# Mechanics
# More about Impulse
# Conservation of Momentum

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# Last time

- center of mass
- linear momentum
- momentum and Newton's second law
- impulse

# Overview

- more about impulse
- conservation of momentum

# Linear Momentum

**Linear momentum**

The linear momentum of an object is the product of the object's mass with its velocity.

$$\mathbf{p} = m\mathbf{v}$$

It is a vector.

Units:

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Units: kg m/s

# Bouncing

Does a ball that strikes a wall and stops dead experience more or less impulse than a ball that bounces? (Assume masses and velocities are the same.)

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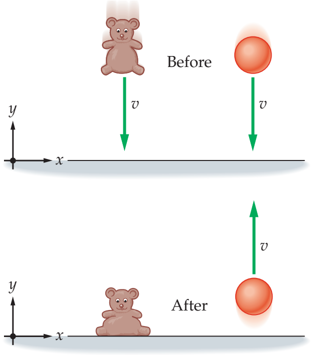
Does a ball that strikes a wall and stops dead experience more or less impulse than a ball that bounces? (Assume masses and velocities are the same.)

The bouncing ball must experience a larger impulse, because its momentum change is bigger!

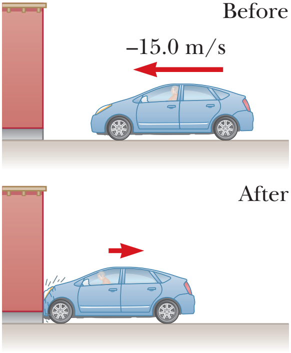



$$\Delta \boldsymbol{p}_{\text{bear}} = mv\mathbf{j} \qquad \Delta \boldsymbol{p}_{\text{ball}} = 2mv\mathbf{j}$$

# Crash test example

In a particular crash test, a car of mass 1500 kg collides with a wall. The initial and final velocities of the car are:

$$\mathbf{v}_i = -15.0\ \mathbf{i}\ \text{m/s}$$
$$\text{and } \mathbf{v}_f = 5.00\ \mathbf{i}\ \text{m/s.}$$




If the collision lasts 0.150 s, find the impulse caused by the collision and the average net force exerted on the car.

What would the net force be if the car stuck to the wall after the collision?

[1]Serway & Jewett, Physics for Scientists and Engineers, 9th ed, page 255.

# Crash test example

Impulse?

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$$\begin{aligned} \mathbf{J} &= \Delta\mathbf{p} \\ &= m(\mathbf{v}_f - \mathbf{v}_i) \\ &= (1500\ \text{kg})(5.00 - (-15.0)\ \text{m/s})\ \mathbf{i} \\ &= \underline{3.00 \times 10^4\ \mathbf{i}\ \text{kg m/s}} \end{aligned}$$

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Average net force?

$$\begin{aligned}\mathbf{F}_{\text{net,avg}} &= \frac{\mathbf{J}}{\Delta t} \\ &= \frac{3.00 \times 10^4\ \mathbf{i} \text{ kg m/s}}{0.150 \text{ s}} \\ &= \underline{2.00 \times 10^5\ \mathbf{i} \text{ N}}\end{aligned}$$

If car does not recoil:

$$\mathbf{F}_{\text{net,avg}} = \underline{1.50 \times 10^5\ \mathbf{i} \text{ N}}$$

# Crash test example

Conclusion: designing a car to deform and not recoil in a collision can reduce the forces involved.

[1]Image from http://northdallasgazette.com

# Impulse Question

**12.** A man claims that he can hold onto a 12.0-kg child in a head-on collision as long as he has his seat belt on. Consider this man in a collision in which he is in one of two identical cars each traveling toward the other at 60.0 mi/h relative to the ground. The car in which he rides is brought to rest in 0.10 s. (a) Find the magnitude of the average force needed to hold onto the child. (b) Based on your result to part (a), is the man's claim valid? (c) What does the answer to this problem say about laws requiring the use of proper safety devices such as seat belts and special toddler seats?

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&= \frac{(12)(60\ \text{mi/h})(1609\ \text{m/mi})}{(0.10\ \text{s})(3600\ \text{s/h})} \\
&= \underline{3.2 \times 10^3\ \text{N}}
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(b) man's claim?
It seems unlikely that he will be able to exert 3200 N of force on the child.

(c) Secure your toddler with a child safety seat!

# Conservation of Momentum

Momentum has a very important property.

It obeys a **conservation law**.

If there is no net external force, the *total momentum* of all objects interacting does not change.

$$\begin{array}{ccc} \text{total momentum} & = & \text{total momentum} \\ \text{before interaction} & & \text{after interaction} \end{array}$$

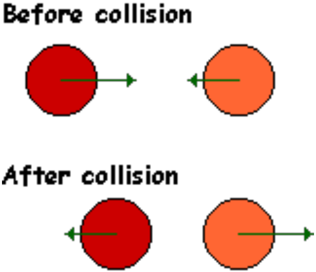
Equivalently, there is zero change in the total momentum:

$$\Delta \mathbf{p}_{\text{net}} = \sum (\Delta \mathbf{p}) = 0$$

# Conservation of Momentum

Even though the total momentum does not change, individual objects may see their momentum change.

For example, consider two colliding balls:

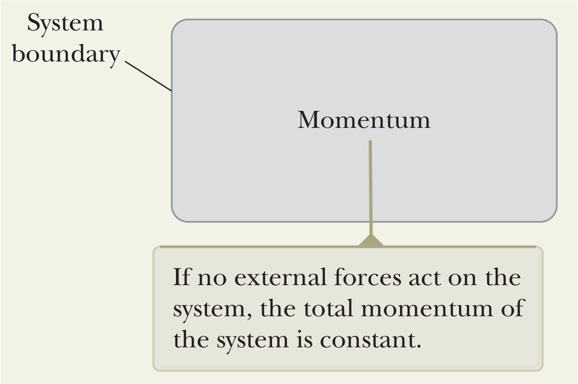



The momentum of each ball changes in the collision, but the sum of their momenta is the same before and after.

[1]Image from http://www.compuphase.com.

# Conservation of Linear Momentum

For an *isolated* system, *ie.* a system with **no external forces**, total linear momentum is *conserved*.




[1]Figures from Serway & Jewett.

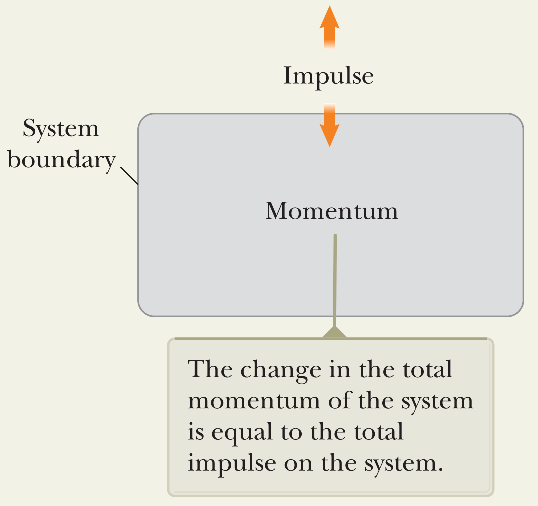
# Conservation of Linear Momentum

For an *isolated* system, *ie.* a system with no external forces, total linear momentum is *conserved*:

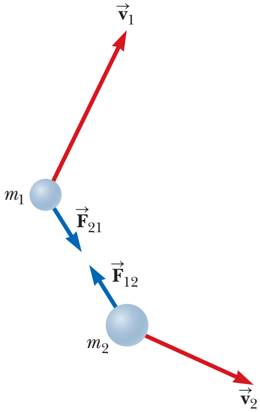
$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_i \mathbf{p}_i\right) = 0$$

(Note: before when speaking of energy "isolated" meant "not exchanging energy" now, for momentum, it means, no net external force acts on the system.)

# Nonisolated Systems



# Newton's Third Law and Conservation of Momentum




Newton's third law for two interacting particles:

$$\mathbf{F}_{21} = -\mathbf{F}_{12}$$

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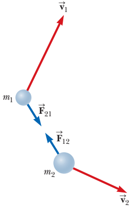


Newton's third law for two interacting particles:

$$\begin{aligned} \mathbf{F}_{21} &= -\mathbf{F}_{12} \\ \frac{d\mathbf{p}_1}{dt} &= -\frac{d\mathbf{p}_2}{dt} \end{aligned}$$

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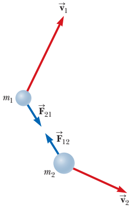


Newton's third law for two interacting particles:

$$\begin{aligned} \mathbf{F}_{21} &= -\mathbf{F}_{12} \\ \frac{d\mathbf{p}_1}{dt} &= -\frac{d\mathbf{p}_2}{dt} \\ \frac{d}{dt}\left(\mathbf{p}_1 + \mathbf{p}_2\right) &= 0 \end{aligned}$$

Implies:

$\mathbf{p}_{\text{total}} = \mathbf{p}_1 + \mathbf{p}_2$ does not change with time. Or, $\Delta\mathbf{p}_{\text{total}} = 0$.

# Newton's Third Law and Conservation of Momentum

**Newton's third law $\Leftrightarrow$ conservation of momentum**

No external forces (only internal action-reaction pairs):

$$\Delta \mathbf{p}_{\text{total}} = 0$$

# Conservation of Momentum Example

A honeybee with a mass of 0.150 g lands on one end of a floating 4.75-g popsicle stick. After sitting at rest for a moment, it runs toward the other end with a velocity $\mathbf{v}_b$ relative to the still water. The stick moves in the opposite direction with a speed of 0.120 cm/s.

What is the velocity of the bee? (Let the direction of the bee's motion be the positive $x$ direction.)

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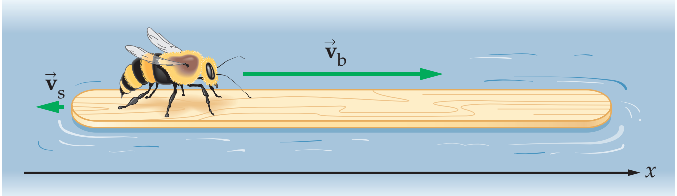
SKETCH:



# Conservation of Momentum Example

The net external force is zero, so $\Delta \boldsymbol{p} = 0$.

$$\begin{aligned} \mathbf{p}_{\text{tot},i} &= \mathbf{p}_{\text{tot},f} \\ 0 &= \mathbf{p}_b + \mathbf{p}_s \\ \mathbf{p}_b &= -\mathbf{p}_s \end{aligned}$$

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0 &= \mathbf{p}_b + \mathbf{p}_s \\
\mathbf{p}_b &= -\mathbf{p}_s \\
m_b \mathbf{v}_b &= -m_s \mathbf{v}_s \\
\mathbf{v}_b &= \frac{m_s v_s}{m_b} \mathbf{i} \\
\mathbf{v}_b &= \underline{3.80 \text{ cm/s } \mathbf{i}}
\end{aligned}
$$

# Summary

- more about impulse
- conservation of momentum

**Quiz** tomorrow, start of class (a chapter 9 HW problem).

**Lab Report** due tomorrow, for Tuesday lab only.

**2nd Test** next week.

**Homework**

- Ch 9 Probs: 19, 21, 25, 27, 39, 40