Mechanics
Energy
Kinetic Energy and Work

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De Anza College

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Last time

- work
- the vector dot product
- net work

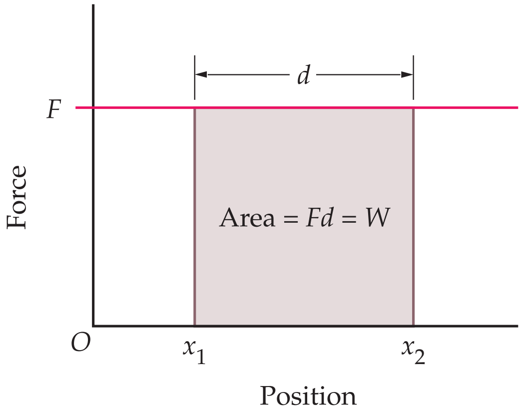
Overview

- work done by a varying force
- kinetic energy
- the work-kinetic energy theorem
- power

Work Done by a Variable Force

We can understand that the work done by a force is the area under the force-displacement curve.

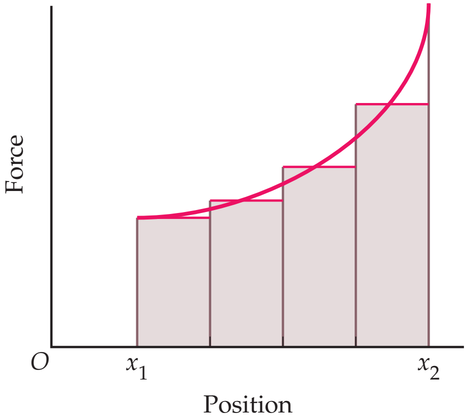
Plotting a constant force F as a function of x ($\Delta x = d$), $F(x)$:



Work Done by a Variable Force

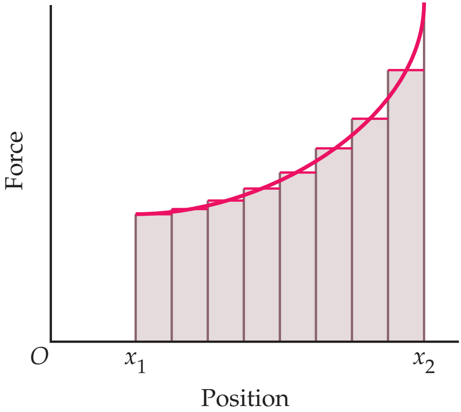
We can also apply this idea when $F(x)$ is not constant.

We can approximate the area under the curve by breaking it up into rectangles and adding the area of each rectangle.



Work Done by a Variable Force

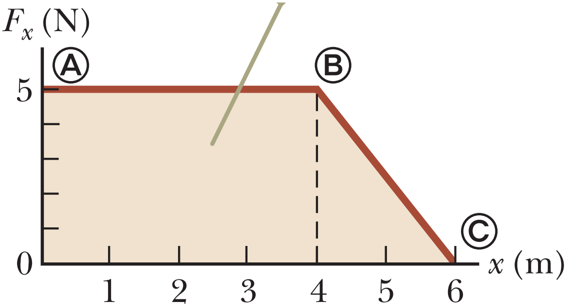
The approximation becomes more accurate when we break it up into more rectangles.



It we break it up into an infinite number of infinitesimally thin rectangles, we will be evaluating the *integral* of the force with respect to the displacement of the object.

Question

What is the work done by the force indicated in the graph as the particle moves from $x = 0$ to $x = 6$ m?



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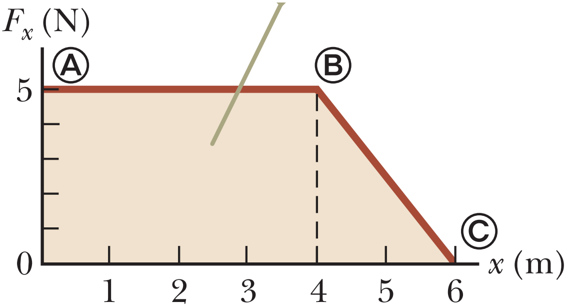
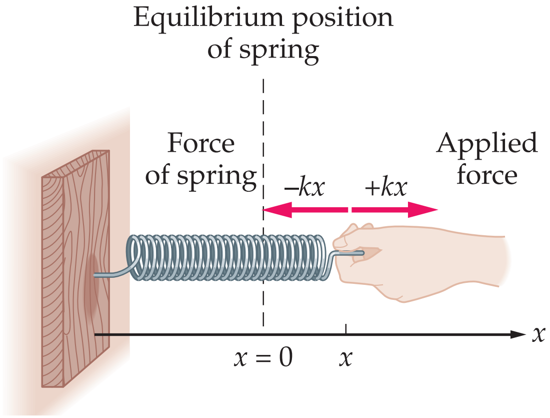


$W = 25$ J.

Work Done stretching a Spring

One important example of a force that varies with an object's displacement is the spring force.



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Work Done stretching a Spring

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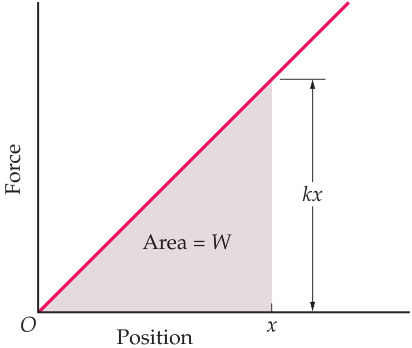

What is the work done by the applied force in stretching the spring a distance x?

Work Done stretching a Spring

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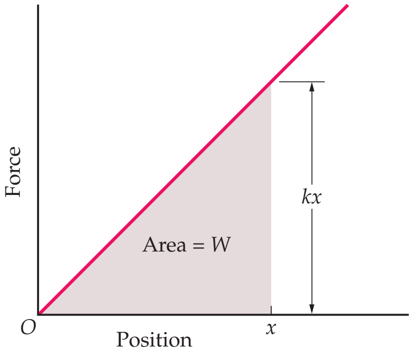

What is the work done by the applied force in stretching the spring a distance x?

$$W_{\text{app}} = \frac{1}{2}kx^2$$

Kinetic Energy

Objects in motion have energy. We call this energy kinetic energy.

Kinetic energy, K

the energy that a system has as a result of its motion, or the motion of its constituent parts.

$$K = \frac{1}{2}mv^2$$

Work and Kinetic Energy

The net work done on a (particle) system is the total energy that is transferred to the system from the environment. Let the system's mass be m.

How much work does the environment do in accelerating the system from from speed v_i to speed v_f?

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$$\begin{aligned} W_{\text{net}} &= F_{\text{net}}\, d \\ &= (ma)d \end{aligned}$$

We can express the acceleration of a particle in terms of the initial and final speeds of the particle and the particle's displacement:

$$v_f^2 = v_i^2 + 2ad$$

So,

$$a = \frac{v_f^2 - v_i^2}{2d}$$

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Using this in our expression for the net work done:

$$\begin{aligned}
W_{\text{net}} &= F_{\text{net}}\ d \\
&= (ma)d \\
&= m\left(\frac{v_f^2 - v_i^2}{2d}\right)d \\
&= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2
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&= K_f - K_i \\
&= \Delta K
\end{aligned}$$

Work-Kinetic Energy Theorem

For accelerating from v_i to v_f

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So,

$$W_{\text{net}} = \Delta K$$

This is the **Work-Kinetic Energy Theorem**, which could also be stated as:

"When the environment does work on a system and the only change in a system is in its speed, the net work done on the system equals the change in kinetic energy of the system."

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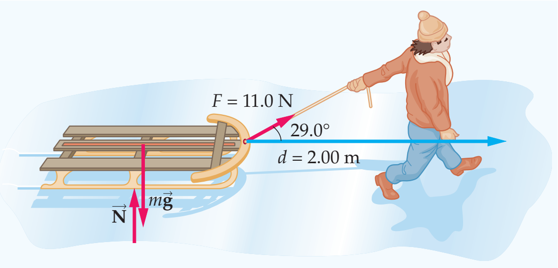
We are treating the system **as a particle**. (There is now way to define a *potential energy* in that case.)

Work-Kinetic Energy Theorem Example

A boy exerts a force of 11.0 N at 29.0° above the horizontal on a 6.40-kg sled. Find the work done by the boy and the final speed of the sled after it moves 2.00 m, assuming the sled starts with an initial speed of 0.500 m/s and slides horizontally without friction.



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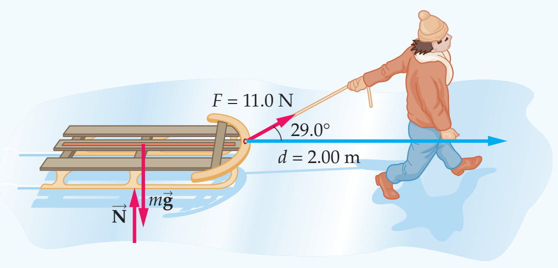

$$W_{\text{boy}} = Fd\cos(29.0^\circ) = (11.0\ \text{N})(2.00\ \text{m})\cos(29.0^\circ) = 19.2\ \text{J}$$

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$$W_{\text{net}} = \cancelto{0}{W_{\mathbf{N}}} + \cancelto{0}{W_{m\mathbf{g}}} + W_{\text{boy}} = 19.2 \text{ J}$$

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$$\begin{aligned}
W_{\text{net}} &= \Delta K \\
&= \frac{1}{2} m (v_f^2 - v_i^2) \\
v_f^2 &= \frac{2W_{\text{net}}}{m} + v_i^2 \\
v_f &= \sqrt{\frac{2(19.2\ \text{J})}{(6.40\ \text{kg})} + (0.500\ \text{m/s})^2} \\
&= \underline{2.50\ \text{m/s}}
\end{aligned}$$

Power

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The *average power* is defined as

$$P = \frac{W}{\Delta t}$$

Unit: the Watt. 1 J/s = 1 W

Power Example

Ch 7, # 44.

•44 A skier is pulled by a towrope up a frictionless ski slope that makes an angle of 12° with the horizontal. The rope moves parallel to the slope with a constant speed of 1.0 m/s. The force of the rope does 900 J of work on the skier as the skier moves a distance of 8.0 m up the incline. (a) If the rope moved with a constant speed of 2.0 m/s, how much work would the force of the rope do on the skier as the skier moved a distance of 8.0 m up the incline? At what rate is the force of the rope doing work on the skier when the rope moves with a speed of (b) 1.0 m/s and (c) 2.0 m/s?

[0] Halliday, Resnick, Walker, 9th ed

Power Example

(a) constant velocity $\Rightarrow \mathbf{F}_{\text{net}} = 0$
Whether the skier moves up a 1 m/s or 2 m/s, the force is the same, the displacement is the same, so the work is the same:

$$W = 900 \text{ J}$$

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(c) $t = \frac{8 \text{ m}}{2 \text{ m/s}} = 4 \text{ s}$

$$P_{\text{avg}} = \frac{W}{t} = \frac{900 \text{ J}}{4 \text{ s}} = \underline{2.3 \times 10^2 \text{ W}}$$

Power

$$P = \frac{W}{t}$$

From the definition of work for a constant force in the direction of the displacement: $W = Fd$:

$$\begin{aligned} P &= \frac{Fd}{t} \\ &= F\left(\frac{d}{t}\right) \end{aligned}$$

This gives another expression for power, since speed $v = \frac{d}{t}$

$$P = \mathbf{F} \cdot \mathbf{v}$$

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Could we use the formula $P = \mathbf{F} \cdot \mathbf{v}$ to solve parts (b) and (c)?

Summary

- work done by a varying force
- kinetic energy
- the work-kinetic energy theorem
- power

Next Test Thurs Nov 15, TBC.

Quiz given Thursday.

Homework

- Ch 7 Ques: 5
- read Ch 7, sections 1–7 and section 9
- Ch 7 Ques: 1; Probs: 1, 5, 9, 15, 19, 27, 29, 43, 45