Mechanics
Energy
Work

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Last time

- more circular motion with forces
  - banked turns
  - vertical loops
- introduced energy

Overview

- work
- the vector dot product
- net work
- work done by a varying force (?)

Work

Work is an amount of energy.

The amount of work, W, done on an object depends on the applied force and the displacement of the object as the force acts.

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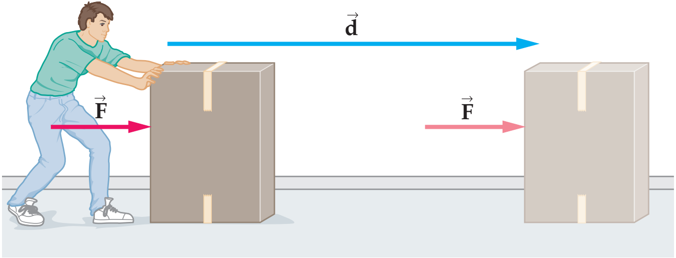
The amount of work, W, done on an object depends on the applied force and the displacement of the object as the force acts.

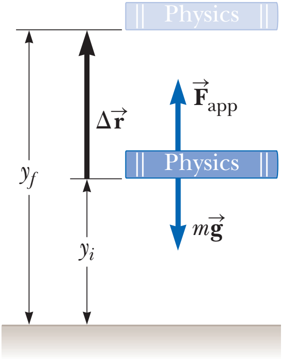
If the force is in the same direction as the displacement,

$$W = Fd$$



Example

What is the work done in lifting a 3.0-kg book 0.50 m?



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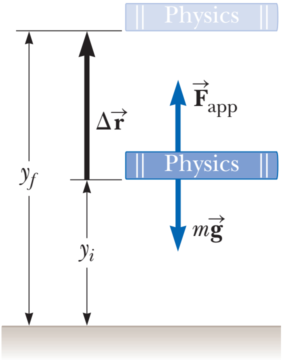


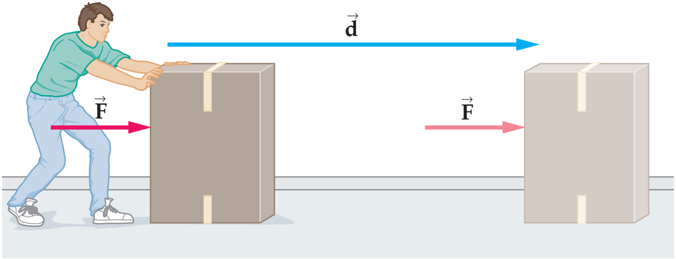
To lift a book with constant velocity, requires $F_{\text{app}} = mg$

$$W = Fd = (3.0\ \text{kg})(9.81\ \text{m/s}^2)(.50\ \text{m}) = 14.7\ \text{J}$$

Work

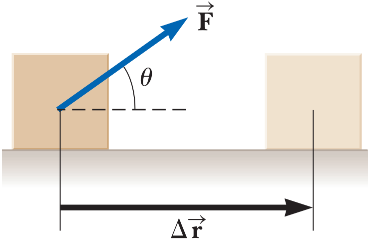
If the force, F, is in the **same direction** as the displacement, the work done by F is given by

$$W = Fd$$



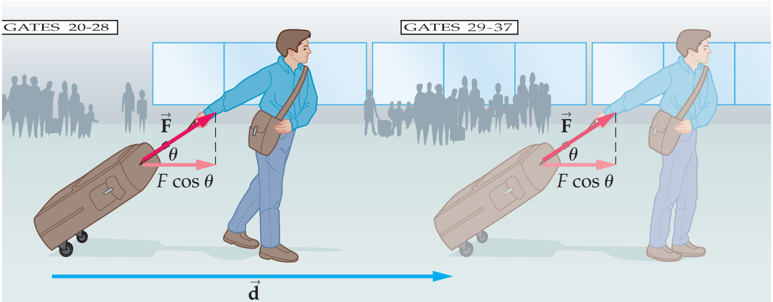
Work

What if the force is not in the direction of the displacement?



We need to extend our definition of work.

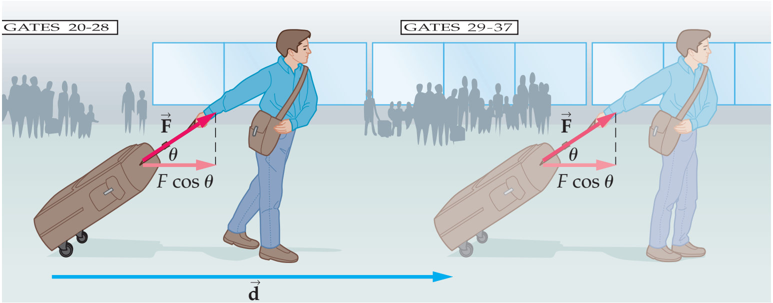
Work



For a constant applied force, *Work* is defined as:

$$W = \mathbf{F} \cdot \mathbf{d} = Fd\cos\theta$$

Work



For a constant applied force, *Work* is defined as:

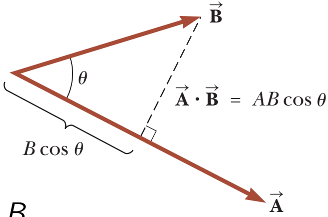
$$W = \mathbf{F} \cdot \mathbf{d} = Fd \cos \theta$$

In the above expression we use something called the **dot product** of two vectors.

Vectors Properties and Operations

Multiplication by a vector:

The Dot Product



Let $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$
$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j}$,

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y$$

The output of this operation is a **scalar**.

Equivalently,

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$$

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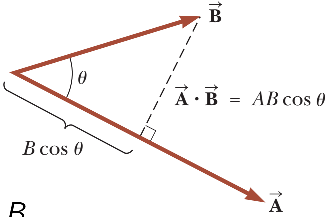

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Properties

- The dot product is commutative: $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
- If $\mathbf{A} \parallel \mathbf{B}$, $\mathbf{A} \cdot \mathbf{B} = AB$.
- If $\mathbf{A} \perp \mathbf{B}$, $\mathbf{A} \cdot \mathbf{B} = 0$.

Vectors Properties and Operations

Multiplication by a vector: The Dot Product

Try it! Find $\mathbf{A} \cdot \mathbf{B}$ when $\mathbf{A}$ is a vector of magnitude 6 N directed at 60° above the x-axis and B is a vector of magnitude 2 m pointed along the x-axis.

Multiplication by a vector: The Dot Product

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$$\mathbf{A} \cdot \mathbf{B} = (6 \text{ N})(2 \text{ m}) \cos(60^\circ) = 6 \text{ J}$$

$(1 \text{ J} = 1 \text{ Nm})$

Vectors Properties and Operations

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Now find $\mathbf{A} \cdot \mathbf{B}$ when:

$$\mathbf{A} = 1\,\mathbf{i} + 2\,\mathbf{j} \; ; \quad \mathbf{B} = -1\,\mathbf{i} - 4\,\mathbf{j}$$

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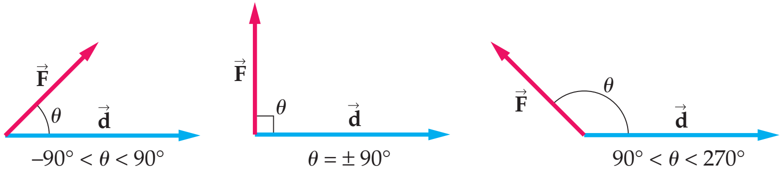
$$\mathbf{A} \cdot \mathbf{B} = (1)(-1) + (2)(-4) = -9$$

Units of Work

Work can be positive or negative!



$W = Fd\cos\theta > 0$ — positive work

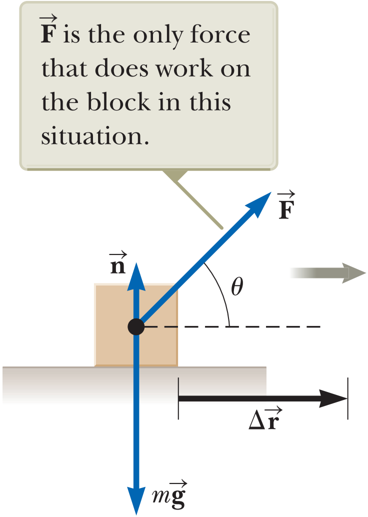
$W = Fd\cos\theta = 0$ — zero work

$W = Fd\cos\theta < 0$ — negative work

For work done *on* a system:

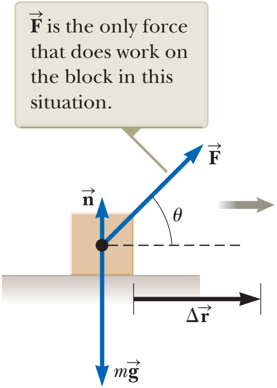
- Positive $\Rightarrow$ energy is transferred *to* the system.
- Negative $\Rightarrow$ energy is transferred *from* the system.

Work done by individual forces



If there are several forces acting on a system, each one can have an associated work.

Work done by individual forces



$$W_n = 0 \qquad W_{mg} = 0 \qquad W_F = Fd\cos\theta$$

In other words, we can ask what is the work done on the system by each force separately.

Net Work

The **net work** is the sum of all the individual works.

$$W_{\text{net}} = \sum_i W_i$$

where $W_i = F_i d \cos\theta$ is the work done by the force F_i.

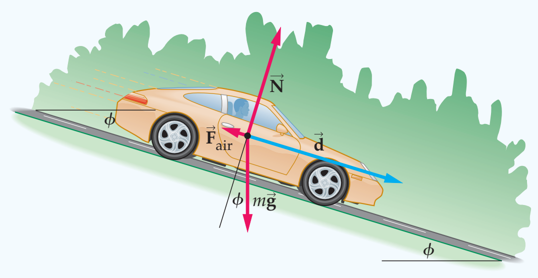
If the system can be modeled as a particle (the only case we consider in this course):

$$W_{\text{net}} = F_{\text{net}} d \cos\theta$$

assuming the net force is constant.

Question

A car speeds up as it coasts down a hill that makes an angle ϕ to the horizontal.



The work done by the weight ($m\mathbf{g}$ force) is

(A) positive

(B) negative

(C) zero

(D) cannot be determined

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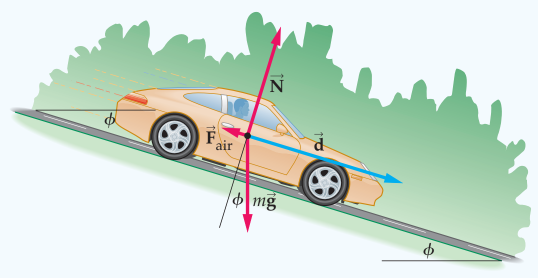

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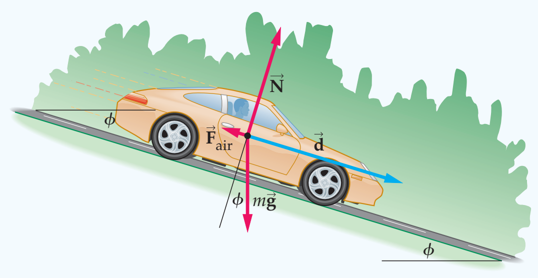

The work done by the normal force, **N**, is

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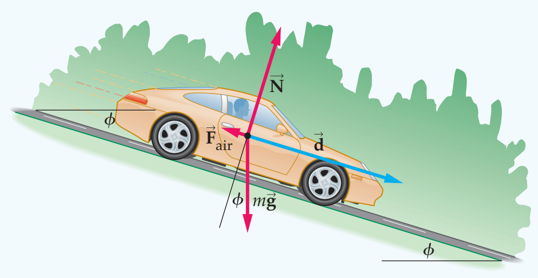
(B) negative

(C) zero

(D) cannot be determined

Question

A car speeds up as it coasts down a hill that makes an angle ϕ to the horizontal.



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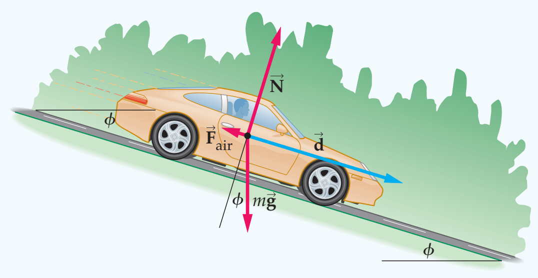

The work done by the air resistance ($\mathbf{F}_{air}$ force) is

(A) positive

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(C) zero

(D) cannot be determined

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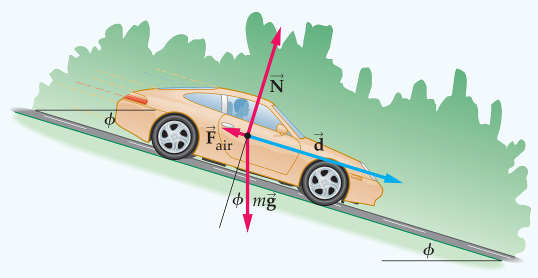

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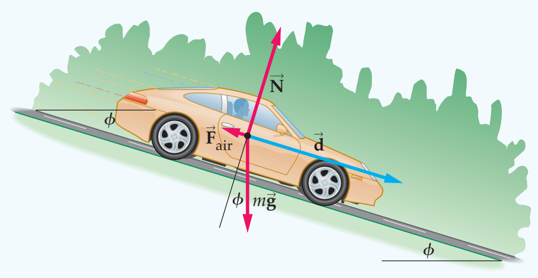

The net (or total) work done by all forces on the car is

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Question

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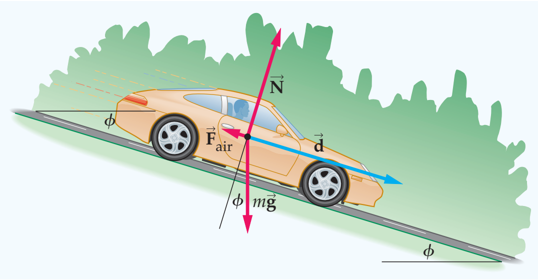

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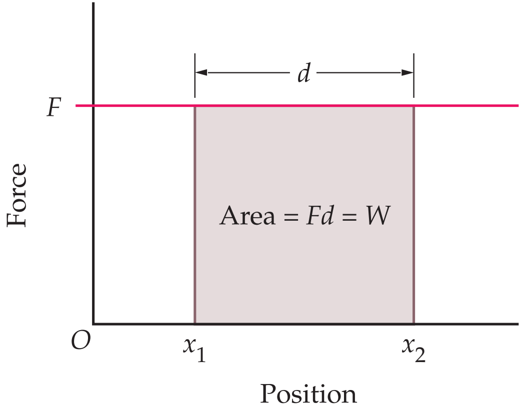
Work Done by a Variable Force

We can understand that the work done by a force is the area under the force-displacement curve.

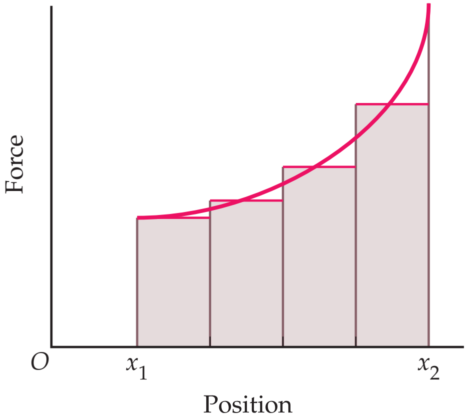
Plotting a constant force F as a function of x ($\Delta x = d$), $F(x)$:



Work Done by a Variable Force

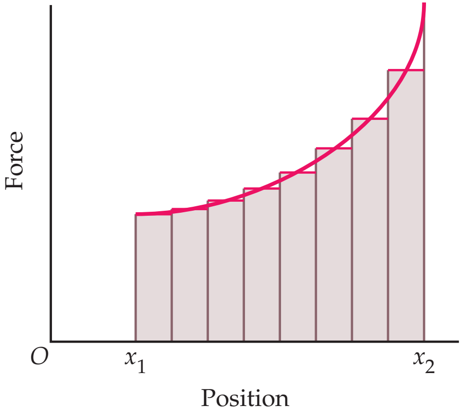
We can also apply this idea when $F(x)$ is not constant.

We can approximate the area under the curve by breaking it up into rectangles and adding the area of each rectangle.



Work Done by a Variable Force

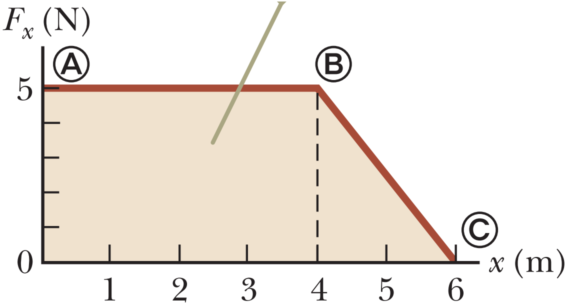
The approximation becomes more accurate when we break it up into more rectangles.



It we break it up into an infinite number of infinitesimally thin rectangles, we will be evaluating the *integral* of the force with respect to the displacement of the object.

Question

What is the work done by the force indicated in the graph as the particle moves from $x = 0$ to $x = 6$ m?



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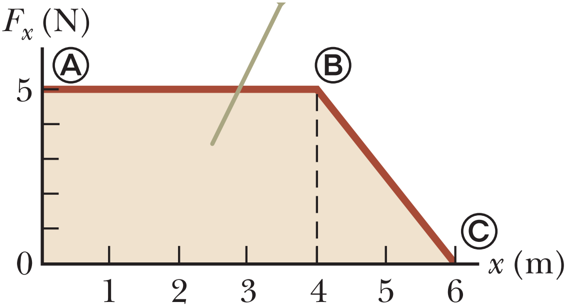
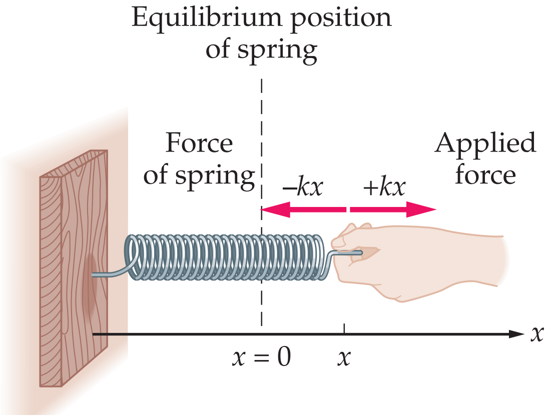


$W = 25$ J.

Work Done stretching a Spring

One important example of a force that varies with an object's displacement is the spring force.



$$F_{\text{app}} = kx$$

Work Done stretching a Spring

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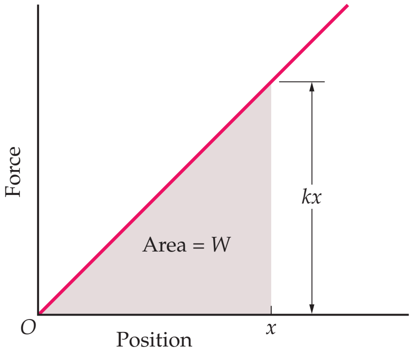
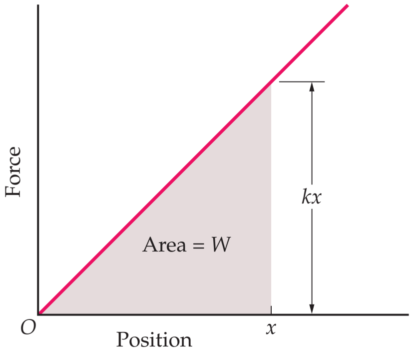


What is the work done by the applied force in stretching the spring a distance x?

Work Done stretching a Spring

$$F_{\text{app}} = kx$$



What is the work done by the applied force in stretching the spring a distance x?

$$W_{\text{app}} = \frac{1}{2}kx^2$$

Summary

- work
- the vector dot product
- net work
- work done by a varying force

Homework

- Ch 7 Ques: 3, 5; Prob: 8, 14, 18, 23

[1]Answers: **8.** 5.0×10^3 J, **14.** 15.3 J, **18.** a) 36 kJ, b) 2.0×10^2 J