Mechanics
Center of Mass
Linear Momentum

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De Anza College

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Last time

- went over quiz 4
- another energy example
- center of mass

Overview

- center of mass
- motion of center of mass
- linear momentum
- impulse

Center of Mass

Expression for the position vector of the center of mass:

$$\mathbf{r}_{\mathrm{CM}} = \frac{1}{M}\sum_i m_i \mathbf{r}_i$$

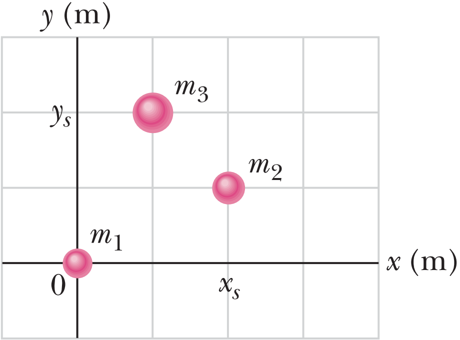
where $\mathbf{r}_i = x_i\mathbf{i} + y_i\mathbf{j} + z_i\mathbf{k}$ is the displacement of particle i from the origin.

Center of Mass Example

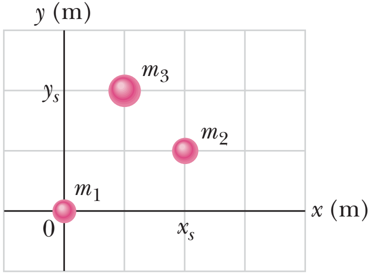
Ch 9, #2

Consider the three-particle system, with masses $m_1 = 3.0$ kg, $m_2 = 4.0$ kg, and $m_3 = 8.0$ kg. The scales on the axes are set by $x_s = 2.0$ m and $y_s = 2.0$ m.

What are the coordinates of the system's center of mass?



Center of Mass Example



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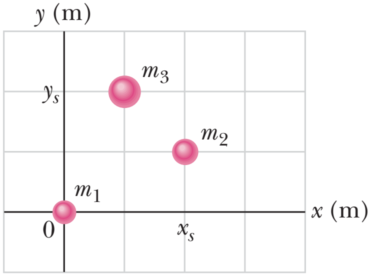

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If m_3 is gradually increased, does the center of mass of the system shift toward or away from that particle, or does it remain stationary?

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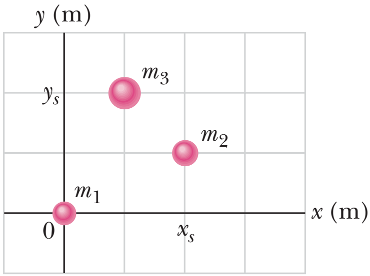

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shifts toward m_3

Center of Mass of Continuous Objects

In solid objects, mass seems to be distributed continuously.[1]

It is possible to calculate the center of mass in this case also, but that requires evaluating integrals.

Nevertheless, we can guess intuitively the rough position of the center of mass in many cases.

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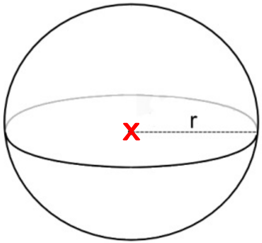

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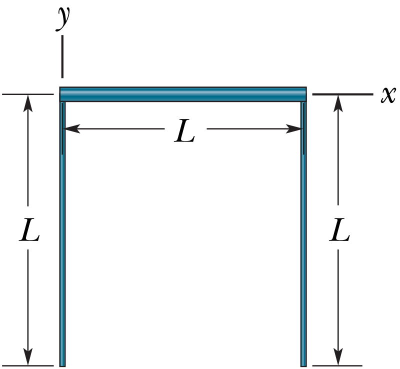
If an object is made up of several different regular shapes, you can find the center of mass by treating each part as point mass at the center of mass of that shape.



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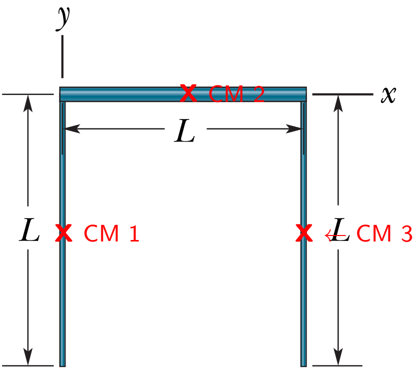

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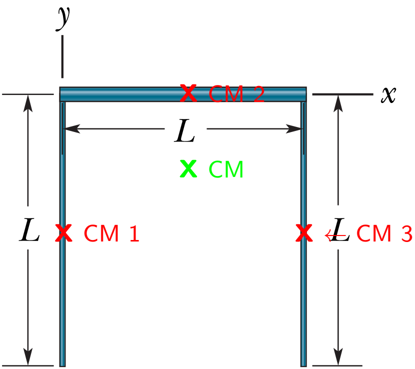
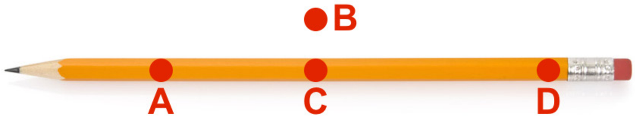


Center of Mass Questions

Where is the center of mass of this pencil?



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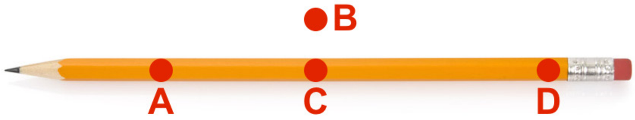

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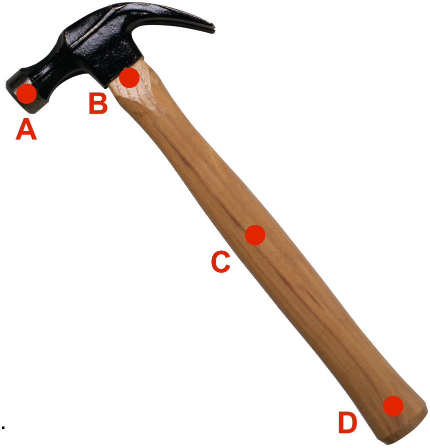

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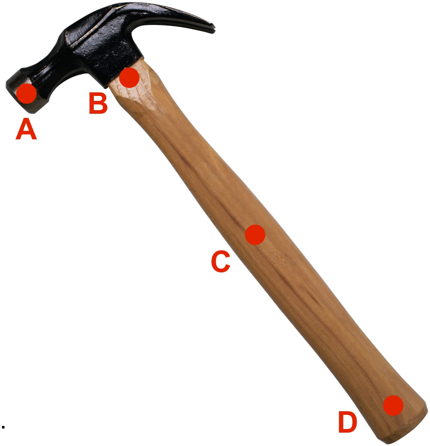

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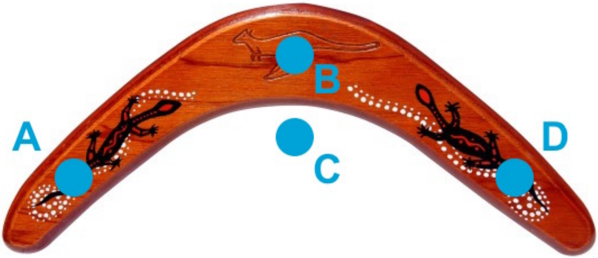

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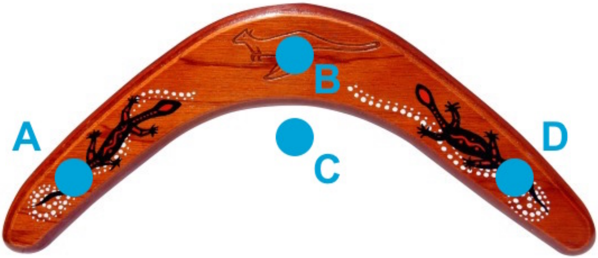

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Center of Gravity

The center of gravity is the single point on an object where we can model the force of gravity as acting on the object.

Near the surface of the Earth, the Earth's gravitational field is uniform, so this is the **same point** as the center of mass.

The center of gravity is the point at which you can balance an object on a single point of support.

[1]Figure from http://dev.physicslab.org/

Center of Mass vs Center of Gravity

For a solid, rigid object:

center of mass

the point for an object where we can model all the mass as being, in order to find the object's trajectory; a freely moving object rotates about this point

center of gravity

the single point for an object where we can model the force of gravity as acting on the object; the point at which you can balance the object.

Motion of the Center of Mass

$$\mathbf{r}_{\text{CM}} = \frac{1}{M}\sum_i m_i \mathbf{r}_i$$

where $\mathbf{r}_i = x_i\mathbf{i} + y_i\mathbf{j} + z_i\mathbf{k}$ is the displacement of particle i from the origin.
Differentiating gives:

$$\begin{aligned}
\frac{\mathrm{d}\mathbf{r}_{\text{CM}}}{\mathrm{dt}} &= \frac{1}{M}\sum_i m_i \frac{\mathrm{d}\mathbf{r}_{\text{i}}}{\mathrm{dt}} \\
\mathbf{v}_{\text{CM}} &= \frac{1}{M}\sum_i m_i \mathbf{v}_i
\end{aligned}$$

And differentiating one more time:

$$\mathbf{a}_{\text{CM}} = \frac{1}{M}\sum_i m_i \mathbf{a}_i$$

Newton's 2nd Law, revisited

$$\mathbf{a}_{\mathrm{CM}} = \frac{1}{M}\sum_i m_i \mathbf{a}_i$$

Newton's 2nd law for a particle i tells us $\sum_j F_{j,i} = m_i \mathbf{a}_i$, where $\sum_j F_{j,i}$ is the sum of all forces on particle i.

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Then,

$$\mathbf{a}_{\text{CM}} = \frac{1}{M} \sum_{i,j} F_{j,i}$$

and $\sum_{i,j} F_{j,i}$ is the sum over all forces on all particles. It's the net force!

Therefore,

$$\mathbf{F}_{\text{net}} = M \mathbf{a}_{\text{CM}}$$

Newton's 2nd law holds for the acceleration of the center of mass. This is good because we've already been assuming it when we treated blocks as point masses.

Linear Momentum

Linear momentum

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Units: kg m/s

Momentum

Momentum was so important for Newton's understanding of motion that he called it *the* "quantity of motion".

Newton's (more general!) version of his second law:

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Momentum is related to force.

This is where our intuition about momentum comes from:

Is it harder to stop a bowling ball or a tennis ball if they are both approaching you at the same speed?

Is it harder to stop a bullet or a tossed marble?

Change in Momentum and Impulse

The bullet and the bowling ball are harder to stop because they have more momentum.

Can we be more precise about “hard to stop”?

Change in Momentum and Impulse

The bullet and the bowling ball are harder to stop because they have more momentum.

Can we be more precise about "hard to stop"?

Yes! "Hard to stop" can be measured as how much force must be applied for how much time to stop the motion.

To stop something, we must change its momentum: $\Delta \mathbf{p}$

If we apply a constant force $\mathbf{F}$ for an amount of time Δt:

$$\Delta \mathbf{p} = \mathbf{F}\,\Delta t$$

Change in Momentum and Impulse

For some reason, we also give the change in momentum a special name:

Impulse

Impulse, $\mathbf{J}$, is the change in momentum.

$$\text{Impulse, } \mathbf{J} = \Delta \mathbf{p}$$

and so

$$\text{Impulse, } \mathbf{J} = \mathbf{F}\,\Delta t$$

(assuming the force $\mathbf{F}$ is constant)

Impulse

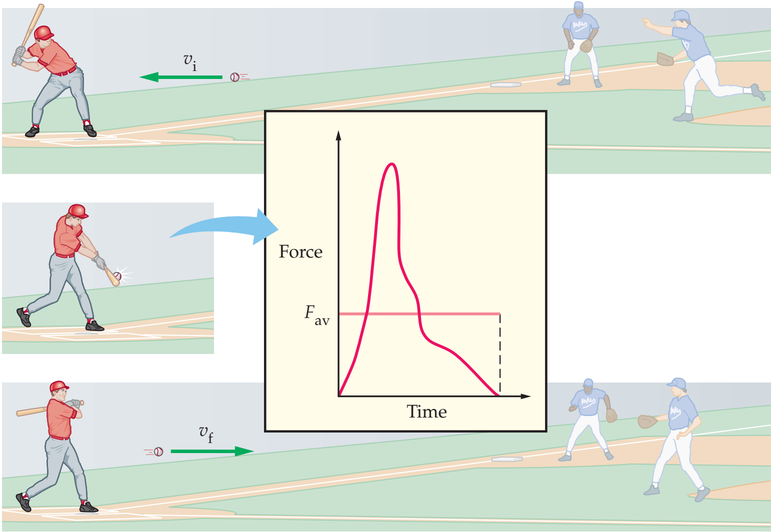
If the force on an object is not constant, we can still write:

$$\mathbf{J} = \mathbf{F}_{\text{avg}}\Delta t$$

where $\mathbf{F}_{\text{avg}}$ is the **average force** on the object over the time interval Δt.

Impulse from Changing Force

$$\text{Impulse, } \mathbf{J} = \mathbf{F}_{\text{avg}} \Delta t$$



[1]Figure from Walker, "Physics".

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A steel cable will bring you to rest in a fraction of a second. That is a force of well over 1500 N!

A bungee line will bring you to rest over a period of several seconds: force $\sim$ 500 N: comparable to the force of gravity on you. A much more pleasant experience.

Force-time trade-off

The idea that changing the momentum of an object over a longer period of time reduces the force on the object is very important in engineering.

In particular, it is a principle used in design to improve safety.

Summary

- momentum
- momentum and Newton's second law
- impulse

Homework

- Ch 9 Ques: 1; Probs: 1, 3, 5
- read about momentum and impulse in chapter 9