!



$$\mathbf{r}_f = \mathbf{v}_i t - \frac{1}{2} g t^2 \mathbf{j}$$

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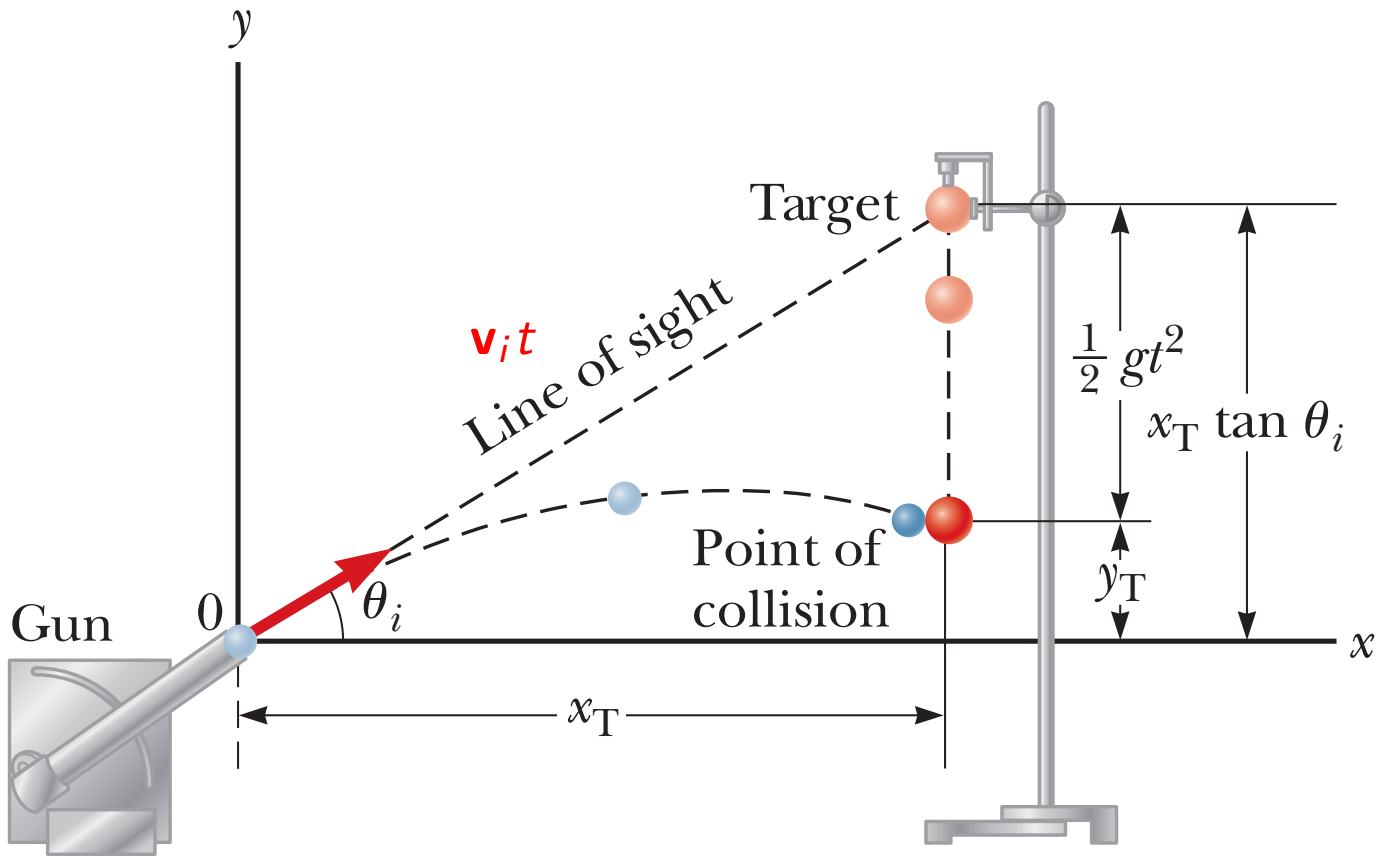


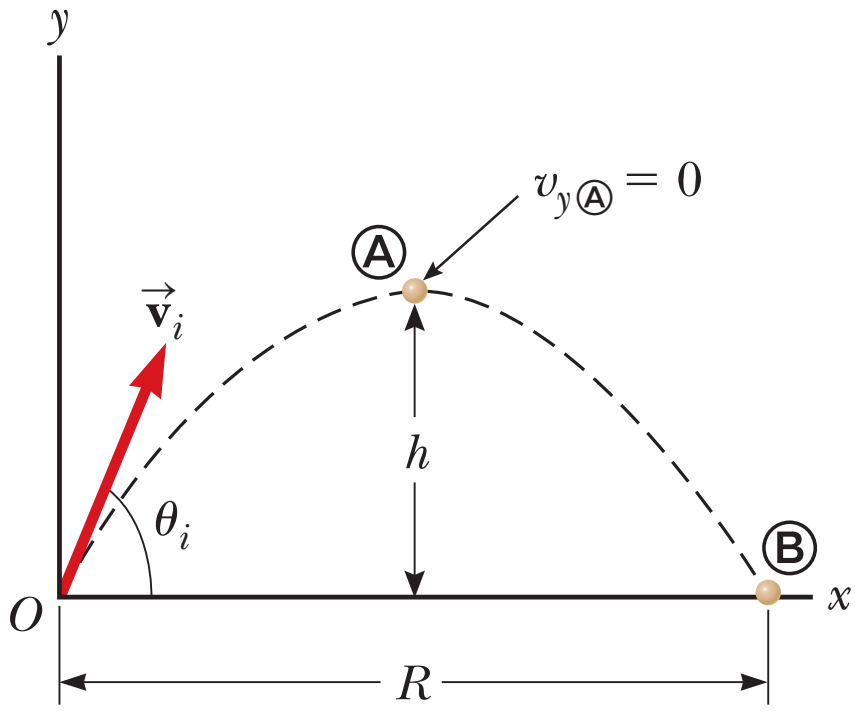
$$\mathbf{r}_f = \mathbf{v}_i t - \frac{1}{2} g t^2 \mathbf{j}$$

Time of Flight of a Projectile

time of flight

The time from launch to when projectile hits the ground.

How can we find the time of flight of a projectile?



Assuming that it is launched from the ground and lands on the ground at the same height...

Time of Flight of a Projectile

One way to find it: notice that just when striking the ground, $\Delta y = 0$.

$$\begin{aligned} \Delta y &= v_{y,i} t + \frac{1}{2} a_y t^2 \\ 0 &= v_i \sin\theta t - \frac{1}{2} g t^2 \end{aligned}$$

Now cancel a factor of t. Warning! This will remove one solution to this equation in t. What is it?

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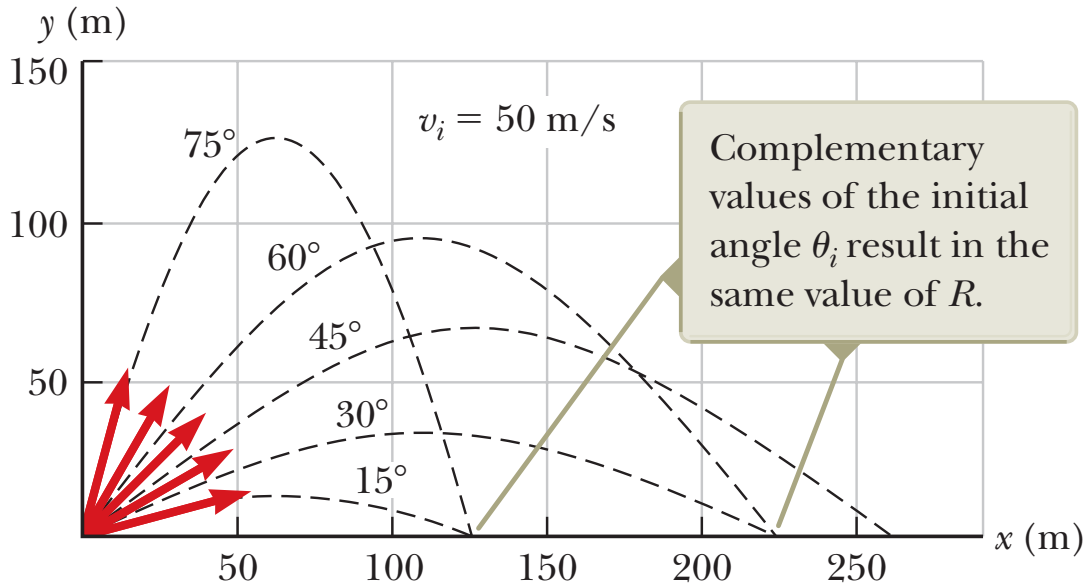
$$\frac{1}{2} g t = v_i \sin\theta$$

$$t_{\text{flight}} = \frac{2 v_i \sin\theta}{g}$$

Time of Flight of a Projectile

Quick Quiz 4.3[1] Rank the launch angles for the five paths in the figure with respect to time of flight from the shortest time of flight to the longest. (Assume the magnitude v_i remains the same.)



A 15°, 30°, 45°, 60°, 75°

B 45°, 30°, 60°, 15°, 75°

C 15°, 75°, 30°, 60°, 45°

D 75°, 60°, 45°, 30°, 15°

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[1] Page 86, Serway & Jewett

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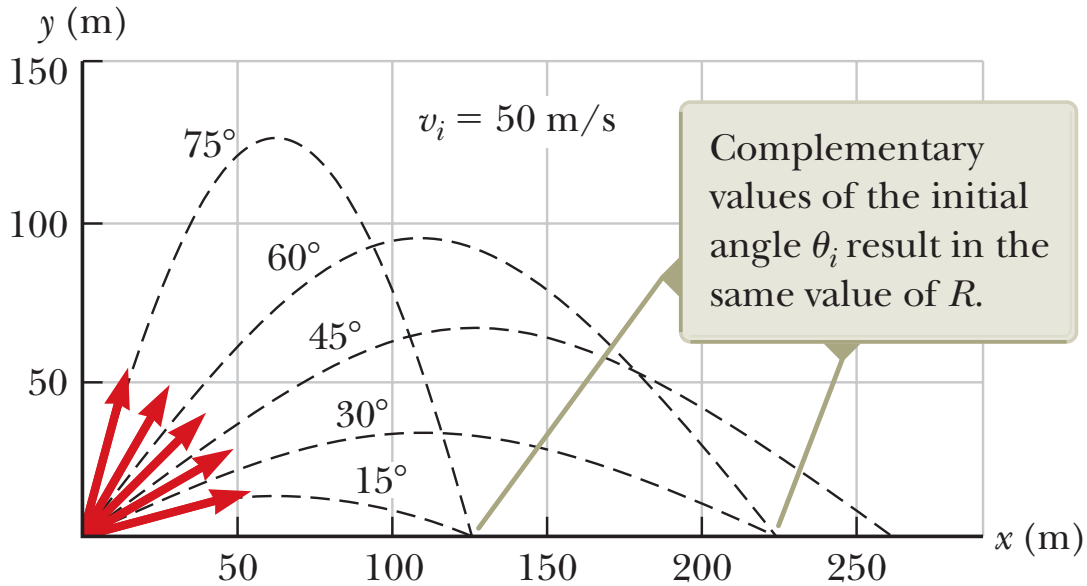

A 15°, 30°, 45°, 60°, 75° ←

B 45°, 30°, 60°, 15°, 75°

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D 75°, 60°, 45°, 30°, 15°

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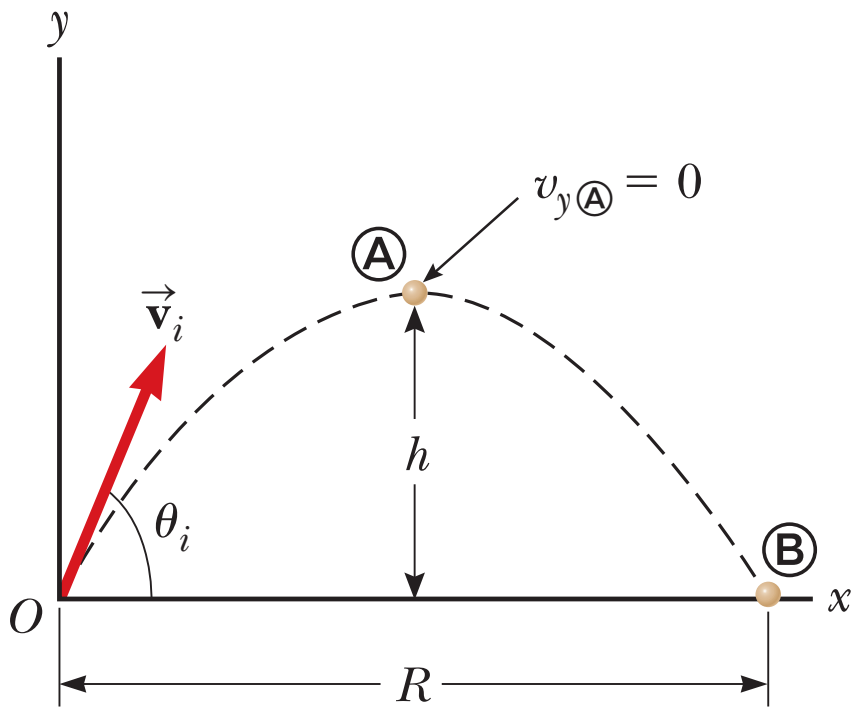
[1] Page 86, Serway & Jewett

Range of a Projectile

range

The distance in the horizontal direction that a projectile covers before hitting the ground.

How can we find the range of a projectile?



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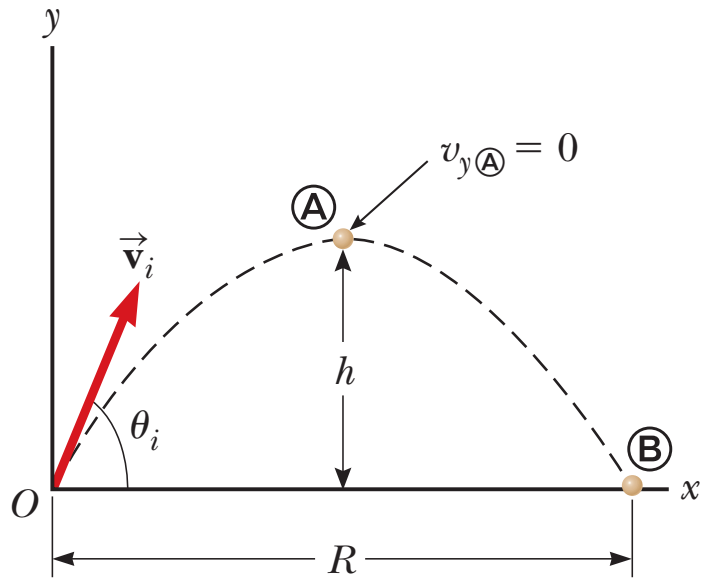
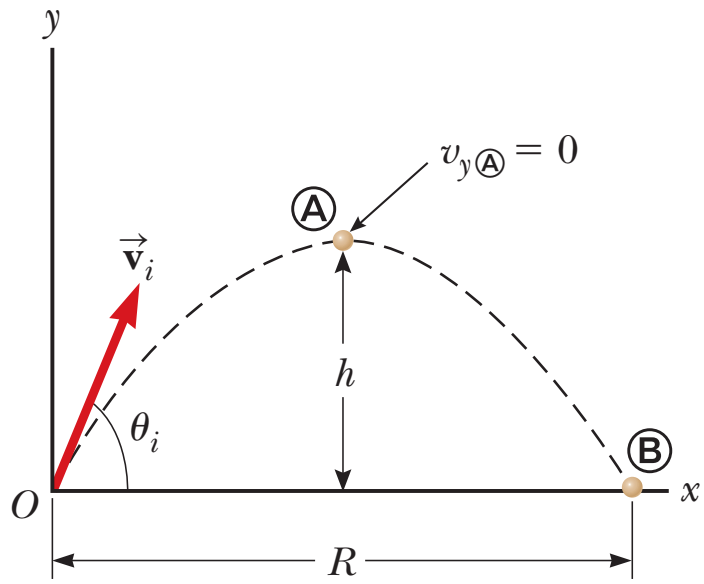


There is no acceleration in the x-direction! $(a_x = 0)$

$$\Delta x = v_x t$$

We just need t. But t is the time of flight!

Range of a Projectile



$$\Delta x = v_x t$$

$$R = v_i \cos\theta \left(\frac{2 v_i \sin\theta}{g} \right)$$

$$R = \frac{2 v_i^2 \sin\theta \cos\theta}{g}$$

$$R = \frac{v_i^2 \sin(2\theta)}{g}$$

Range of a Projectile

A long jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s. How far does he jump in the horizontal direction?

Range of a Projectile

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$$\begin{aligned} R &= \frac{v_i^2 \sin(2\theta)}{g} \\ &= \frac{(11.0 \text{ m/s})^2 \sin(2 \times 20.0)}{9.8 \text{ m/s}^2} \\ &= \underline{7.94 \text{ m}} \end{aligned}$$

Maximizing Range

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

What angle maximizes the range of the projectile?

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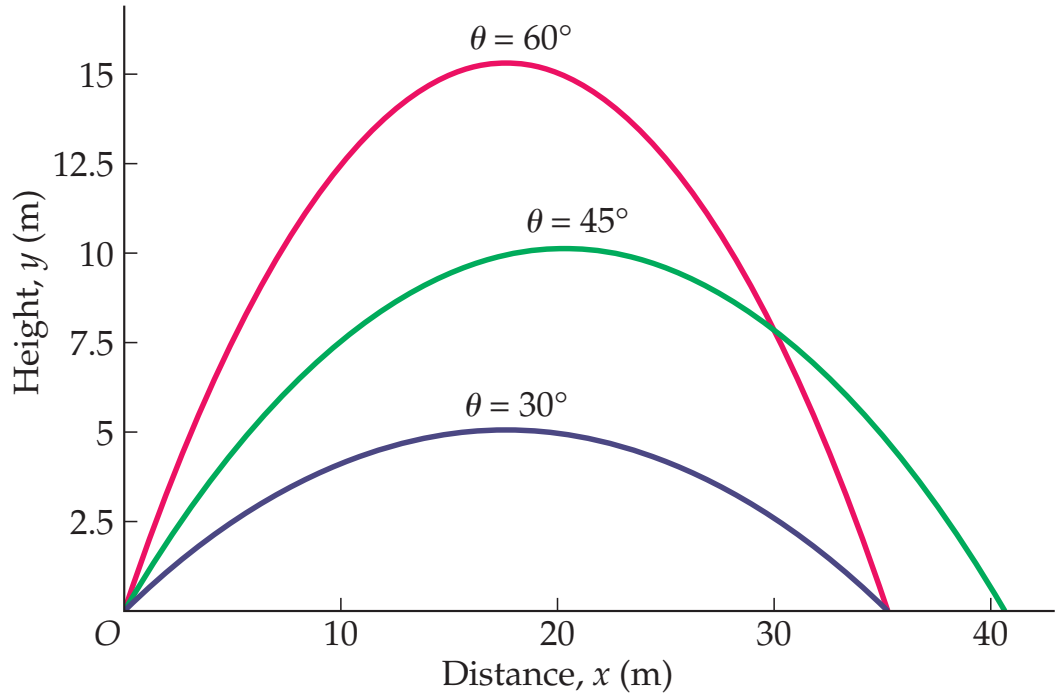
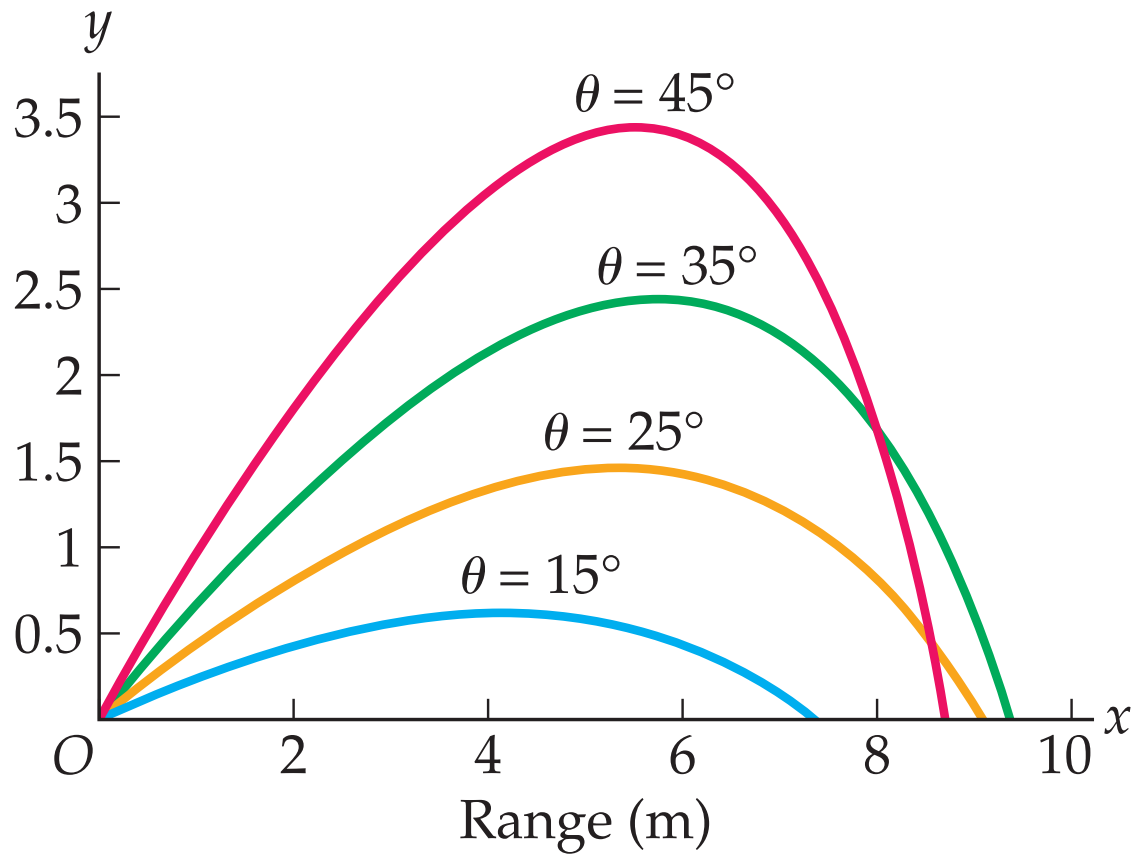


R is maximized when $\sin(2\theta) = 1 \quad \Rightarrow \quad \theta = 45°$

What Happens with Air Resistance?

The projectile's path without air resistance is a symmetrical parabola.

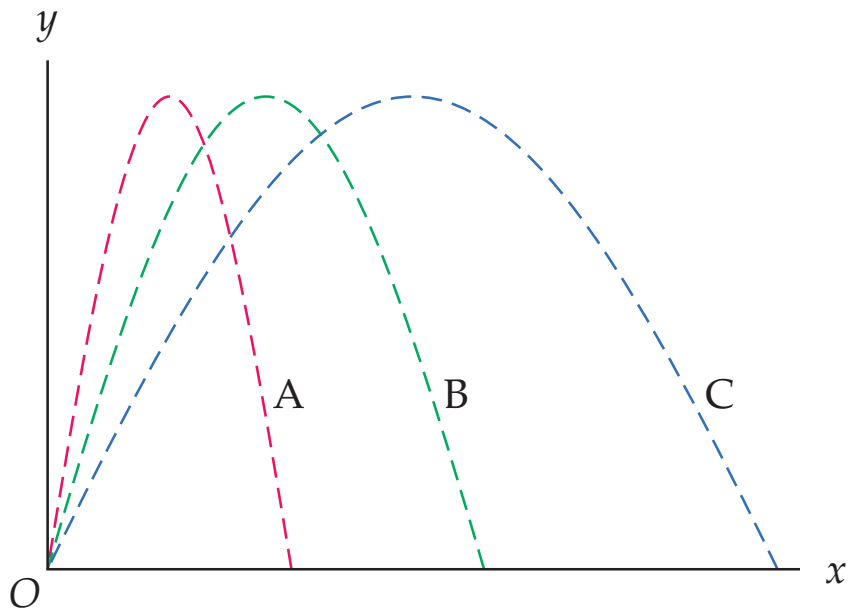
With air resistance, this is no longer the case.



[1]Figure from Walker, "Physics", page 97.

Height and initial speed conceptual question

Three projectiles (A, B, and C) are launched with different initial speeds so that they reach the same maximum height, as shown. List the projectiles in order of increasing time of flight.



(A) A, B, C

(B) C, B, A

(C) B, C, A

(D) all the same

[1]Walker, "Physics", page 106, prob 28.

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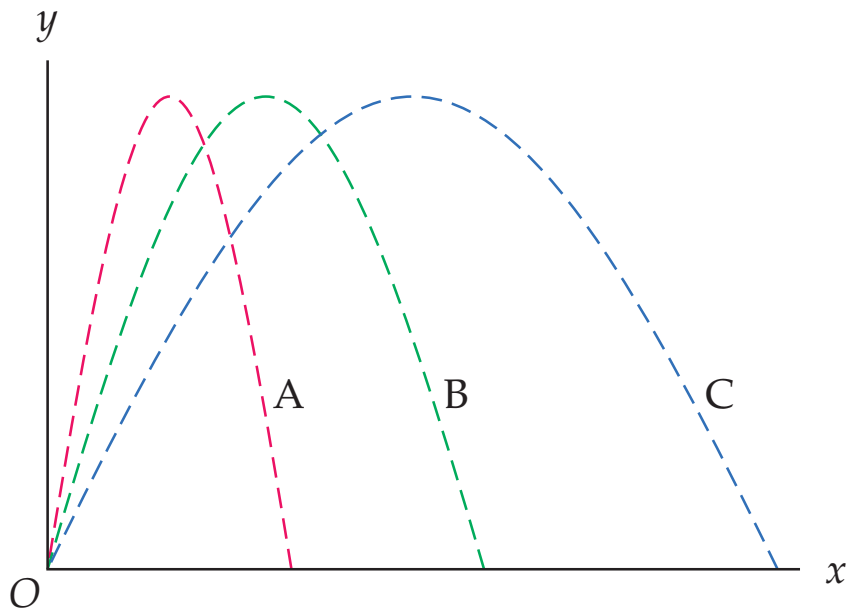

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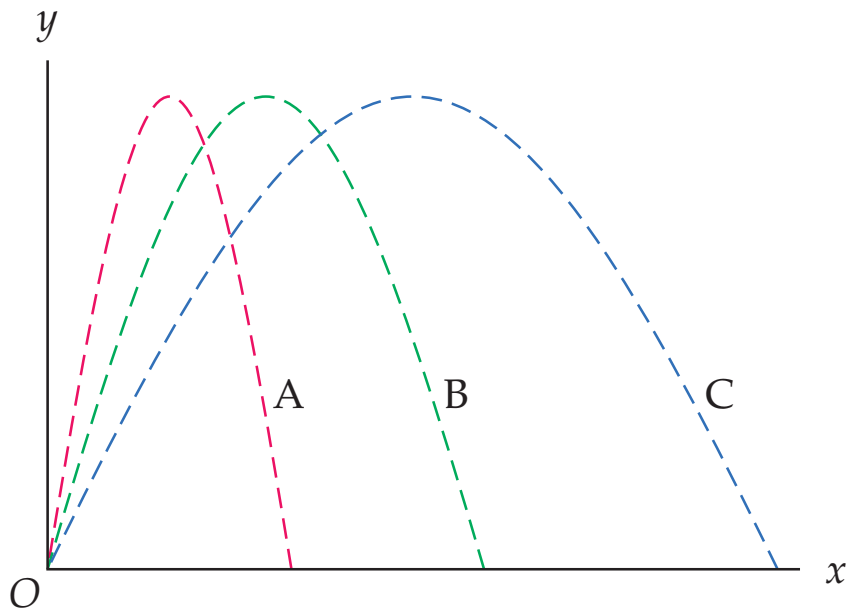

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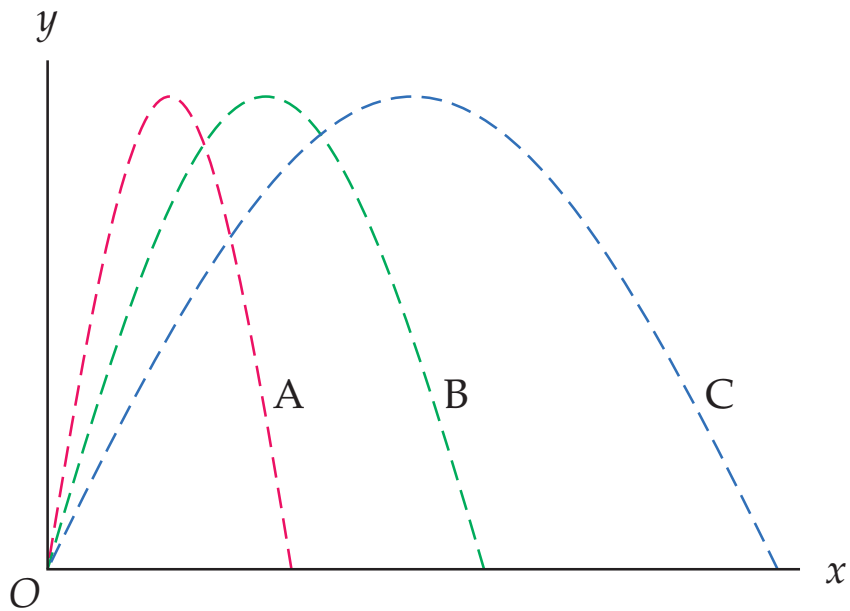

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Summary

- projectile motion

Homework

- Ch 4 Ques: 5, 7; Probs: 21, 23, 25, 27, 29, 35.