Mechanics
Relative Motion and Projectile Motion

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Last time

- projectile motion

Overview

- projectile trajectory
- projectile examples
- relative motion and projectile motion

Projectile Trajectory

Suppose we want to know the height of a projectile (relative to its launch point) in terms of its x coordinate. Suppose it is launched at an angle θ above the horizontal, with initial velocity v_i.

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For the x-direction:

$$x = v_i \cos\theta t \Rightarrow t = \frac{x}{v_i \cos\theta}$$

y-direction:

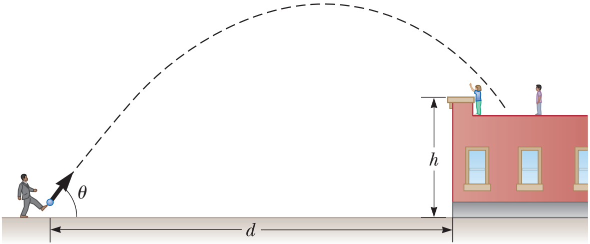
$$y = v_i \sin\theta t - \frac{1}{2} g t^2$$

Substituting for t gives:

$$y = (\tan\theta)x - \frac{g}{2v_i^2 \cos^2\theta} x^2$$

Projectile Motion Example: #25, page 103

25. A playground is on the flat roof of a city school, 6.00 m above the street below (Fig. P4.25). The vertical wall of the building is $h = 7.00$ m high, forming a 1-m-high railing around the playground. A ball has fallen to the street below, and a passerby returns it by launching it at an angle of $\theta = 53.0°$ above the horizontal at a point $d = 24.0$ m from the base of the building wall. The ball takes 2.20 s to reach a point vertically above the wall. (a) Find the speed at which the ball was launched. (b) Find the vertical distance by which the ball clears the wall. (c) Find the horizontal distance from the wall to the point on the roof where the ball lands.



[1] Serway & Jewett

Projectile Motion Example

Part (a)

Given: $\Delta x = 24.0$ m, $t = 2.20$ s, $\theta = 53.0^{\circ}$
Asked for: v_0

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Strategy: we know $\Delta x = v_{0x} t$ and $v_{0x} = v_0 \cos\theta$.

Rearranging:

$$\begin{aligned}
\Delta x &= v_0 \cos\theta t \\
v_0 &= \frac{\Delta x}{\cos\theta t} \\
&= \frac{(24.0 \text{ m})}{\cos(53^\circ)(2.20 \text{ s})} \\
&= \underline{18.1 \text{ m/s}}
\end{aligned}$$

Projectile Motion Example

Part (b)

Given: $\Delta x = 24.0$ m, $t = 2.20$ s, $\theta = 53.0^{\circ}$, $v_0 =$ m/s, $h = 7.00$ m
Asked for: height above the wall, $\Delta y - h$

Projectile Motion Example

Part (b)

Given: $\Delta x = 24.0$ m, $t = 2.20$ s, $\theta = 53.0^{\circ}$, $v_0 =$ m/s, $h = 7.00$ m
Asked for: height above the wall, $\Delta y - h$

Strategy: there are a couple of ways to solve it. One way:

$$\Delta y = v_{0y} t - \frac{1}{2} g t^2$$

$$\begin{aligned}
\Delta y - h &= v_0 \sin\theta\, t - \frac{1}{2} g t^2 - h \\
&= (18.1 \text{ m/s}) \sin(53^{\circ})(2.20 \text{ s}) - \frac{1}{2}(9.81)(2.20)^2 - 7\text{m} \\
&= \underline{1.13 \text{ m}}
\end{aligned}$$

Projectile Motion Example

Part (c)

Given: $\theta = 53.0^\circ$, $v_0 = \text{m/s}$, $\Delta y = 6.00\text{ m}$
Asked for: distance behind the wall, $\Delta x - d$

Projectile Motion Example

Part (c)

Given: $\theta = 53.0^\circ$, $v_0 =$ m/s, $\Delta y = 6.00$ m
Asked for: distance behind the wall, $\Delta x - d$

Strategy: Could find t, then Δx. Or, use trajectory equation:

$$y = (\tan\theta)x - \frac{g}{2v_i^2\cos^2\theta}x^2$$

Projectile Motion Example

Solve

$$\frac{g}{2v_i^2\cos^2\theta}x^2 - (\tan\theta)x + y = 0$$

for x.

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Putting in the numbers in the question:

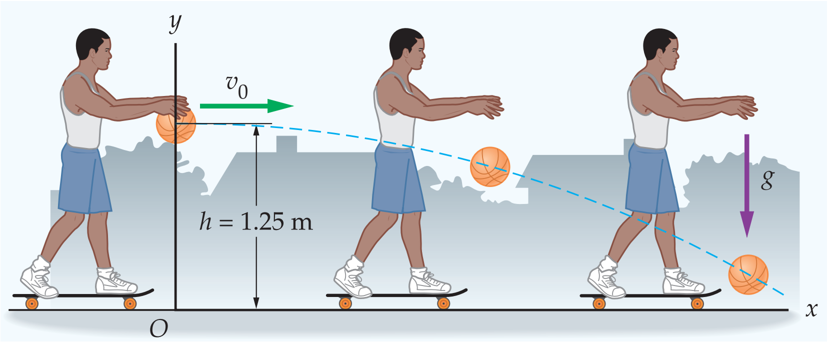
$$x = 26.79 \text{ m} \quad \text{or} \quad x = 5.44 \text{ m}$$

Want the solution that is larger than 24 m, since ball makes it onto roof.

Distance behind wall: 26.79 m - 24 m = $\underline{2.79 \text{ m}}$.

Relative Motion And Projectile Motion

Observer on the skateboard sees the ball fall straight down.



Another observer on the sidewalk sees the ball as a horizontally launched projectile.

Relative Motion And Projectile Motion

To decide who pays for lunch, a passenger on a moving train tosses a coin straight upward with an initial speed of 4.38 m/s and catches it again when it returns to its initial level. From the point of view of the passenger, then, the coin's initial velocity is $(4.38 \text{ m/s})\,\mathbf{j}$. The train's velocity relative to the ground is $(12.1 \text{ m/s})\,\mathbf{i}$.

(a) What is the minimum speed of the coin relative to the ground during its flight? At what point in the coin's flight does this minimum speed occur? Explain.

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12.1 m/s, At the top of its path, where the y-component of velocity is zero.

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$\mathbf{v}_0 = 12.9$ m/s, at 19.9° above the horizontal

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Summary

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- relative motion and projectile motion

Homework

- previously set: Ch 4 Ques: 5, 7; Probs: 21, 23, 25, 27, 29, 35.
- new: Ch 4 Ques: 9; Probs: 39, 43, 49.
- new: Ch 4 Probs: 57, 59, 67 (circular motion - wait to do)