Mechanics
Collisions

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Last time

- impulse
- isolated and nonisolated systems
- conservation of momentum

Overview

- inelastic and elastic collisions

Collisions

A major application of momentum conservation is studying collisions.

This is not just useful for mechanics but also for statistical mechanics, subatomic physics, etc.

For our purposes, there are two main kinds of collision:

- elastic
- inelastic

Collisions

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Internal forces obey Newton's third law $\Rightarrow$ Momentum is conserved.

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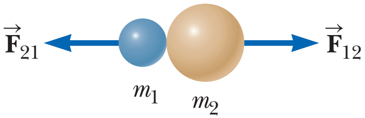
Internal forces obey Newton's third law $\Rightarrow$ Momentum is conserved.

This is **true for both** elastic and inelastic collisions. (So long as there is no external net force.)

Collisions

Collisions can occur in macroscopic systems through contact.



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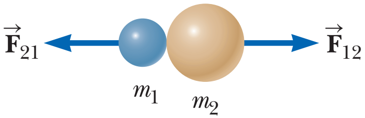
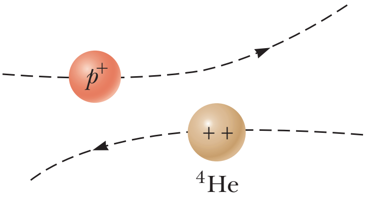


And collisions can occur through purely repulsive forces, even if two particles never make contact.



Types of Collision

There are two different types of collisions:

Elastic collisions

are collisions in which none of the kinetic energy of the colliding objects is lost. $(K_i = K_f)$

- truly elastic collisions do not occur at macroscopic scales
- many collisions are close to elastic, so we can model them as elastic

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are collisions in which energy is lost as sound, heat, or in deformations of the colliding objects.

- all macroscopic collisions are somewhat inelastic
- when the colliding objects stick together afterwards the collision is *perfectly inelastic*

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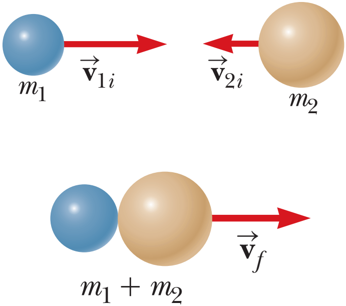
$$\mathbf{p}_i = \mathbf{p}_f \quad \Rightarrow \quad m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

However, in a special case of *perfectly* inelastic collisions, we have more information.

Perfectly Inelastic Collisions



Now the two particles stick together after colliding $\Rightarrow$ same final velocity!

$$\mathbf{p}_i = \mathbf{p}_f \quad \Rightarrow \quad m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$$

Perfectly Inelastic Collisions

In this case it is straightforward to find an expression for the final velocity:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$$

So,

$$\mathbf{v}_f = \frac{m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i}}{m_1 + m_2}$$

Inelastic Collision Example

Two freight rail cars collide and lock together. Initially, one is moving at 10 m/s and the other is at rest. Both have the same mass. What is their final velocity?

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Suppose the moving train travels in the x-direction.

$$\begin{aligned} \mathbf{p}_{\text{net},i} &= \mathbf{p}_{\text{net},f} \\ m\mathbf{v}_i &= (2m)\mathbf{v}_f \\ \mathbf{v}_f &= \frac{m\mathbf{v}_i}{2m} \\ v_f &= 5 \text{ m/s } \mathbf{i} \end{aligned}$$

The final mass is twice as much, so the final speed must be only half as much: $v_f = 5$ m/s.

Collision Question

Two objects collide and move apart after the collision. Could the collision be inelastic?

(A) Yes.

(B) No.

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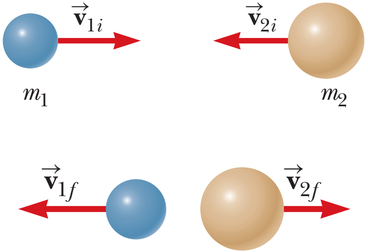
Elastic Collisions



For two particles involved in an elastic collision, we can write two equations:

$$\mathbf{p}_i = \mathbf{p}_f \quad \Rightarrow \quad m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

$$K_i = K_f \quad \Rightarrow \quad \frac{1}{2}m_1(v_{1i})^2 + \frac{1}{2}m_2(v_{2i})^2 = \frac{1}{2}m_1(v_{1f})^2 + \frac{1}{2}m_2(v_{2f})^2$$

(Assume the masses of the two particles remain unchanged.)

Elastic Collisions

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These are independent equations $\Rightarrow$ can solve for multiple unknowns.

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These are independent equations $\Rightarrow$ can solve for multiple unknowns.

However, the terms with squares in the KE equation make this a bit tedious in practice.

Elastic Collisions

One convenient trick is replace the KE equation with another one that doesn't have the quadratic terms.

This only works in 1-dimensional collisions.

$$\begin{aligned} m_1 v_{1i} + m_2 v_{2i} &= m_1 v_{1f} + m_2 v_{2f} \\ \frac{1}{2} m_1 (v_{1i})^2 + \frac{1}{2} m_2 (v_{2i})^2 &= \frac{1}{2} m_1 (v_{1f})^2 + \frac{1}{2} m_2 (v_{2f})^2 \end{aligned}$$

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From those two, this equation can be derived:

$$(v_{1i} + v_{1f}) = (v_{2f} + v_{2i})$$

(The v's are assumed to lie along a single direction and can be positive or negative)

Elastic Collisions - in 1 Dimension only

Simplify the kinetic energy equation:

$$\begin{aligned}\frac{1}{2}m_1(v_{1i})^2 + \frac{1}{2}m_2(v_{2i})^2 &= \frac{1}{2}m_1(v_{1f})^2 + \frac{1}{2}m_2(v_{2f})^2 \\ m_1(v_{1i})^2 + m_2(v_{2i})^2 &= m_1(v_{1f})^2 + m_2(v_{2f})^2\end{aligned}$$

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Collect the m_1 terms separately:

$$\begin{aligned} m_1(v_{1i}^2 - v_{1f}^2) &= m_2(v_{2f}^2 - v_{2i}^2) \\ m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) &= m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \qquad (1) \end{aligned}$$

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But notice, we can collect the m_1 terms in the momentum equation also (notice v's may be negative in this expression!):

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \qquad (2)$$

Elastic Collisions - in 1 Dimension only

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \qquad (1)$$

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Divide equation 1 by equation 2 :

$$(v_{1i} + v_{1f}) \quad = \quad (v_{2f} + v_{2i})$$

Gives us a different pair of independent equations to solve from:

$$\begin{cases} m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \\ v_{1i} + v_{1f} = v_{2f} + v_{2i} \end{cases}$$

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$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \tag{1}$$

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(where the v's are assumed to lie along a single direction and can be positive or negative)

1-D Elastic Collision Example

Ch 9, #62

Two titanium spheres approach each other head-on with the same speed and collide elastically. After the collision, one of the spheres, whose mass is 300 g, remains at rest.

(a) What is the mass of the other sphere?

(b) What is the speed of the two-sphere center of mass if the initial speed of each sphere is 2.00 m/s?

1-D Elastic Collision Example

(a) What is the mass of the other sphere?

use:

$$\begin{cases} m_1 v_{1i} + m_2 v_{2i} = m_1 \cancelto{0}{v_{1f}} + m_2 v_{2f} \\ v_{1i} + \cancelto{0}{v_{1f}} = v_{2f} + v_{2i} \end{cases}$$

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$$m_2 = 100 \text{ g}$$

(b) What is the speed of the two-sphere center of mass if the initial speed of each sphere is 2.00 m/s?

use:

$$\mathbf{v}_{\text{CM}} = \frac{1}{M} \sum_i m_i \mathbf{v}_i$$

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use:

$$\mathbf{v}_{\text{CM}} = \frac{1}{M} \sum_i m_i \mathbf{v}_i$$

$$v_{\text{CM}} = 1.00 \text{ m/s}$$

1-D Elastic Collision Example, More detail

(a)
2nd equation:

$$\begin{aligned} v_{1i} + \cancelto{0}{v_{1f}} &= v_{2f} + v_{2i} \\ v + 0 &= v_{2f} + (-v) \\ v_{2f} &= 2v \end{aligned}$$

Conservation of momentum (1st equation):

$$\begin{aligned} m_1 v_{1i} + m_2 v_{2i} &= m_1 \cancelto{0}{v_{1f}} + m_2 v_{2f} \\ m_1 v + m_2(-v) &= 0 + m_2(2v) \\ m_1 - m_2 &= 2m_2 \\ m_2 &= \frac{m_1}{3} \end{aligned}$$

$$m_2 = 100 \text{ g}$$

Summary

- collisions

2nd Test Thursday, Nov 15.

Homework

- Ch 9 Ques: 3; Prob: 51, 60[1], 61, 63

[1] Ans: (a) 1.9 m/s, (b) right, (c) yes.