Mechanics
More Circular Motion

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Last time

- spring force
- circular motion

Overview

- banked turn
- vertical loop
- introducing energy

A Banked Turn

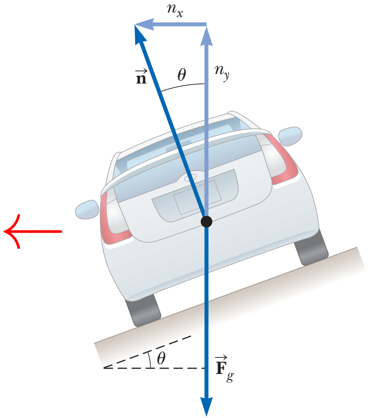
Curved roadways are often not flat. The are often **banked**, that is sloped at an angle to the horizontal.

This is so that a component of the normal force on the car can help provide some or all of the centripetal force.

[0]Photo from Walker, "Physics".

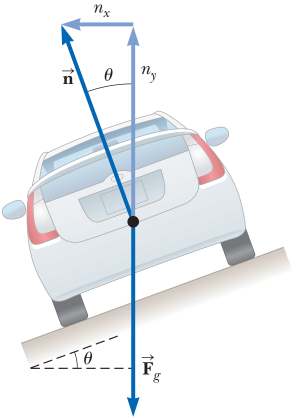
A Banked Turn

Sharp turns in roads are often banked inwards to assist cars in making the turn: the centripetal force comes from the normal force, not friction.



A Banked Turn

A turn has a radius r. What should the angle θ be so that a car traveling at speed v can turn the corner without relying on friction?



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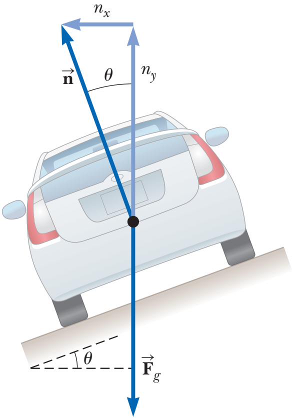
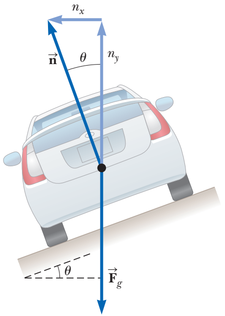


Hint: consider what the net force vector must be in this case.

A Banked Turn



y-direction (vertical, positive up):

$$\begin{aligned} F_{y,\text{net}} &= 0 \\ n_y - mg &= 0 \end{aligned}$$

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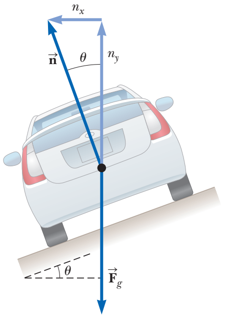

y-direction (vertical, positive up):

$$\begin{aligned} F_{y,\text{net}} &= 0 \\ n_y - mg &= 0 \\ n\cos\theta &= mg \\ n &= \frac{mg}{\cos\theta} \end{aligned}$$

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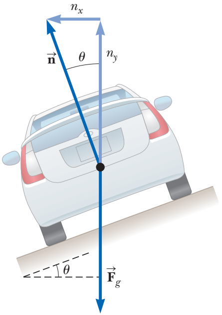
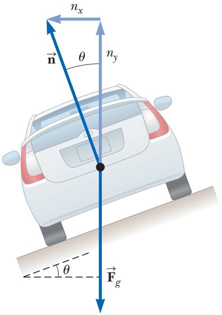


x-direction (horizontal, positive left):

$$F_{x,\text{net}} = ma_x$$
$$n_x = \frac{mv^2}{r}$$

A Banked Turn



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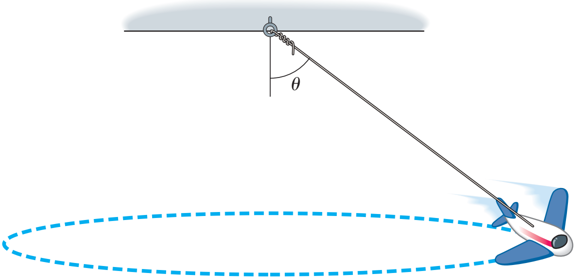
$$\begin{aligned}
F_{x,\text{net}} &= ma_x \\
n_x &= \frac{mv^2}{r} \\
n \sin\theta &= \frac{mv^2}{r} \\
\frac{mg}{\cos\theta}\sin\theta &= \frac{mv^2}{r} \\
\tan\theta &= \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)
\end{aligned}$$

Banked Turn Related Problems

This situation is called a "conical pendulum". But notice, it is actually a banked-turn-style problem in disguise!

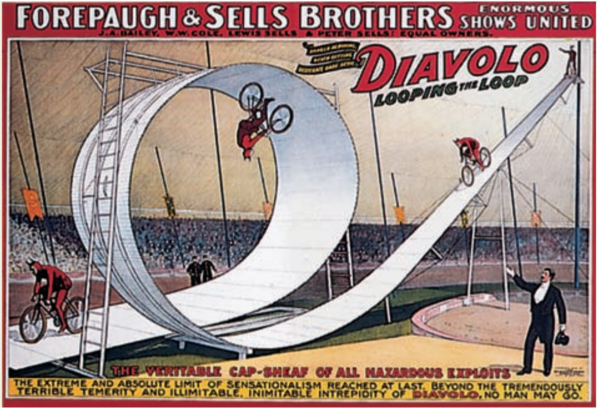


The role that was played by the normal force in the banked turn problem is now played by the tension in the string.

[1]See probs 51, 70, Ch 6.

Vertical Loop

How can the daredevil ride through the loop upside down without falling?



[1]Picture from Halliday, Resnick, Walker, 9th ed.

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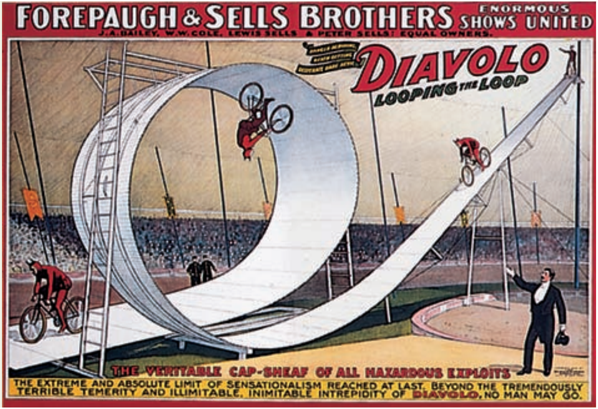


If the daredevil's speed is high enough, the centripetal acceleration needed to *keep* on the circle can be greater than g.

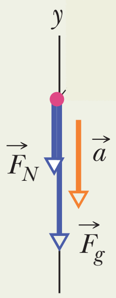
[1]Picture from Halliday, Resnick, Walker, 9th ed.

Vertical Loop

If (s)he is on the verge of being able to make it around, the place (s)he would just start to fall is the top of the loop. At the top of the loop:

System:

daredevil + bicycle



y-direction:

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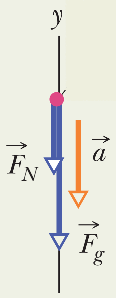

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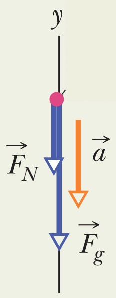

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$$\begin{aligned} -mg &= -\frac{mv^2}{r} \\ v &= \sqrt{rg} \end{aligned}$$

Vertical Loop

As long as (s)he manages to keep a speed of $v = \sqrt{rg}$ at the top of the track, the daredevil will not fall.

It doesn't depend on the daredevil's mass!

Energy

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One way that energy is often described is that it represents **the ability of a system to do work**.

We need to know what work is!

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The amount of work, W, done on an object depends on the applied force and the displacement of the object as the force acts.

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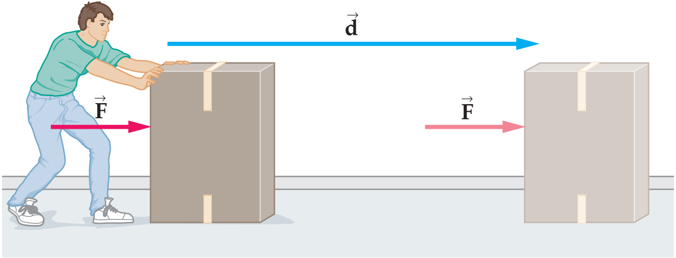
If the force is in the same direction as the displacement,

$$W = Fd$$



Work

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Units of Work?

They have a special name: Joules, symbol J.

1 joule = 1 J = 1 Nm

Work is not a vector. Work is a scalar.

Summary

- banked turn
- vertical loop
- introduced energy

Homework

- Ch 6 Prob: 48[1], 51, 57, 70

[1]Prob 48, assume the car is locked onto the rails so that the "normal force" from the track on the car could up or down. Answers: (a) 3.7×10^3 N, (b) up, (c) 1.3×10^3 N, (d) down.