# Mechanics 

Units, Dimensional Analysis, and Unit
Conversion

Lana Sheridan<br>De Anza College

Sept 25, 2018

## Last time

- introduced the course
- basic ideas about science and physics


## Overview

- introduce SI units
- unit conversion
- dimensional analysis


## Quantities, Units, Measurement

If we want to make precise quantitative statements we need to agree on measurements: standard reference units.

We will mostly use SI (Système International) units:
Length meter, m
Mass kilogram, kg
Time second, s
and many more!

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and many more!

In all your answers, make sure you include the appropriate units!

## Dimensional Analysis

All measurement values have associated units.

Equations relating measurable physical values also relate units.

This means units on each side of the equals sign must be equal.

Dimensional analysis allows us to:

- check our equation is correct
- check our calculation
- figure out the final units of an answer if we can't remember what they should be

This is very important!

## Dimensional Analysis Examples

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Equation:

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x=v t
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Speed is a length covered in an amount of time: $[\mathrm{m}] /[\mathrm{s}]$.

## Dimensional Analysis Examples

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The dimensions of our equation:

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[\mathrm{m}]=\frac{[\mathrm{m}]}{[\mathrm{s}]}[\mathrm{s}]
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Speed is a length covered in an amount of time: $[\mathrm{m}] /[\mathrm{s}]$.
The dimensions of our equation:

$$
[\mathrm{m}]=\frac{[\mathrm{m}]}{[8]}[8]
$$

So,

$$
[\mathrm{m}]=[\mathrm{m}] \checkmark
$$

## Dimensional Analysis Examples

The other way to do dimensional analysis is directly in terms of the SI units that will be used.

$$
v^{2}=v_{0}^{2}+2 a x
$$

$a$ is an acceleration, the rate of change of speed.
Units: meters per second squared, $\mathrm{m} / \mathrm{s}^{2}$
Speed, $v$, has units $\mathrm{m} / \mathrm{s}$.

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& {\left[\mathrm{m}^{2} / \mathrm{s}^{2}\right]=\left[\mathrm{m}^{2} / \mathrm{s}^{2}\right]+\left[\mathrm{m}^{2} / \mathrm{s}^{2}\right]}
\end{aligned}
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## Dimensional Analysis Question

Which of the following equations are dimensionally correct?
(1) $v_{f}=v_{i}+a x$
(2) $y=(2 m) \cos (k x)$, where $k=2 m^{-1}$.

A (1) only
B (2) only
C both
D neither
${ }^{1}$ Serway \& Jewett, Page 16, \# 9.

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## Dimensional Analysis Question

(1) Units of $v_{f}=v_{i}+a x$ :

$$
\begin{aligned}
{\left[\mathrm{ms}^{-1}\right] } & =\left[\mathrm{ms}^{-1}\right]+\left[\mathrm{ms}^{-2}\right] \times[\mathrm{m}] \\
{\left[\mathrm{ms}^{-1}\right] } & =\left[\mathrm{ms}^{-1}\right]+\left[\mathrm{m}^{2} \mathrm{~s}^{-2}\right]
\end{aligned}
$$

No. (1) is not dimensionally correct.

## Dimensional Analysis Question

(2) Units of $y=(2 m) \cos (k x)$

$$
\begin{aligned}
{[\mathrm{m}] } & =[\mathrm{m}] \times \cos \left(\left[\mathrm{m}^{-1}\right] \times[\mathrm{m}]\right) \\
{[\mathrm{m}] } & =[\mathrm{m}]
\end{aligned}
$$

Yes. (2) is dimensionally correct.

## Scale of Units



## Unit Scaling/Conversion Examples

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What is $3 \mathrm{~g} / \mathrm{cm}^{3}$ in $\mathrm{kg} / \mathrm{m}^{3}$ ?

$$
\left(3 \mathrm{~g} / \mathrm{cm}^{3}\right) \times\left(\frac{1 \mathrm{~kg}}{1000 \mathrm{~g}}\right) \times\left(\frac{100 \mathrm{~cm}}{1 \mathrm{~m}}\right)^{3}=3000 \mathrm{~kg} / \mathrm{m}^{3} .
$$

## Scientific Notation

An alternate way to write numbers is in scientific notation.
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In scientific notation, we could write this as:

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In scientific notation, we could write this as:

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$$

This is the same thing.

$$
10^{8}=100,000,000
$$

so,

$$
3.00 \times 100,000,000=300,000,000 \mathrm{~m} / \mathrm{s}
$$

## Scientific Notation: One digit only before decimal!

One reason to use scientific notation is to clearly convey the number of significant figures in a value.

There is one digit, followed by a decimal point, followed by more digits, if there is more than one significant figure.

Here there are two significant figures:

$$
3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}
$$

Here there are 4 significant figures:

$$
\begin{aligned}
& 2.998 \times 10^{8} \mathrm{~m} / \mathrm{s} \\
& \text { one digit }
\end{aligned}
$$

one digit before the decimal +3 digits after the decimal $=4$ s.f.s

## Scientific Notation vs Unit Scaling Prefixes

In scientific notation,

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3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}
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Alternatively, we could write this with a unit prefix:

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where 1 Mm is one mega-meter,

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Alternatively, we could write this with a unit prefix:

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where 1 Mm is one mega-meter, or use kilometers:

$$
300,000 \mathrm{~km} / \mathrm{s}
$$

or use a prefix with scientific notation:

$$
3.00 \times 10^{5} \mathrm{~km} / \mathrm{s}
$$

## Unit Conversion Examples

It may be necessary to change units several times to get to the unit you need.

What is $60.0 \mathrm{mi} / \mathrm{hr}$ in $\mathrm{m} / \mathrm{s}$ ? ( mi is miles, hr is hours)

## Unit Conversion Examples

It may be necessary to change units several times to get to the unit you need.

What is $60.0 \mathrm{mi} / \mathrm{hr}$ in $\mathrm{m} / \mathrm{s}$ ? ( mi is miles, hr is hours)
$1 \mathrm{mi}=1.609 \mathrm{~km}$

## Summary

- units and dimensional analysis
- scientific notation
- unit conversion

Quiz start of class this Thursday.

## Homework

- Get the textbook: Fundamentals of Physics Extended, Halliday, Resnick, and Walker (9th Edition).
- Ch 1, Problems: 1, 3, 9, 23, 27.

